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Strategy use, the development of automaticity, and working memory involvement in complex multiplication

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Participants practiced a set of complex multiplication problems (e.g., 3×18) in a pre-/postpractice design. Before, during, and after practice, the participants gave self-reports of problem-solving strategies. At prepractice, the most common strategy was a mental version of the standard multidigit algorithm, and dual tasks revealed that working memory load was high and heavier for problems solved via nonretrieval strategies. After practice, retrieval was used almost exclusively, and participant variability, automaticity level of problems (proportion of trials on which retrieval was used over the entire experiment), and error rates were significant predictors of problem-solving latencies. Practice reduced working memory involvement to minimal levels, and there was no relationship between automaticity level and working memory load. The commonalities between the present findings and findings related to automaticity development in simple arithmetic are discussed.

Most theories and models of arithmetic processing have been constructed with the implicit or explicit assumption that adults retrieve most, if not all, basic arithmetic facts from long-term memory (Ashcraft & Battaglia, 1978; Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Campbell & Oliphant, 1992; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; Koshmider & Ashcraft, 1991; Lemaire, Abdi, & Fayol, 1996). In recent years, there has been a proliferation of research in which strategy use and development in children's and adults' simple mental arithmetic problem solving have been examined (e.g., Geary, 1996; Hecht, 2002; Kirk & Ashcraft, 2001; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996; Lemaire & Siegler, 1995; Seyler, Kirk, & Ashcraft, 2003; Siegler, 1988). One of the more interesting findings from this research is that a large percentage of adults continue to use strategies to solve basic addition, subtraction, multiplication, and division problems (e.g., Campbell & Xue, 2001; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996; Seyler et al., 2003; Tronsky & Shneyer, 2004). This widespread finding has made it necessary to reexamine some of the empirical effects in basic arithmetic research in order to clarify and qualify some of those findings, so that more accurate models of arithmetical cognition can be developed; research related to this has just begun, at least in the domain of simple arithmetic (Campbell & Xue, 2001; Hecht, 2002; Seyler et al., 2003; Tronsky, Anderson, & McManus, 2005).

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Investigations of adults' complex mental arithmetic skills also have become more numerous, especially recently (e.g., Ashcraft, Donley, Halas, & Vakali, 1992; Fürst & Hitch, 2000; Geary, Frensch, & Wiley, 1993; Logie, Gilhooly, & Wynn, 1994; Seitz & Schumann-Hengsteler, 2000; Trbovich & LeFevre, 2003). At present, few researchers have examined the strategies that adults use to solve these problems (with the exception of Geary et al., 1993), how the use of strategies impacts working memory (WM), and how the development of automaticity impacts WM involvement. The purpose of the present investigation is to examine adults' initial strategy use in complex mental multiplication and corresponding WM involvement, to document how problem-solving processes and WM involvement change with practice, and to examine the factors that govern performance after retrieval from long-term memory (automaticity) has been established. In order to set the context for the present investigation, it is necessary to provide a review of theory and research related to strategy use and development in mental arithmetic, the structure of WM, and the implications that strategy use and automaticity have for WM's role in arithmetic.

Strategy Development in Mental Arithmetic: The Adaptive Strategy Choice Model

Siegler and his colleagues (e.g., Lemaire & Siegler, 1995; Shrager & Siegler, 1998; Siegler & Shipley, 1995) have developed a model of how strategy use develops with arithmetic experience, called the *adaptive strategy choice model* (ASCM; see also the SCADS model, Shrager & Siegler, 1998). In this model, four dimensions of strategic change are outlined, each of which can lead to overall speed and accuracy improvements in problem solving. These dimensions are (1) what strategies are available,

(2) when each strategy is used (both the relative frequency of the use of the strategy and types of problems on which it is used), (3) how each strategy is executed (the efficiency with which each is executed), and (4) how strategies are chosen (what governs which strategy is chosen in a particular context). This model assumes that learners implicitly accumulate data about the answers generated to problems and about the speed and accuracy with which they are generated when particular strategies are used. These data on the effectiveness of strategies are represented at a global level (all problems), at the level of categories of problems (e.g., problems that share similar features, such as all problems with an operand of 2), and at the level of individual problems (e.g., 6×7). When a problem is presented, information related to these three types of data determines the strength of each strategy on each problem (i.e., the probability that a particular strategy will be selected). Information related to something called the *novelty* dimension (how often the strategy has been used in the past) also accumulates. This allows for new strategies to be tried and developed even when current strategies are fairly efficient. The model is self-modifying, since the speed, accuracy, and novelty information attributed to each strategy changes as more problems are encountered and solved. This self-modification results in the decreased use of less effective strategies and the increased use of more effective ones.

A further division of ASCM is the strategy execution phase. Once a strategy has been selected, it is carried out. An important aspect of this phase is that produced answers (correct or incorrect) are associated with each problem, and the increase in the strength of the problem–answer association is greater for correct than for incorrect answers (usually through some sort of internal or external reinforcement). Another important assumption (backed by empirical data) is that retrieval of answers from long-term memory becomes the dominant strategy, since it eventually yields the shortest response times (RTs) and the highest accuracy rates. After retrieval becomes the sole strategy, RTs continue to drop as the associations between problems and their correct answers become stronger. Three interesting predictions of the model will be one focus of the present investigation. The first two predictions are that the initial accuracy of strategy execution will predict how soon (number of problem exposures before) retrieval strategies will be used and that it will lead to more accurate and faster retrieval in the future. The third prediction is that initial retrieval accuracy will lead to faster and more accurate future retrieval. The first two predictions follow from the model because initial accuracy of strategy execution should lead to stronger problem–answer associations, since fewer and weaker incorrect problem–answer associations are formed that compete with the correct association. The third prediction is based on a similar argument. If initial retrieval is accurate, the correct problem–answer association will become even stronger, relative to incorrect ones, leading to future use of retrieval that is increasingly both more accurate and faster (more automatic). All three of the aforementioned predictions were confirmed in Lemaire and Siegler's (1995) study of second graders' simple multiplication and will be examined in the present investigation.

As was mentioned previously, strategy use and the development of automaticity (problem solving after retrieval is the sole strategy used) are important for the understanding of simple arithmetic performance. For example, Delaney, Reder, Staszewski, and Ritter (1998), Klapp, Boches, Trabert, and Logan (1991), and Rickard (1997) have demonstrated that RTs continue to drop after participants have switched to retrieval. In other words, automaticity continues to build after retrieval has been established, and degree of automaticity may predict overall and problem-by-problem response latencies and WM load after practice (e.g., Hecht, 2002; Klapp et al., 1991; Seyler et al., 2003). Since this is the second major focus of the present investigation, I will now briefly outline WM and what is currently known about its role in arithmetic problem solving.

Working Memory Involvement in Mental Arithmetic

Most researchers have used Baddeley's (1992, 1996, 2001) model to study WM involvement in arithmetic. This model includes a four-part system: the central executive, phonological loop, visuospatial sketchpad, and episodic buffer. The central executive is an attentional control system involved in many processes, such as when one must select and execute strategies, retrieve (weak) information from long-term memory, inhibit irrelevant information, monitor input, simultaneously store and process information, and coordinate and allocate resources to other parts of the WM system (e.g., Ashcraft, 1995; Baddeley, 1992, 1996; Szmalec, Vandierendonck, & Kemps, 2005; Vandierendonck, De Vooght, & Van der Goten, 1998). Auditory-based information is stored and rehearsed using the phonological loop, whereas visual and spatial information is stored and rehearsed using the visuospatial sketchpad. The episodic buffer is a recently added and largely untested component that is hypothesized to provide, among other things, an interface between the subcomponents of WM and long-term memory under the control of the central executive.

Much of the evidence for the model above has been collected using dual-task methodologies in which participants perform two tasks simultaneously. An example of a dual task is having a person tap a random rhythm on a computer keyboard while solving arithmetic problems (De Rammelaere & Vandierendonck, 2001). WM load is determined by comparing performance on each of the tasks when they are executed together (dual task) with performance on each of the tasks when they are done alone (single task), with poorer performance on either/both of the tasks in the dual-task condition indicating WM involvement.

There is a growing research literature whose focus is on WM involvement in both simple and complex arithmetic (see DeStefano & LeFevre, 2004, for a review).

The general findings of these investigations have been that simple mental arithmetic requires central executive resources (e.g., Ashcraft et al., 1992; De Rammelaere et al., 1999, 2001; De Rammelaere & Vandierendonck, 2001; Deschuyteneer & Vandierendonck, 2005; Hecht, 2002). Investigations that have focused on the phonological loop and visuospatial sketchpad have produced mixed findings. Some studies have shown that adding a phonological or visuospatial dual task results in poorer arithmetic and/or dual-task performance, whereas others have not (e.g., Ashcraft et al., 1992; De Rammelaere et al., 1999, 2001; Hecht, 2002; Klapp et al., 1991; Lee & Kang, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000; 2002).

Most relevant to the present investigation are the results of Klapp et al. (1991) and Hecht (2002). Klapp et al. examined WM involvement by using two secondary tasks, either verbal repetition of one of the months of the year (May, May, May, etc.) or sequential retrieval and verbalization of the 12 months of the year (January, February, March, etc.) while verifying answers to alpha-arithmetic problems (e.g., $A + 3 = D$). Klapp et al. found that both of the dual tasks interfered when counting was used to solve problems, only sequential month saying interfered once retrieval was used (automaticity), and sequential month-saying interference was further reduced when participants were trained beyond automaticity. Hecht (2002) collected strategy use data on simple addition problems and had participants solve the problems while performing a task that loaded the phonological loop (repeating a letter of the alphabet) or a task that loaded the central executive and the phonological loop (random letter generation). Analyses showed that use of nonretrieval strategies loaded both the phonological loop and the central executive, whereas problems solved by retrieval did not load either. These studies are important because they establish a link between WM load and degree of automaticity and between WM load and strategy use, at least in the domains of alpha- and simple arithmetic.

Fewer researchers, however, have examined the role of WM in complex arithmetic. The consensus so far has been that the phonological loop, visuospatial sketchpad, and central executive are all involved during complex mental arithmetic (e.g., Ashcraft et al., 1992; Fürst & Hitch, 2000; Geary & Widaman, 1992; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002; Trbovich & LeFevre, 2003), and the degree to which each is involved may depend on such variables as presentation format, presentation orientation, mode of response, and strategy use (DeStefano & LeFevre, 2004; Trbovich & LeFevre, 2003).[1] Also, it has been shown that WM capacity/efficiency is related to certain aspects of complex arithmetic problem solving, such as carrying (e.g., Geary & Widaman, 1992; Seitz & Schumann-Hengsteler, 2000). Two aspects of complex mental arithmetic research yet to be examined are how WM involvement differs across strategy use and across practice and the development of automaticity.

The Present Investigation

To investigate the development of strategy use in complex multiplication and related WM effects, a pre-/postpractice design was used. Participants solved number multiplication problems beyond the basic facts (e.g., 6×17). The practice portion of the experiment consisted of three 1-h sessions, during which the participants were presented 10 complex multiplication problems and their commuted pairs via computer to solve. Presentation frequency across problems was equal, and strategy data were collected for each participant before, during, and after practice. The participants also completed WM tasks (single and dual) at both pre- and postpractice, to examine changes in WM involvement.

Because strategy use in complex arithmetic is largely a neglected topic and research on strategy use across practice does not exist, much of the present investigation is exploratory. Adults use a wide variety of strategies to solve simple arithmetic problems, although strategies are more limited for multiplication (e.g., Kirk & Ashcraft, 2001; LeFevre, Bisanz, et al., 1996; LeFevre, Sadesky, & Bisanz, 1996). One goal of the present research was to document inter- and intraparticipant strategy use on a set of complex multiplication problems and the qualitative and quantitative nature of participants' strategy shifts as they move toward automaticity. The second goal was to apply the ASCM to complex arithmetic by examining the relationship between initial accuracy of nonretrieval strategies and indices of later problem-solving performance. The third goal was to determine the nature and extent of WM involvement in complex multiplication across strategies and across levels of automaticity.

METHOD

Participants

Twenty-five undergraduates from a Massachusetts university took part in the study. After the prepractice session, 2 participants dropped out, leaving 23 participants (6 males and 17 females). These remaining 23 participants completed all the experimental sessions and earned six experimental credits that counted as extra credit in their psychology class(es).

Materials

Each participant was administered tasks using the Computer-Based Academic Assessment System (CAAS). CAAS allows one to create computer tasks to collect individual item and aggregate RTs and accuracies. A detailed account of the construction of each of the tasks and a brief description of how CAAS operates follow in the Procedure section.

Single Tasks

Simple multiplication task. Seventy-two simple multiplication problems were presented at pre- and postpractice. The problems were whole number multiplication problems from 2×2 to 9×9 (tie problems—2×2, 3×3, etc.—appeared twice) in a horizontal orientation; the stimuli for all the remaining tasks also were presented horizontally. Problems were split into two blocks, so that one problem was randomly selected for the first block and its commuted pair was relegated to the second block, with each tie problem presented once in each block. Within blocks, problems were presented in random order. These problems were included in the battery of tasks because participants' simple multiplication-problem–solving efficiency, which is one subcomponent of most complex multiplication problem-solving strategies, should be predictive of initial complex multiplication-problem–solving latencies.

Prepractice complex multiplication task. Thirty complex multiplication problems and their commuted pairs were presented at prepractice. These problems were composed of a single digit number between 2 and 9 and one double-digit number between 13 and 19 (e.g., 8×15). This set of 60 problems was separated into two blocks; a random problem from each commuted pair was selected for the first block, and its counterpart was selected for the second block, with random presentation of problems within blocks.

Practice task. Of the 60 problems from the prepractice multiplication task, a subset of 20 problems (10 problems and their commuted pairs) was selected for the practice task: 2×16, 2×19, 3×15, 3×18, 4×16, 4×19, 6×17, 7×13, 7×15, and 8×14. These problems were selected to represent a wide range of operands and problem size (answers ranged from 32 to 112). Eight blocks of 10 problems each were constructed such that 10 problems were presented in the odd-numbered blocks and their 10 commuted pairs were presented in the even-numbered blocks. Problem presentation within blocks was random.

Complex multiplication strategy assessment task. The 20 practice problems described above were presented in this task. After solving each problem, the participants reported the strategies that they had used. The problems were divided into two blocks. If a problem was selected for the first block, its commuted pair was selected for the second block, and problem presentation within blocks was random.

Postpractice complex multiplication task. In this task, the 10 practice problems and their commuted pairs were presented three times each. These stimuli were presented in six blocks of 10 problems each. Odd blocks contained 10 practice problems, and even blocks contained those problems' 10 commuted pairs; problem presentation within blocks was random.

The full complement of complex problems that was presented at prepractice was not administered at postpractice. Studies in which transfer from practiced to unpracticed arithmetic problems has been examined have shown these effects to be negligible (e.g., Pauli, Bourne, & Birbaumer, 1998; Rickard, Healy, & Bourne, 1994). To include nonpracticed problems at postpractice would have required additional stimuli in the postpractice complex multiplication and strategy assessment tasks. This would have made an already long experiment even longer for the participants, and in light of the research outlined above, most likely for little or nothing in terms of transfer effects.

Working Memory Tasks

Random letter sequences. The specific dual tasks and methodologies used in this study were modified from those in a recent study by Ashcraft and Kirk (2001) and had been used in a previous simple arithmetic investigation (Tronsky, 2003). A total of 120 (quasi-) randomly selected and ordered six-letter consonant strings were presented before the multiplication problems in the WM dual- and single-task conditions. These consonant strings were constructed while controlling for phonological similarity, articulation rate, and meaningfulness of the sequence. Sixty of the letter strings were used in the multiplication dual-task condition, and 30 were used in each of two control conditions. Each set of letter strings was approximately equally difficult to remember, on the basis of results from a previous investigation (Tronsky, 2003).

Complex multiplication dual task. The prepractice version of this task contained the same 60 problems as the prepractice complex multiplication task and was divided into blocks in the same manner. Problems were randomly ordered within blocks, but this order was the same across participants (due to the nature of the task; see below). The postpractice version of the task was the same, with one modification: Only the 10 practiced problems and their commuted pairs were included. Thus, in the postpractice version of this task, there were six blocks of problems (10 per block), and odd blocks contained 10 practice problems, whereas even blocks contained their commuted pairs.

Procedure

Each task was administered using a PC and the CAAS software (for additional information about the system and its use in other investigations, see Cisero, Royer, Marchant, & Jackson, 1997; Royer, 1997; Royer & Tronsky, 1998). In CAAS, test stimuli appear in black typeface in the middle of a computer screen against a white background. The appearance of each stimulus immediately triggers a timing mechanism in the computer, and each stimulus remains on the screen until a participant voices an answer into a microphone. Voicing stops the timing mechanism, thereby recording the RT for each trial, accurate to ± 1 msec. A metal scorer's box that has two buttons is interfaced with the computer. The scorer pushes one button to indicate a correct response and the other to indicate an incorrect response. Problem-by-problem RT and accuracy data for each participant are written to a computer file.

Before each task, directions appeared on the screen that explained the nature of the task and instructed the participants to place equal emphasis on speed and accuracy. After the directions, problems not used in the experiment were administered to ensure that each participant understood what he or she would see and was expected to do. Upon completion of these practice problems, the participants proceeded to the test stimuli, for which RTs and accuracies were recorded. The tasks were presented in a pseudorandom order. The set of three non-WM tasks was presented either before or after the set of three WM tasks, and within sets, tasks were presented in random order.

Non–Working Memory Tasks

For the simple multiplication, complex multiplication, and strategy assessment tasks, the screen initially went blank for 1,000 msec. Then a stimulus appeared, and the participant's response stopped the computer's timing mechanism. The scorer pressed a button on the response box to record the accuracy of the response, and the screen went blank for another 1,000 msec. This cycle was repeated until all the stimuli had been presented.

In the strategy assessment task, after solving each of the 20 complex problems, the participants gave verbal reports of the strategies that they had used. Problems with the methods employed to assess strategy use in mental arithmetic have come to the forefront of recent research (e.g., Kirk & Ashcraft, 2001; Robinson, 2001; Seyler et al., 2003), which influenced the strategy assessment procedure used here. Instead of the participants being given descriptions and examples of strategy categories, they were allowed to verbally explain their solution strategies. These verbal descriptions were then classified by the experimenter and resulted in the creation of the following categories: retrieval, decomposition (e.g., decomposing the problem 2×19 into $2 \times 16 = 32$; $32 + 6 = 38$), standard algorithm (mentally carrying out the standard multidigit multiplication algorithm), repeated addition (e.g., $2 \times 16 = 16 + 16 = 32$), tens (multiplying the tens column, then the ones column, and then adding the two products), or other (the most common report was that the participants started to use one of the strategies above and then retrieved the answer after partial solution of the problem). Once classified, the participants were given the category label for the strategy. For example, if a participant reported on the first strategy trial that he or she had "multiplied the single digit number by the ones digit in the two-digit problem, then multiplied the single digit by the tens digit, and then added the two answers," the strategy was classified as the *standard algorithm*. He or she was then allowed to use that label if the same strategy was used for another problem in the strategy assessment task.

Working Memory Tasks

Before each multiplication problem, a six-letter consonant string appeared. The participants were instructed to read the letters out loud at a rate of approximately two letters per second. Once the participants had finished reading the letters, the screen went blank for 1,000 msec, and a multiplication problem appeared. After the participants had voiced an answer, the screen went blank for another 1,000 msec. Six dashes then appeared on the screen that prompted the participants to recall the letters from the first screen, in the correct order, while the researcher wrote down their responses. After consonant recall, the screen went blank for 1,000 msec before a new consonant string appeared, and the process was repeated until all the stimuli had been presented. The participants were instructed at the outset not to use strategies to impose meaning on the consonant strings, such as filling in vowel(s) between the consonants to make words. This was done to ensure that the participants were not using long-term memory to aid letter recall.

Two single-task (control) conditions were employed that involved testing both the multiplication and the letter recall tasks individually, keeping total vocalization time constant (Ashcraft & Kirk, 2001). In the multiplication single task, the participants read the letters shown on the screen, solved the arithmetic problem, and then were *shown* the consonant string to *read*. In the letter recall single task, the consonant string was shown, the arithmetic problem appeared with its *answer*, which the participants *read* off the screen, and recall of the correct letter string was prompted by six dashes. Therefore, in both single tasks, the WM load was removed, but the procedure and vocalization of the task remained very similar to the procedure and vocalization in the dual-task condition.

Practice

Each practice session began with the strategy assessment task, followed by the practice task. This procedure was continued with brief breaks between tasks until four blocks of practice had been completed. At this point, the strategy assessment task was administered again, and the practice session ended. The participants completed a total of three 1-h practice sessions. During each session, they solved each one of the 10 practice problems 42 times (32 times in the four practice blocks and 10 times in the five strategy assessments) for a total of 126 presentations of each problem. Including pre- and postpractice tasks, the participants solved each problem a total of 150 times by the end of the experiment.

Postpractice

Upon finishing the practice sessions, the participants completed the postpractice versions of the tasks in the same order as they had at prepractice. Each participant completed the full battery of prepractice, practice, and postpractice tasks in approximately 6 h over a 3- to 6-week period. Due to the complexity of the experiment, a summary of the tasks and the frequency and order in which they were administered is presented in the Appendix.

RESULTS

Strategy Use in Complex Multiplication

Strategy Use Across Problems and Participants at Prepractice

As can be seen in Table 1, variability of strategy choices was somewhat limited across problems and is best summarized by grouping the problems into two sets. The first three problems in Table 1 were solved using similar patterns of strategies. Many of the participants solved these problems by using retrieval and, when not using retrieval, relied on the standard algorithm. The seven other problems were usually solved with the standard algorithm (63%–74% of the time) and were solved via retrieval infrequently (with the lone exception of the problem 4×16). These larger problems were solved via the tens method somewhat frequently as well (7%–17% of the time).[2]

Variability of strategy use both within and across participants was lower than expected. Twenty of the 23 participants reported using either the standard algorithm or the tens method strategies more than 50% of the time, and 17 participants used one of these two strategies 70% of the time or more. Only 2 participants used retrieval as the dominant strategy, and only 1 participant used decomposition on the majority of problems. Therefore, most of the participants used a strategy (algorithm or tens method) that required the solving of two simple multiplication problems, temporary storage of the simple problems' answers, and recombination (addition) of those answers to arrive at the final answer.

Because most of the participants reported using these solution methods initially, it was possible to construct variables that would capture the processes involved in executing these strategies and to use those variables to predict participants' RTs. These variables and the processes they represent are similar to those from a previous complex multiplication study that did not include strat-

Table 1
Strategy Use Across Complex Multiplication Problems at Prepractice

Problem	Retrieval	Standard Algorithm	Tens Method	Decomposition	Addition	Other
2×16	46	35	4	3	11	0
2×19	28	54	4	4	7	2
3×15	54	37	2	2	2	2
3×18	7	76	13	4	0	0
4×16	20	63	9	7	2	0
4×19	4	74	7	15	0	0
7×13	7	72	17	4	0	0
6×17	4	74	15	4	0	2
7×15	4	74	17	2	0	2
8×14	4	74	17	4	0	0

Note—Values in each column are the percentage of times each strategy was used across the sample, collapsing across commuted pairs within problems.

egy self-report measures (Geary & Widaman, 1987). The first variable was the prepractice simple multiplication RTs (in milliseconds) for each participant. This was used to predict the efficiency of solving the multiplication of the single digit operand by the ones digit of the double-digit operand. The product of the single digit operand and the tens digit of the double-digit operand was used as a measure of the efficiency of the single-digit × tens-digit product. The simple multiplication task could not be used, because 10-operand problems, such as 4 × 10, were not included in that task. The final variable was a carry variable that indexed the efficiency of carrying when the additions of the aforementioned partial products were performed (e.g., 17 × 6 would have a code of 4 for this variable related to the product of the ones digits 7 and 6).

The variables above and a set of variables that were dummy coded to estimate between-participant variability were used to predict complex multiplication RTs. The dummy-coded variables were included so that problem variability would not be confounded with participant variability (e.g., Hecht, 1999; LeFevre & Liu, 1997; Lorch & Myers, 1990; see Lorch & Myers for a more in-depth treatment of repeated measures regression analyses). Before conducting regression analyses, the RT data were trimmed, removing spoiled (e.g., microphone malfunctions), incorrect answer, and outlier RTs (RTs more than three standard deviations from an individual's mean RT). With these criteria, 21.1% of the prepractice complex multiplication RT trials were removed (10.9% spoiled, 9.0% incorrect, and 1.2% outlier). After the data had been trimmed, the group mean RT was 3,712 msec, with a range of 7,135 msec, indicating a wide range of initial problem-solving speed in this sample. Not surprisingly, the error rates were fairly high (9%) and variable (range: 2%–27%) as well.

Hierarchical regression analyses were conducted with the dummy-coded participant variables entered together in one block and the simple, carry, and tens variables entered together as a block; the results of these analyses are presented in Tables 2 and 3. Whether entered first or second, both blocks of variables accounted for a significant portion of complex RT variance. As can be seen in Table 2, the participant variable accounted for more variance in complex RTs when entered first (almost 40%), whereas the three process variables accounted for an additional 8.7%. Turning to Table 3, simple, carry, and tens slopes were all significantly greater than zero [$ts(22) > 3.07$, $ps < .05$].

Development of Retrieval With Practice

As was mentioned previously, few studies have been performed to investigate complex mental arithmetic performance, and only Geary et al. (1993) have examined problem-by-problem strategy use in adults (for complex subtraction). The present investigation is the first to document complex multiplication strategy choices and how they change with practice. Figure 1 depicts how strategy use changed over practice, collapsing across problems, and Figure 2 depicts how quickly retrieval developed as a strategy for each problem. Keep in mind that strategy assessments were conducted at prepractice, at the beginning and end of each practice session, and in between each block of 80 practice problems within a practice session (see the Appendix). This explains why there are more than 12 strategy assessments in Figures 1 and 2.

It also explains the large dip in retrieval use at Strategy Assessment 7 in Figure 2, since it was right before the beginning of Practice Session 2 for most of the participants. Practice Session 2 was always at least 1 day after the first practice session, most likely allowing for some decay of problem–answer associations in long-term memory.

Table 2
Pre- and Postpractice Complex Multiplication Hierarchical Regression Analyses

Time	Independent Variable	Model	Block	Total R^2	Incremental R^2
Prepractice	Participants	1	1	.397	.397
	Simple, carry, and tens		2	.484	.087
	Simple, carry, and tens	2	1	.189	.189
	Participants		2	.484	.295
Postpractice	Participants	1	1	.347	.347
	Automaticity		2	.426	.079
	Errors		3	.435	.009
	Simple, carry, and tens		4	.440	.005

Note—The dependent measure was complex multiplication response time (RT). Participants was a dummy-coded variable to estimate between-participants RT variability. The simple variable was the RT to solve the single-digit problems imbedded within the complex problems; the carry variable was the carried value after a single-digit multiplication was performed within a complex problem (e.g., 4 is the carry value for 17 × 6); the tens variable was the value of the tens digit multiplied by single digit (e.g., 6 was the tens value for 17 × 6); automaticity was the proportion of times retrieval strategies were used over the course of practice for each problem; error was the number of errors over the course of practice on each problem. Simple, carry, and tens were entered together in one block, whereas participants, automaticity, and errors were entered individually. All R^2 and R^2 increments are significant at $p < .05$.

Table 3
Slope Values of Variables Used to Predict Complex Multiplication Response Times at Pre- and Postpractice

Time	Predictor	Slope: Unstandardized	Slope: Standardized
Prepractice	Simple	692.4**	.15**
	Carry	217.9*	.11*
	Tens	1.6*	.13*
Postpractice	Simple	45.4*	.07*
	Carry	−2.2	−.01
	Tens	−1.0	−.06
	Automaticity	−944.4**	.35**
	Errors	23.1**	.11**

Note—The variables are the same as those reported in Table 2 and are the result of regression analyses where variables were simultaneously entered with the participant variable described in the text and Table 2. The first column under slope represents unstandardized coefficients, whereas the second slope column represents standardized coefficients. $^*p < .05$. $^{**}p < .001$.

As is shown in Figure 1 (and as was mentioned above), most of the participants at prepractice reported using either the algorithm or the tens strategy on the majority of problems, whereas only 2 participants reported using retrieval at least 50% of the time. Retrieval developed fairly rapidly with practice, however, increasing from 29% to 61% of the trials after one block of practice (Strategy Assessments 2 and 3) and reaching 90% by the end of the 1st hour of practice (Strategy Assessment 6). Although retrieval use was very high after the 1st hour of practice, it was not uniformly high across individuals. Whereas 14 participants used retrieval 100% of the time at this point, 6 participants, over one quarter of the sample, still used retrieval less than 75% of the time. After all the practice sessions had been completed (Strategy Assessment 16), only 3 participants reported using strategies (on 20%, 5%, and 5% of the trials, respectively), and average retrieval use across the sample was 99%.

Additional aspects of these figures are worth noting. In general, the strategy curves for each problem were quite similar; the use of retrieval on each problem started out fairly low and increased rapidly over the first few practice sessions. Second, although the aforementioned similarities exist, the development of retrieval with practice was not homogeneous across problems. For the three problems with the smallest products (2×16, 2×19, 3×15), retrieval use was higher than for other problems from the outset, more rapidly approached 100%, and declined less between Practice Sessions 1 and 2 (Strategy Assessment 7 in Figure 2). The problems 3×18 and 4×16 more closely resembled the five largest problems in terms of development of retrieval.

Application of the Adaptive Strategy Choice Model

Before analyses involving the postpractice data were conducted, the same data-trimming procedure as that employed at prepractice was used, resulting in the removal of 9.4% of the trials (6.6% spoiled, 0.7% incorrect, and 2.1% outlier), yielding a group mean RT of 999 msec.[3] It is interesting to note that this mean RT is equal to or slightly shorter than *simple* multiplication RTs for recent North American samples (e.g., Campbell & Xue, 2001; Koshmider & Ashcraft, 1991; LeFevre, Bisanz, et al., 1996; LeFevre & Liu, 1997; Tronsky & Anderson, 2005). As was reviewed earlier, even after retrieval has been established, associative strength between a problem and its answer continues to build and is reflected in shorter RTs (Klapp et al., 1991; Rickard, 1997). Klapp et al. used the term *automaticity* to reflect the switch from counting to memory retrieval to solve

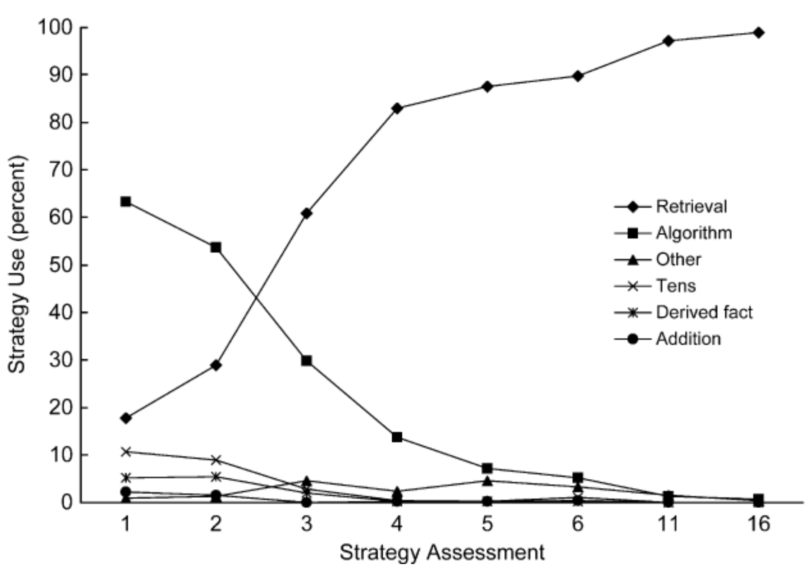

Figure 1. Types of strategies and how use of those strategies changed with practice.

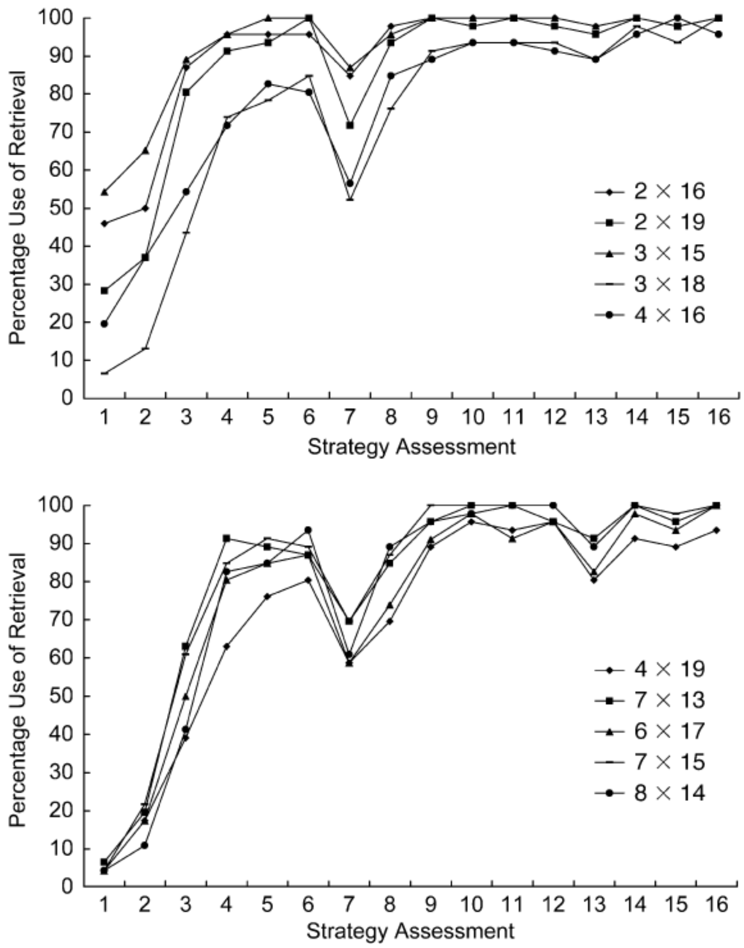

Figure 2. Average percentages of participants using retrieval at each strategy assessment for small (top panel) and large (bottom panel) problems. Assessments 7 and 12 correspond to the first strategy assessments for Practice Sessions 2 and 3, respectively. The dips in the graph likely reflect decay of some problem–answer associations in long-term memory, since there was at least a full day between the participants' practice sessions.

problems and used the term *training beyond automaticity* to reflect practice after retrieval (automaticity) had been established. A similar terminology will be used here. Because repeated strategy assessments were conducted, when and how often retrieval was used across problems and participants could be quantified. An automaticity-level variable was constructed for each problem and its commuted pair for each participant. This was done by assigning a value of 2 to a problem when retrieval was used and a value of 1 to a problem when a nonretrieval strategy was used in each strategy assessment. These values were then averaged across strategy assessments within problems. Therefore, a person who was able to retrieve the answer to a problem early during practice had an automaticity level close to 2 for that problem, whereas a person who was not able to retrieve an answer to a problem until the end of Practice Session 3 had a level close to 1 for that problem.

Recall from the introduction that Siegler's (Lemaire & Siegler, 1995; Shrager & Siegler, 1998; Siegler & Shipley, 1995) ASCM predicts that the more accurately one executes initial nonretrieval strategies, the sooner one should switch to using retrieval and the faster and more accurate one should be when using retrieval. Also recall that initial accuracy of retrieval should result in faster and more accurate later retrieval. Both predictions result from the proposal that initial accuracy should lead to the correct problem–answer associations becoming stronger, relative to incorrect associations. Because the strength of competing associations decreases, it is less likely that incorrect answers will be retrieved in the future, and the stronger activation of the correct association will result in shorter RTs. To test these predictions, several correlations were performed. Surprisingly, initial accuracy of nonretrieval strategies (error rates for each participant over the first two practice sessions) did not predict how quickly people developed retrieval strategies, as measured by the automaticity-level variable ($r = .16$, n.s.). As was predicted, however, initial accuracy of nonretrieval strategies was significantly correlated with postpractice complex multiplication RT ($r = -.60, p < .01$). Error rates during Practice Blocks 3–6 were used for the *initial retrieval errors* variable, because most of the participants had just begun to use retrieval on most problems (about 85%) during these blocks, whereas error rates in Practice Blocks 7–12 were used for the *later retrieval errors* variable. Significant relationships were found between initial nonretrieval accuracy and overall retrieval accuracy ($r = .47, p < .05$), between initial retrieval accuracy and later retrieval accuracy ($r = .57, p < .01$), and between initial retrieval accuracy and later (postpractice) RTs ($r = -.44$, $p < .05$). In summary, most of the predictions of ASCM that hold for simple arithmetic held for complex arithmetic as well.

Automaticity Analyses

Although the variables reflecting the processing involved in the two most common strategies used at prepractice were significant predictors of complex multiplication RTs, these variables should not capture much of the variance in RTs after automaticity has been developed, whereas automaticity level should. Therefore, another set of hierarchical regression analyses was run, using the postpractice complex multiplication RTs as the criterion measure. Because the preceding ASCM analysis revealed that both nonretrieval and retrieval errors were correlated with postpractice RTs, a variable called *errors* that reflected the total number of errors for each participant on each problem was also included in the analyses.

In the postpractice hierarchical regression analyses, participant, prepractice process (simple multiplication RT, tens, and carry), error, and automaticity variables were the predictors, each entered in different blocks, with the dummy-coded participant variables always entered first. After practice, it was expected that the automaticity and error variables would be much better predictors of complex multiplication RTs than would the process variables and that the slopes associated with each process variable would be much smaller than at prepractice, most likely not significantly different from zero. Between-participant variability again captured a large portion of the variance (34.7%). There were six different analyses (i.e., six different orders in which the repeated measures variables could be entered), and therefore, only the analysis in which the most incremental variance was explained with each additional block is presented in Table 2. The process variables, whether entered on the second, third, or fourth block, explained a much smaller proportion of the variance (0.5% to 1.7%) than at prepractice. In contrast, the automaticity variable always explained the largest proportion, regardless of entry order (5.7% to 7.9%), whereas the error variable explained anywhere from 0.7% (entered last) to 2.7% (entered second) of the variance in RTs. Table 3 summarizes the unstandardized and standardized slopes of each of the within-subjects predictors. The process variable slopes were smaller at postpractice than at prepractice, and although all three process variable slopes were significantly larger than zero at prepractice, only simple multiplication RT was at postpractice. As was expected, the automaticity variable slope ($b_{\text{unstd.}} = -944.4$) was significantly greater than zero, which shows that, at postpractice, those problems that were least highly automatized were solved just under a second slower than those that were most highly automatized. In summary, the results of the hierarchical regression analyses indicate that variables that capture the processes involved in algorithmic problem solving explained significant variance in complex multiplication RTs at prepractice. After practice, when use of retrieval from long-term memory was virtually the only strategy used, these process variables captured very little variance in RTs whereas error rates and, to a greater extent, level of automaticity captured a larger portion of RT variance. I will now turn to a discussion of the effect of strategy use and automaticity on WM involvement in complex multiplication.

Working Memory Analyses

General Working Memory Analyses

To test whether WM was involved in complex multiplication problem solving, single task performance was compared with dual-task performance for both the RT and the letter recall measures. For the RT analyses, only the trimmed data were used, and for the letter recall analyses, only those stimuli that corresponded to the trimmed RT data set were used (spoiled, incorrect, and outlier percentages were similar to those in the previous sections). WM involvement would be indicated if there were a significant difference in RT and/or letter recall accuracy when dual-task were compared with single-task conditions. Because complex multiplication error rates did not differ across WM load conditions at prepractice (9.2% for single task vs. 9.3% for dual task) or postpractice (0.4% for both single and dual tasks), they were not analyzed further.

Within-subjects ANOVAs with the variables of practice (pre- vs. post-) and WM load (dual vs. single task) were conducted on both the complex multiplication RT and the letter recall data. Table 4 shows the means by condition. RTs were significantly shorter after practice [$F(1,22) = 58.43$, $MS_e = 2{,}923{,}537$, $p < .001$], but the main effect of WM load and the practice $\times$ WM load interaction were not significant [$Fs(1,22) = 2.85$ and 0.96, respectively; MS_es = 166,699 and 127,405, respectively]. At postpractice, complex multiplication RTs were 71 msec longer in the dual-task than in the single-task condition [$F(1,22) = 6.38$, $MS_e = 8{,}999$, $p < .05$].

For the letter recall data, there were significant effects of practice and WM load that were qualified by a practice $\times$ WM load interaction [$Fs(1,22) = 175.44$, 38.10, and 47.73, respectively; MS_es = 25.26, 54.69, and 29.60, respectively; $ps < .001$]. The interaction was due to a

Table 4
Working Memory Load Imposed by Complex Multiplication Problems Across Practice

	Complex Multiplication RT (msec)			Letter Recall (%)		
Time	Dual Task	Single Task	Difference	Dual Task	Single Task	Difference
Prepractice	3,884	3,667	217	60	78	18
Postpractice	1,086	1,015	71	82	84	2

Note—Working memory load was calculated as dual task − single task for complex multiplication response time (RT) and single task − dual task for letter recall. This was done so that positive values would indicate working memory involvement in both tasks. Letter recall is the percentage of letters recalled in their correct serial position.

greater increase in letter recall from pre- to postpractice in the dual-task condition (22% increase in the dual task vs. 5% in the single task). Planned tests revealed that the letter recall difference across WM load conditions at prepractice (18%) was significant [$F(1,22) = 56.65$, $MS_e = 61.15$, $p < .001$], but the corresponding difference of 2% was not significant at postpractice [$F(1,22) = 1.41$, $MS_e = 23.14$, n.s.]. The magnitude and pattern of RT and letter recall differences across WM load conditions at postpractice were very similar to those from an analogous study of WM involvement in simple multiplication after training (Tronsky, 2003).

Strategy Use and Working Memory Analyses

Because many of the participants used both nonretrieval and retrieval strategies at prepractice, it was possible to evaluate whether WM involvement at prepractice differed across strategies. To do this, the letter recall and complex multiplication RT data in dual- and single-task conditions were divided into the categories of *retrieval* and *nonretrieval* on the basis of the participants' initial strategy assessment. This division of problems resulted in 12 individuals who had multiple retrieval and nonretrieval trials in both the single- and the dual-task conditions. Only 12 of the 23 participants were in this subgroup, because many of the participants either did not use retrieval during the initial strategy assessment (7 participants) or had too few retrieval trials left for analysis after the data-trimming procedure (4 participants). Simple arithmetic research has shown that nonretrieval strategies are executed more slowly and rely more on WM resources than do retrieval strategies (e.g., Hecht, 2002; Seyler et al., 2003; Tronsky et al., 2005; Tronsky & Shneyer, 2004). Given that strategy use in complex arithmetic involves execution of more subprocesses than in simple arithmetic, RT and letter recall differences across strategies (retrieval vs. nonretrieval) should be large for complex multiplication problems as well.

The RT data from the 12 participants were subjected to a repeated measures ANOVA with the variables of strategy (retrieval vs. nonretrieval) and WM load (dual task vs. single task). The main effect of strategy was significant, since the participants were 1,769 msec faster when solving a problem via retrieval [$F(1,11) = 28.65$, $MS_e = 1{,}311{,}800$, $p < .001$]. Neither the main effect of WM load [$F(1,11) = 2.21$, $MS_e = 668{,}492$, n.s.] nor the WM load × strategy interaction [$F(1,11) = 0.29$, $MS_e = 407{,}427$, n.s.] was significant. Letter recall in the dual-task condition, however, was 22% higher for retrieval trials than for nonretrieval trials [$F(1,11) = 11.36$, $MS_e = 246.6$, $p < .001$]. These data show that the execution of nonretrieval strategies was slower and more WM demanding than memory retrieval execution and are consistent with existing simple arithmetic and alpha-arithmetic research (Hecht, 2002; Klapp et al., 1991).

Automaticity and Working Memory Analyses

In addition to comparing WM involvement across strategies, it is necessary to compare WM involvement within strategies. It was not possible to make such a comparison within nonretrieval strategies, given that, within participants, nonretrieval use was often limited to one type of strategy. It was possible, however, to examine whether automaticity level was related to WM involvement in complex multiplication.

To do this, the automaticity variable was used to construct a dichotomous variable. For each participant, postpractice problems were divided in half, into problems that were of low versus high automaticity, leaving a total of 30 possible RTs in each category. One participant's data were not used, because automaticity values were the same for 80% of the problems. ANOVAs were performed with the factors automaticity (low vs. high) and WM load (single vs. dual task) for both the RT and the letter recall data. The effect of interest in each analysis was whether there was a significant interaction between automaticity and WM load. The RT difference for low versus high automaticity problems was 6 msec larger in the dual-task condition, a nonsignificant difference [$F(1,21) = 0.53$, $MS_e = 7{,}009$, n.s.]. Letter recall in the dual-task condition was 3% better for the high-automaticity problems, also a nonsignificant difference [$F(1,21) = 3.74$, $MS_e = 193.0$, $p > .05$]. Thus, level of problem automaticity was not related to WM involvement.

DISCUSSION

The first major goal of the present study was to document strategy use in complex multiplication before, during, and after extensive practice. Before practice, strategies were variable, although less variable than was expected. Most of the participants reported using the standard right-to-left algorithm for multidigit multiplication, although the tens method (left-to-right multiplication) was used for larger problems somewhat frequently and retrieval was used somewhat frequently for the three problems with the smallest product. So far, researchers have focused on WM involvement in complex arithmetic and how it varies across problem characteristics and presentation format (e.g., carry vs. noncarry problems in complex addition; Fürst & Hitch, 2000; Seitz & Schumann-Hengsteler, 2000; Trbovich & LeFevre, 2003), although none, to my knowledge, has examined how WM involvement changes across strategy use. The present investigation shows that there is individual variability in complex arithmetic strategy use, and future investigations in which the relative involvement of the subsystems of WM across strategy use is explored need to collect strategy use data. For example, the visuospatial sketchpad may be more heavily loaded when a mental version of the standard algorithm is used (because people visualize the problem in a vertical format and solve the problem visually) than when the tens method or the decomposition method is used (strategies that may be verbal and load the phonological loop more). Trbovich and LeFevre found indirect support for this; a visuospatial dual task interfered more with complex addition problems (e.g., 5 + 47) that were presented vertically than with those presented horizontally, whereas the opposite pattern of interference was found when a phonological dual task was employed. Their explanation was that the vertical format activated visually based solution strategies, whereas the horizontal format activated strategies that relied more heavily on phonological codes; Trbovich and LeFevre, however, did not conduct strategy assessments to further support their argument.

Another important finding from the present investigation was that timing of strategy shifts varied across problems and participants. Although the majority of the participants had switched to using retrieval for all the problems after the first practice session (over 40 presentations per problem), a large minority, almost 40% of the sample, did not switch to retrieval for all the problems until the middle of the second practice session or well into the third practice session. Use of retrieval on problems across practice (collapsing across participants) was not uniform either. For example, the automaticity measure showed that the proportion for use of retrieval was highest for the 3 × 15 problem (92% of the problem presentations across strategy assessments) and was lowest for the 4 × 19 problem (76% of the problem presentations across strategy assessments). Regression analyses indicated that a large proportion of variance in problem RTs was explained by between-subjects variability and variables that represented the processes involved in carrying out the most common strategies (standard algorithm and tens method). Between-subjects variability at postpractice explained the most variance in complex multiplication RTs (similar to the prepractice level), and the variance explained by the prepractice process variables dropped precipitously, indicating that a different process (i.e., retrieval) was used to solve the problems. Also, the automaticity variable explained a significant proportion of RT variance, which was similar in magnitude to that explained by the process variables at prepractice. At the same time, however, over 50% of the variance in postpractice complex RTs remained unexplained.

The second major goal of the present investigation was to apply the ASCM to examine individual differences. Recall that the model predicts that individual differences in initial nonretrieval accuracy should correlate with participants' strategy shift to retrieval, initial accuracy of retrieval, and later speed and accuracy of retrieval; initial retrieval accuracy should correlate with later accuracy and speed of retrieval. All but one of these predictions was supported: Only initial accuracy of nonretrieval failed to predict the timing of strategy shifts to retrieval. These results largely mirror those of Lemaire and Siegler (1995), who examined French second graders' multiplication development. This is particularly interesting given that adults in the present sample were fairly skilled in simple multiplication and made far fewer errors at the outset of problem solving, as compared with the second graders in Lemaire and Siegler. It appears that ASCM can be generalized to the development of complex multiplication skills in adults.

The third major goal of the present investigation was to establish the load that complex multiplication puts on WM before and after practice. More specifically, I sought to test the hypotheses that WM is heavily loaded when complex multiplication problems are solved before practice, when use of nonretrieval strategies is high, that WM is more heavily loaded when strategies are used rather than retrieval, and that at postpractice, when retrieval is the sole method of problem solution, WM is minimally loaded and any load most likely stems from low-automaticity problems.

As was hypothesized, WM was heavily loaded at prepractice, since use of nonretrieval strategies was high; letter recall was significantly poorer in the dual-task than in the single-task condition. Also at prepractice, those problems that were solved via nonretrieval strategies loaded WM significantly more than did those that were solved via retrieval, as evidenced by poorer letter recall for nonretrieval trials. The third WM hypothesis was partially supported. At postpractice, when retrieval was used almost exclusively, there was no effect of WM load on letter recall, but WM load did cause a small but significant slowing of complex multiplication RTs. Also, degree of problem automaticity was not related to WM load, since RTs and letter recall did not differ across level of problem automaticity.

The majority of the WM findings also parallel the results from simple arithmetic. These investigations have established that use of nonretrieval strategies is more resource demanding than use of retrieval for addition (Hecht, 2002), subtraction (Seyler et al., 2003; Tronsky et al., 2005), multiplication (Tronsky & Anderson, 2005), and division (Tronsky & Shneyer, 2004), although retrieval use still loads WM significantly (Deschuyteneer &

Vandierendonck, 2005; Hecht, 2002; Tronsky et al., 2005). At present, the consensus is that processes central to arithmetic retrieval most likely are not responsible for increases in RTs across load conditions (Hecht, 2002; Rusconi, Galfano, Speriani, & Umiltà, 2004; Tronsky et al., 2005). For example, Hecht (2002) found that a variable that indexed rate of retrieval of addition answers did not vary across load conditions, even though absolute RTs did. A recent interpretation for this increase in RTs across load for retrieval trials is that such processes as encoding of the digits in a problem or other processes peripheral to activation of problems' candidate answers require WM resources (Deschuyteneer & Vandierendonck, 2005; Hecht, 2002; Rusconi et al., 2004). This interpretation also could be applied to the null finding of the effects of level of automaticity on WM involvement here. As in Klapp et al. (1991), the participants in the present investigation were trained to automaticity and beyond. It could be that once a certain level is reached for a problem, automaticity level is not associated with increased WM involvement.

A second possible explanation for the lack of an automaticity-level/WM-load relationship concerns the secondary task used in this experiment. The extent and type of central executive involvement in the letter recall task is still somewhat unclear, although there is some preliminary evidence that it does load WM (Szmalec et al., 2005; Vandierendonck et al., 1998). Central executive/attentional resources may be required when participants switch from rehearsing the letters to solving the arithmetic problem (i.e., task switching) or while monitoring and maintaining the correct order of the letters. If retrieval of lower automaticity answers is central executive demanding and the letter recall task only minimally loaded this subcomponent of WM, the null findings are less surprising. In future research, it will be important to see whether similar results can be obtained with tasks that load specific subcomponents of WM and, more particularly, the central executive.

CONCLUSION

In summary, the present investigation shows that the development of automaticity in complex multiplication may be similar to the development of automaticity in simple arithmetic. Some possible commonalities suggested by the present research and by previous research on simple arithmetic are as follows.

1. Initial accuracy of strategy-based problem solving predicts the development of various aspects of skilled problem-solving (e.g., initial and later retrieval-based accuracy and later retrieval speed; Lemaire & Siegler, 1995).

2. Before practice, variables that index the subprocesses involved in strategy-based problem solving predict problem-solving latencies (e.g., LeFevre, Sadesky, & Bisanz, 1996; Siegler, 1988; Widaman & Little, 1992).

3. After retrieval has been established, level of automaticity (conceptually similar to such variables as associative strength and probability of retrieval used in other studies) is predictive of solution latencies (e.g., Klapp et al., 1991; LeFevre, Bisanz, et al., 1996; Mabbott & Bisanz, 2003).

4. WM load is greater in strategy-based problem solving (e.g., Hecht, 2002; Seyler et al., 2003), is greatly reduced with practice (Klapp et al., 1991; Tronsky, 2003), and for retrieval, may be confined to peripheral processes, rather than processes central to activation of candidate answers (Deschuyteneer & Vandierendonck, 2005; Hecht, 2002; Rusconi et al., 2004).

Additional research is needed to bolster and fully examine the possibilities outlined above. It may be that complex arithmetic practice investigations will allow us to study the processes and empirical effects related to complex arithmetic and will allow us to speculate about some aspects of simple arithmetic that are not easily examined. For example, there is still great debate about the nature of the representations that are used in simple arithmetic processing. Some claim that nonverbal, spatial-based magnitude representations are used to store and retrieve answers to arithmetic problems, whereas others insist these representations are verbal in nature and/or depend on the arithmetic operation in question and the method of learning or instruction (e.g., Butterworth et al., 2001; Dehaene & Cohen, 1997; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Whalen, 1997). Using complex arithmetic problems and a practice paradigm similar to the one used here would allow one to vary learning methods (rote verbal learning of answers versus strategy-/magnitude-based learning methods) to examine their effects on the representations used in arithmetic processing, while controlling important variables, such as problem presentation frequency. This and other related investigations are currently being developed.

REFERENCES

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition*, **1**, 3-34.

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning & Memory*, **4**, 527-538.

Ashcraft, M. H., Donley, R. D., Halas, M. A., & Vakali, M. (1992). Working memory, automaticity, and problem difficulty. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 301-329). Amsterdam: Elsevier.

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, **130**, 224-237.

Baddeley, A. D. (1992). Working memory. *Science*, **255**, 556-559.

Baddeley, A. D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology*, **49A**, 5-28.

Baddeley, A. D. (2001). Is working memory still working? *American Psychologist*, **56**, 851-864.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *Quarterly Journal of Experimental Psychology*, **54A**, 1005-1029.

Campbell, J. I. D., & Oliphant, M. (1992). Number-fact retrieval: A model and computer simulation. In J. I. D. Campbell (Ed.), *The na-*

ture and origins of mathematical skills (pp. 331-364). Amsterdam: Elsevier.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, **130**, 299-315.

Cisero, C. A., Royer, J. M., Marchant, H. G., & Jackson, S. J. (1997). Can the computer-based academic assessment system (CAAS) be used to diagnose reading disability in college students? *Journal of Educational Psychology*, **89**, 599-620.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, **33**, 119-150.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, **284**, 970-974.

Delaney, P. F., Reder, L. M., Staszewski, J. J., & Ritter, F. E. (1998). The strategy-specific nature of improvement: The power law applies by strategy within task. *Psychological Science*, **9**, 1-7.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (1999). The contribution of working memory resources in the verification of simple mental arithmetic sums. *Psychological Research*, **62**, 72-77.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory & Cognition*, **29**, 267-273.

De Rammelaere, S., & Vandierendonck, A. (2001). Are executive processes used to solve simple arithmetic production tasks? *Current Psychology Letters: Behaviour, Brain, & Cognition*, **5**, 79-90.

Deschuyteneer, M., & Vandierendonck, A. (2005). Are "input monitoring" and "response selection" involved in solving simple mental arithmetical sums? *European Journal of Cognitive Psychology*, **17**, 343-370.

DeStefano, D., & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, **16**, 353-386.

Fürst, A. J., & Hitch, G. J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory & Cognition*, **28**, 774-782.

Geary, D. C. (1996). The problem-size effect in mental addition: Developmental and cross national trends. *Mathematical Cognition*, **2**, 63-93.

Geary, D. C., Frensch, P. A., & Wiley, J. G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology & Aging*, **8**, 242-256.

Geary, D. C., & Widaman, K. F. (1987). Individual differences in cognitive arithmetic. *Journal of Experimental Psychology: General*, **116**, 154-171.

Geary, D. C., & Widaman, K. F. (1992). Numerical cognition: On the convergence of componential and psychometric models. *Intelligence*, **16**, 47-80.

Hecht, S. A. (1999). Individual solution processes while solving addition and multiplication math facts in adults. *Memory & Cognition*, **27**, 1097-1107.

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition*, **30**, 447-455.

Kirk, E. P., & Ashcraft, M. H. (2001). Telling stories: The perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **27**, 157-175.

Klapp, S. T., Boches, C. A., Trabert, M. L., & Logan, G. D. (1991). Automatizing alphabet arithmetic: II. Are there practice effects after automaticity is achieved? *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **17**, 196-209.

Koshmider, J. W., & Ashcraft, M. H. (1991). The development of children's mental multiplication skill. *Journal of Experimental Child Psychology*, **51**, 53-89.

Lee, K. M., & Kang, S. Y. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition*, **83**, B63-B68.

LeFevre, J.-A., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General*, **125**, 284-306.

LeFevre, J.-A., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, **3**, 31-62.

LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **22**, 216-230.

Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, **8**, 73-103.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General*, **124**, 83-97.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, **22**, 395-410.

Lorch, R. F., & Myers, J. L. (1990). Regression analyses of repeated measures data in cognitive research. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **16**, 149-157.

Mabbott, D. J., & Bisanz, J. (2003). Developmental change and individual differences in children's multiplication. *Child Development*, **74**, 1091-1106.

Pauli, P., Bourne, L. E., Jr., & Birbaumer, N. (1998). Extensive practice in mental arithmetic and practice transfer over a ten-month retention interval. *Mathematical Cognition*, **4**, 21-46.

Rickard, T. C. (1997). Bending the power law: A CMPL theory of strategy shifts and the automatization of cognitive skills. *Journal of Experimental Psychology: General*, **126**, 288-311.

Rickard, T. C., Healy, A. F., & Bourne, L. E. (1994). On the cognitive structure of basic arithmetic skills: Operation, order, and symbol transfer effects. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **20**, 1139-1153.

Robinson, K. M. (2001). The validity of verbal reports in children's subtraction. *Journal of Educational Psychology*, **93**, 211-222.

Royer, J. M. (1997). A cognitive perspective on the assessment, diagnosis and remediation of reading skills. In G. D. Phye (Ed.), *Handbook of academic learning: Construction of knowledge* (pp. 199-234). San Diego: Academic Press.

Royer, J. M., & Tronsky, L. N. (1998). Addition practice with math-disabled students improves subtraction and multiplication performance. In T. E. Scruggs & M. A. Mastropieri (Eds.), *Advances in learning and behavioral disabilities* (pp. 185-218). Greenwich, CT: JAI Press.

Rusconi, E., Galfano, G., Speriani, V., & Umiltà, C. (2004). Capacity and contextual constraints on product activation: Evidence from task-irrelevant fact retrieval. *Quarterly Journal of Experimental Psychology*, **57A**, 1485-1511.

Seitz, K., & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology*, **12**, 552-570.

Seitz, K., & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. *Psychologische Beiträge*, **44**, 275-302.

Seyler, D. J., Kirk, E. P., & Ashcraft, M. H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **29**, 1339-1352.

Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, **9**, 405-410.

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General*, **117**, 258-275.

Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31-76). Hillsdale, NJ: Erlbaum.

Szmalec, A., Vandierendonck, A., & Kemps, E. (2005). Response selection involves executive control: Evidence from the selective interference paradigm. *Memory & Cognition*, **33**, 531-541.

Trbovich, P. L., & LeFevre, J.-A. (2003). Phonological and visual working memory in mental addition. *Memory & Cognition*, **31**, 738-745.

Tronsky, L. N. (2003). *Practice, problem size, and working memory resource use in mental arithmetic: Group and individual differences*. Unpublished manuscript.
Tronsky, L. N., & Anderson, E. C. (2005). *Strategy use in mental multiplication determines central executive load*. Manuscript in preparation.
Tronsky, L. N., Anderson, E. C., & McManus, M. (2005). *Strategy use in mental subtraction determines central executive load*. Manuscript submitted for publication.
Tronsky, L. N., & Shneyer, D. (2004, October). *Strategy execution in simple mental division puts a heavy load on the central executive of working memory*. Poster presented at the 13th Annual Meeting of the New England Psychological Association, Providence, RI.
Vandierendonck, A., De Vooght, G., & Van der Goten, K. (1998). Interfering with the central executive by means of a random interval repetition task. *Quarterly Journal of Experimental Psychology*, **51A**, 197-218.
Whalen, J. (1997, August). *The influence of semantic magnitude representations on arithmetic: Theory, data, and simulation.* Paper presented at the Nineteenth Annual Cognitive Science Society Meeting, Stanford University.
Widaman, K. F., & Little, T. D. (1992). The development of skill in mental arithmetic: An individual differences perspective. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 189-253). Amsterdam: Elsevier.

NOTES

1. Complex multiplication is particularly interesting in that people most likely use multiplication and addition of partial products to solve problems. Thus, any researchers who wish to tease apart what processes tax which subcomponent(s) of WM need to systematically examine the strategies people use to solve these problems (Seitz & Schumann-Hengsteler, 2000).

2. One might have expected initial retrieval use to be closer to 0%; however, some participants were given the strategy assessment after some or all of the other prepractice tasks, which may have given them enough practice on some of the problems that a retrieval strategy was developed. For example, a participant who completed the strategy assessment task last at prepractice would have already seen each of the 10 problems of interest six times (twice each in the complex and dual tasks, and once in each of the two control/single tasks). Furthermore, it may be that some people had practiced and developed retrieval strategies for some of the smaller problems before entering the study—most likely, the 2 and 3 operand problems. I would like to thank Brenda Smith-Chant for the second suggestion.

3. The trimming criteria were also applied to all of the complex multiplication practice and strategy assessment RTs. The percentages of spoiled, incorrect, and outlier RTs for each of these sessions are not reported but are available from the author upon request.

APPENDIX

Prepractice tasks (the order of Sets 1 and 2 was randomized)
- Set 1 (single tasks, presented in random order within this set)
  - Simple multiplication
  - Complex multiplication
  - Complex Multiplication Strategy Assessment 1
- Set 2 (WM tasks, presented in random order within this set)
  - Complex multiplication control
  - Letter recall control
  - Complex multiplication and letter recall dual task

Practice Session 1 (complex multiplication)
- Strategy Assessment 2 (20 problems)
- Practice Block 1 (80 problems)
- Strategy Assessment 3
- Practice Block 2
- Strategy Assessment 4
- Practice Block 3
- Strategy Assessment 5
- Practice Block 4
- Strategy Assessment 6

Practice Session 2
- Strategy assessments (7–11) and practice blocks (5–8) alternated as in Practice Session 1

Practice Session 3
- Strategy assessments (12–16) and practice blocks (9–12) alternated as in Practice Sessions 1 and 2

Postpractice tasks
- Set 1 (single tasks)
  - Simple multiplication
  - Complex multiplication (only practiced problems)
  - Complex Multiplication Strategy Assessment 17 (only practiced problems)
- Set 2 (WM tasks)
  - Complex multiplication alone (only practiced problems)
  - Letter recall alone
  - Complex multiplication (only practiced problems) and letter recall dual task

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