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Correction to "Reminder and 2AFC tasks provide similar estimates of the difference limen: A reanalysis of data from Lapid, Ulrich, and Rammsayer (2008) and a discussion of Ulrich and Vorberg (2009)"

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We recently published an article (García-Pérez & Alcalá-Quintana, 2010) reanalyzing data presented by Lapid, Ulrich, and Rammsayer (2008) and discussing a theoretical argument developed by Ulrich and Vorberg (2009). The purpose of this note is to correct an error in our study that has some theoretical importance, although it does not affect the conclusion that was raised. The error lies in that asymptote parameters reflecting lapses or finger errors should not enter the constraint relating the psychometric functions that describe performance when the comparison stimulus in a two-alternative forced choice (2AFC) discrimination task is presented in the first or second interval. To demonstrate the error, let

$$\Psi_i(x; a_i, b_i) = \frac{1}{1 + \exp[-(x - a_i)/b_i]} \tag{1}$$

be the *latent psychometric functions* (i.e., those in the absence of lapses or finger errors), where i stands for the 2AFC interval (1 or 2) in which the comparison stimulus is presented, a_i is a location parameter (the 50% point on Ψ_i), and b_i is a spread parameter. The location and spread of Ψ_1 and Ψ_2 may differ as a result of "order effects" discussed by Ulrich and Vorberg; they argued that the psychometric function for data pooled across presentation orders is given by

$$\Psi_{2AFC}(x) = \frac{\Psi_1(x; a_1, b_1) + \Psi_2(x; a_2, b_2)}{2} \tag{2}$$

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Furthermore, when the comparison and standard stimuli are identical except as to their magnitude along the dimension of comparison, this function must satisfy $\Psi_{2AFC}(s) = .5$, where s is the standard level. The obvious reason is that $x = s$ renders a comparison stimulus that is identical in all respects to the standard stimulus, so that, regardless of order effects, $\Psi_1(s; a_1, b_1) + \Psi_2(s; a_2, b_2) = 1$. In other words, in these conditions, the point of subjective equality (PSE) must lie at the point of objective equality (POE). Thus, Ψ_1 and Ψ_2 must be jointly estimated under this constraint, which implies that

$$a_2 = s + \frac{b_2}{b_1}(s - a_1). \tag{3}$$

Figure 1a shows psychometric functions Ψ_1 and Ψ_2 reflecting order effects but satisfying this constraint so that the 50% point on Ψ_{2AFC} lies at the standard level (s = 500) and denotes the PSE.

Lapses and finger errors alter the lower and upper asymptotes of the observed psychometric functions. A more general form for the psychometric functions to accommodate these events is

$$\begin{aligned} \Psi_i^*(x; a_i, b_i, \kappa_i, \lambda_i) &= \kappa_i + (1 - \lambda_i - \kappa_i)\Psi_i(x; a_i, b_i) \\ &= \kappa_i + \frac{1 - \lambda_i - \kappa_i}{1 + \exp[-(x - a_i)/b_i]}, \end{aligned} \tag{4}$$

where κ_i determines the lower asymptote and λ_i determines the upper asymptote. Theoretical and practical reasons justify distinguishing upper and lower asymptote parameters that may also differ across presentation orders. For instance, lapses of attention make the observers miss a stimulus for one or another reason, and these lapses may occur more often during the first or second 2AFC interval: Ulrich (2010,

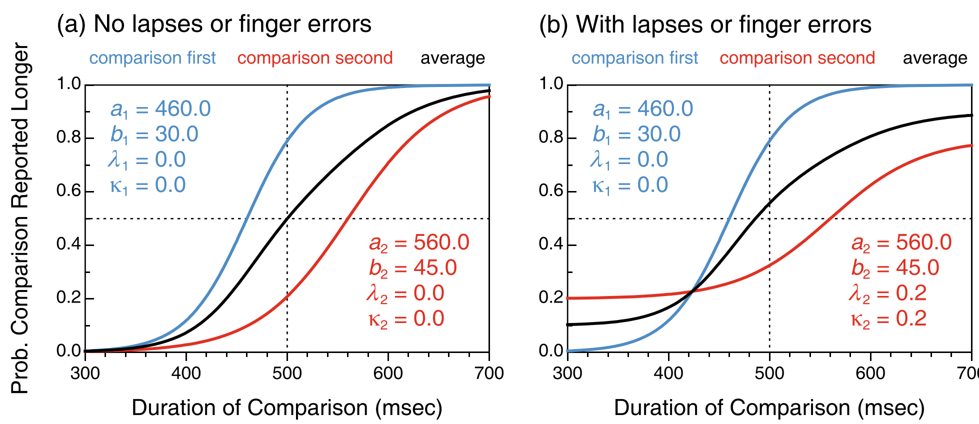

Fig. 1 Psychometric functions showing order effects when (a) there are no lapses or finger errors or (b) with exaggerated and disproportionate lapses or finger errors. The assumed standard level is s = 500. The shapes described by the functions vary as a result of lapses or finger errors, but parameters a_i and b_i are the same in both cases

see his Fig. 12) discussed a model implying an extreme version of this imbalance, which makes the upper and lower asymptotes differ across presentation orders. On the other hand, finger errors (i.e., hitting an unintended response key) are generally thought of as occurring randomly and independently of stimulus level or presentation order; however, the characteristics of the response interface may make observers more prone to misreporting, for instance, an "Interval 1" response for an "Interval 2" response than the other way around, which would also make asymptotes differ across presentation orders (García-Pérez & Alcalá-Quintana, 2012). Finally, even in the absence of such true differences in asymptote parameters across presentation orders, lapses or finger errors that occur purely at random and infrequently may accidentally affect one of the presentation orders more often than the other, rendering stray data points that will bias estimates of the spread and location of the psychometric function for that presentation order (Wichmann & Hill, 2001; see their Fig. 1) unless different asymptote parameters for each presentation order are included to absorb the differential effects. Note that the latter statement is only a practical justification for different asymptote parameters: The true parameters are not purported to differ across presentation orders, and this allowance is only meant to ensure unbiased estimates of location and slope.

Figure 1b shows how the latent psychometric functions in Fig. 1a change when $\kappa_1 = \lambda_1 = 0$ whereas $\kappa_2 = \lambda_2 = .2$ (a deliberately exaggerated example), while all of the remaining

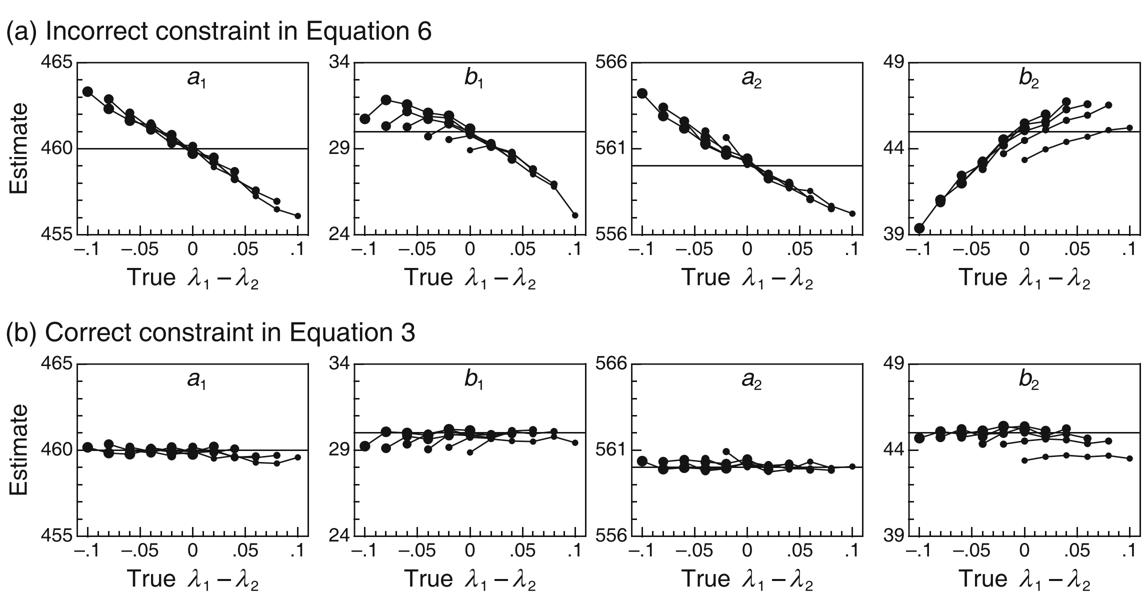

Fig. 2 Recovery of parameters a_i and b_i when the incorrect (a) or correct (b) constraints are imposed. In each panel, the symbols represent the average estimates of the parameter indicated in the inset across 1,000 replicates; the true value of the corresponding parameter is indicated by the ordinate of the horizontal line across each panel. Different strands of symbols pertain to different true values of λ_2, with symbol size progressively increasing as λ_2 increases from 0 to .1. Irregularities in the pattern of results are due to sampling error (only 50 trials per presentation order at each stimulus level), and they progressively disappeared as the numbers of trials increased (results not shown). Parameters λ_1 and λ_2 were also estimated concurrently, but no results are presented for these nuisance parameters, which are never accurately estimated and whose role is only instrumental to help obtain unbiased estimates of the remaining parameters

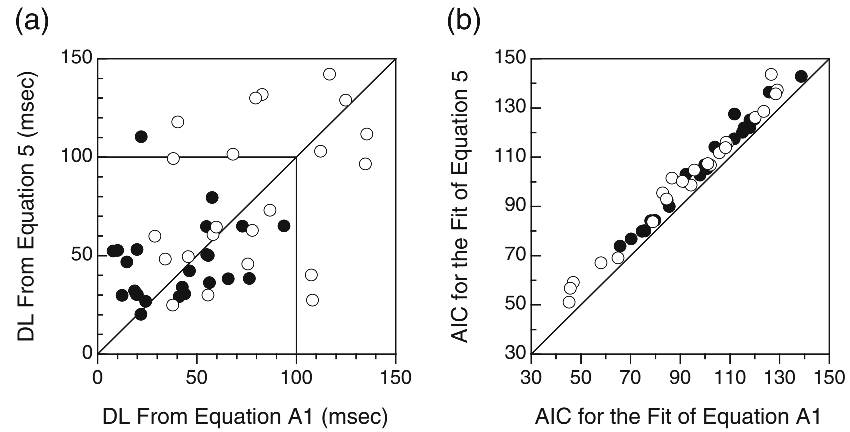

Fig. 3 Replacement for Fig. 13 in García-Pérez and Alcalá-Quintana (2010). With respect to the data in the left panel, in the original article we stated that the "DLs . . . respectively average 49.00 ± 4.03 and 46.69 ± 4.92 in Experiment 5 and 60.10 ± 5.19 and 59.28 ± 6.56 in Experiment 6"; our reanalysis here rendered instead averages of 43.43 ± 3.10 and 40.43 ± 4.93 in Experiment 5, and 56.21 ± 5.87 and 54.34 ± 5.59 in Experiment 6. As in the original analysis, the differences were not significant in either of the two experiments after the reanalysis

parameters are unchanged. The average function is also given by

$$\Psi^*_{2AFC}(x) = \frac{\Psi^*_1(x; a_1, b_1, \kappa_1, \lambda_1) + \Psi^*_2(x; a_2, b_2, \kappa_2, \lambda_2)}{2}, \tag{5}$$

but it is evident that now $\Psi^*_{2AFC}(s) \neq .5$, despite the fact that the PSE is still at $x = s$. To account for the shift in the location of the 50% point on Ψ^*_{2AFC} (under the simplifying assumption that $\kappa_i = \lambda_i$), García-Pérez and Alcalá-Quintana (2010) replaced the constraint in Eq. 3 with

$$a_2 = s + b_2 \ln\left[\frac{(\lambda_1 - \lambda_2)\exp(a_1/b_1) + (1 - \lambda_1 - \lambda_2)\exp(s/b_1)}{(\lambda_1 - \lambda_2)\exp(s/b_1) + (1 - \lambda_1 - \lambda_2)\exp(a_1/b_1)}\right]. \tag{6}$$

And this is where the error lies, because a constraint arising from the theoretical location of the PSE holds for the latent psychometric functions Ψ_i (which express pure perceptual effects and, hence, the PSE) and not for the observed functions Ψ^*_i, which are corrupted by lapses and finger errors that shift the 50% point on Ψ^*_{2AFC} away from the PSE. The events causing the asymptote parameters do not alter the constraint in Eq. 3, and they simply prevent the *observed* psychometric functions Ψ^*_i from matching the *latent* psychometric functions Ψ_i, which could only have been observed in the absence of lapses or finger errors. In other words, the PSE (in the psychological sense of reflecting *perceptual* indistinguishability) must remain at the POE when the comparison and standard stimuli are identical except in magnitude along the dimension of comparison,

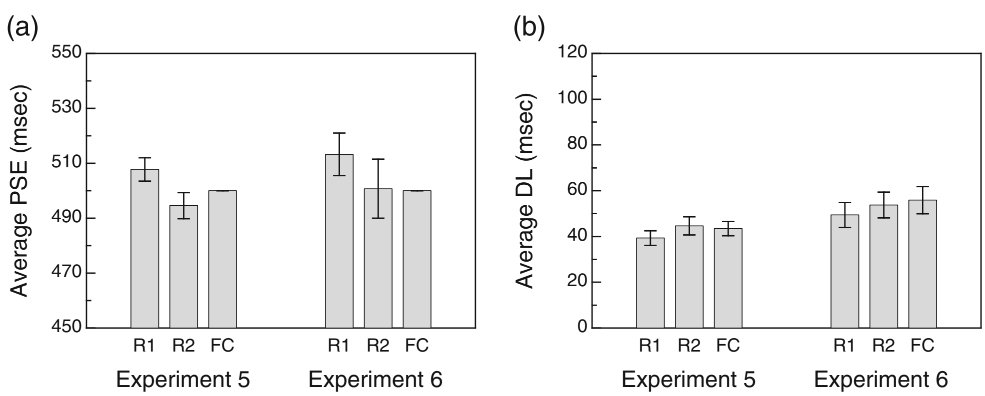

Fig. 4 Replacement for Fig. 14 in García-Pérez and Alcalá-Quintana (2010). With respect to the data in the left panel, the estimates are virtually identical to those reported earlier, and the conclusion remains the same. With respect to the data in the right panel, in the original article we stated that "the differences are only statistically significant between the R1 task and the two other tasks in Experiment 5 [for the difference between R1 and R2, $t(19) = -2.34$, $p < .05$, two-tailed; for the difference between R1 and FC, $t(21) = -4.35$, $p < .0005$, two-tailed] and between the R1 and FC tasks in Experiment 6 [$t(12) = -3.27$, $p < .001$, two-tailed]. Significant as they are, these differences are indeed very small . . . , yielding DLs that are less than 12% larger in the 2AFC than in the reminder task." Our reanalysis here implies that the differences are only statistically significant between the R1 and FC tasks in Experiment 5 [$t(22) = -3.06$, $p < .01$, two-tailed]. On average, across Experiments 5 and 6, DLs are less than 12% larger in the FC than in the R1 task, and 1% larger in the FC than in the R2 task

whereas the 50% point on Ψ^*_{2AFC} will generally not occur at the PSE (as shown in Fig. 1b). Thus, even when lapses and finger errors are considered, the functions Ψ^*_1 and Ψ^*_2 must still be jointly fitted under the constraint in Eq. 3: a_2 is still related to s, a_1, b_1, and b_2 through Eq. 3, and *not* through Eq. 6. For analogous reasons, the difference limen (DL) must be defined as $\mathrm{DL}_i = b_i \log(3)$, also excluding from this expression the asymptote parameters.

Similar considerations apply to the more general case in which the PSE cannot be assumed to lie at the POE (see García-Pérez & Alcalá-Quintana, 2011), and the constraint to be imposed on the joint fit of Ψ^*_1 and Ψ^*_2 in such cases is

$$a_2 = x_{\mathrm{PSE}} + \frac{b_2}{b_1}(x_{\mathrm{PSE}} - a_1), \tag{7}$$

where x_{PSE} is the PSE that must also be estimated.

Although our Fig. 1 has exaggerated the problem with the goal of providing a clear illustration, the consequences of this error are negligible when the values of asymptote parameters are as small as they typically are, or when they are somewhat larger but similar across presentation orders. To illustrate these consequences, we conducted a simulation study in which 1,000 data sets were generated through Eq. 4, with a_i and b_i always as in Fig. 1, but with $\kappa_1 = \lambda_1$ ranging between 0 and .1 in steps of .02 and with $\kappa_2 = \lambda_2$ varying independently with the same set of values. The stimulus levels ranged from 300 to 700 ms in steps of 50 ms, and 50 trials were administered with each presentation order at each stimulus level. The parameters were estimated by jointly fitting the psychometric functions in Eq. 4 (although with $\kappa_i = \lambda_i$) for each presentation order under the (theoretically correct) constraint in Eq. 3 and also under the (theoretically incorrect) constraint in Eq. 6. Figure 2a shows that parameter estimates obtained by imposing the incorrect constraint vary systematically with the difference between λ_1 and λ_2, although the estimation error is generally small: Spread b_i is misestimated within 5% of its true value unless the absolute difference between λ_1 and λ_2 is large (which implies that either λ_1 or λ_2 is atypically large); on the other hand, location a_i is also misestimated within $b_i/20$ of its true value unless the absolute difference between λ_1 and λ_2 is large. Figure 2b shows that imposing the correct constraint recovers the true location and spread parameters very accurately, regardless of the values of λ_1 and λ_2.

Although these estimation errors seem small, there is no reason to ignore them when the correct constraint is equally easy to implement. Thus, we repeated the analyses presented in Figs. 13 and 14 of García-Pérez and Alcalá-Quintana (2010) by fitting again the psychometric functions incorporating asymptote parameters, but now using the correct constraint in Eq. 3. Figs. 3 and 4 are replacements for Figs. 13 and 14 in García-Pérez and Alcalá-Quintana (2010), and note that the conclusion of the original analysis is reinforced by the new analysis (details are given in the captions to Figs. 3 and 4). In sum, the error corrected here does not alter the conclusion raised in our previous article: The data reported by Lapid et al. (2008) do not show evidence of significantly larger estimates of the DL from 2AFC tasks relative to estimates obtained from the reminder task. In any case, order effects differentially affect 2AFC tasks and make them generally more prone to rendering artifactually larger estimates of the DL in comparison to those obtained with the reminder task, but this only reflects a method bias and not a perceptual effect accompanying the 2AFC task.

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