A note on the estimation of the rate of learning in statistical learning models¹

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Abstract

Three estimation techniques for the Estes-Straughan learning rate parameter are discussed and restrictions on their application are indicated.

Much of the work with statistical learning models has been concerned with the prediction of asymptotic response probabilities. However, for a complete description of the behavioral change that occurs in choice situations, the rate of learning is an important variable. Unfortunately, the estimation techniques for the learning rate parameter in most situations are somewhat more complex than is the prediction of asymptotic response probabilities.

This note is concerned with only one model—the Estes-Straughan model (Estes & Straughan, 1954). They suggest that Equation (1) can be used to estimate the value of their rate of learning parameter, θ .

$$(1) \sum_{m=1}^K \bar{P}(m) = K\pi - [\pi - \bar{P}(1)] \frac{[1 - (1-\theta)^{20K}]^n}{1 - (1-\theta)^{20}}$$

(Estes & Straughan, 1954, p. 230.)

In this equation the terms are defined as follows:

$P(m)$ = the proportion of A_1 responses for all subjects in the m th block of 20 trials;

K = the number of blocks of 20 trials in the experiment;

π = the probability of an E_1 event;

θ = the learning rate parameter.

Unfortunately, certain restrictions must be placed on the application of Equation (1). The acceptable values of θ are 0 to 1 as indicated by the description of θ as a stimulus sampling parameter. To facilitate discussion, Equation (1) is shown in modified form as Equation (2).

$$(2) [\pi - \bar{P}(1)] (1-\theta)^{20K} + [\sum \bar{P}(m) - K\pi] (1-\theta)^{20} - \sum \bar{P}(m) + K\pi - \pi + \bar{P}(1) = 0$$

Equation (2) is a polynomial of degree $20K$ in $(1-\theta)$ and thus there are potentially $20K$ solutions. However, since there are only three $(1-\theta)$ terms $[(1-\theta)^{20K}, (1-\theta)^{20}, (1-\theta)^0 = 1]$, the equation has, according to Descartes' Rule of Signs (Brink, 1951), at most two positive solutions. Taking the first derivative of Equation (2) with respect to $(1-\theta)$ yields:

$$20(1-\theta)^{19} [(K\pi - K\bar{P}(1)) (1-\theta)^{20K-20} + \sum \bar{P}(m) - K] = 0.$$

One positive solution of Equation (2) is $\theta = 0$. However,

a solution that is less than 1 but greater than 0 is required. To have such a solution in the open interval $(0, 1)$ Equation (3) must hold.

$$(3) K\pi > \sum \bar{P}(m) > K\bar{P}(1).$$

Thus, unless the sum of the block proportions is greater than $K\bar{P}(1)$, no acceptable solution exists.

It may be suggested that since the $P(A_1) + P(A_2) = 1$, a simple relabeling will result in an acceptable solution. For a single group, this is perhaps acceptable, but if there are several experimental conditions in which both increases and decreases are likely, relabeling the response would not constitute acceptable scientific procedure, since it would of necessity be an ad hoc analysis. Thus, it seems that Equation (3) is a meaningful restriction.

Bush & Mosteller (1955) suggested an approximation technique to evaluate equations in the form of Equation (1). As K becomes very large $(1-\theta)^{20K}$ approaches zero. Thus an approximation to Equation (2) is given as Equation (4).

$$(4) [\sum \bar{P}(m) - K\pi] (1-\theta)^{20} - \sum \bar{P}(m) + K\pi - \pi + \bar{P}(1) = 0.$$

Even this equation is applicable only if Equation (5) or (6) holds.

$$(5) \sum \bar{P}(m) - K\pi > \bar{P}(1) - \pi > 0.$$

$$(6) K\pi - \sum \bar{P}(m) > \pi - \bar{P}(1) > 0.$$

And this is an even more limiting restriction.

A final estimation technique suggested by Estes & Straughan (1954) is given in Equation (7).

$$(7) P(A_1 | E_1) - P(A_1 | E_2) = \theta.$$

Again, the equation applies only if

$$P(A_1 | E_1) > P(A_1 | E_2).$$

As before, mere relabeling is a questionable procedure.

On the basis of the foregoing, it may be suggested that previously found agreements between predictions from the Estes-Straughan model and empirical data may be fortuitous. Each estimation procedure is a straight forward derivation from the model. Yet additional restrictions must be placed on the estimation procedures.

To conclude then, three learning rate parameter estimation procedures have been discussed. Limitations on each equation were noted.

References

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- ESTES, W. K., & STRAUGHAN, J. H. Analysis of a verbal conditioning situation in terms of statistical learning theory. *J. exp. Psychol.*, 1954, 47, 225-234.

Note

1. The author wishes to thank Dr. John Koehler for his critical reading of this note and for his helpful comments.

Comment

Cross, Rankin & Wilson (*Psychon. Sci.*, 1964, 1, 275-276) have shown that albino rats are superior to hooded rats in learning a Y-maze. They ascribed the difference in performance to differences in learning. Before this assumption is accepted, however, it will be necessary to rule out at least two other variables that may be contributing to their result; these variables are emotionality and visual acuity.

In a recent study, Routtenberg & Glickman (*J. comp. physiol. Psychol.*, 1964, 58, 140-142) found that hooded rats were superior to albino rats on the visual cliff. In general, hooded rats consistently chose the optically shallow side, while albino rats usually distributed their choices to both the deep and shallow sides, showing little preference for either side of the cliff. These results were interpreted as supporting the view of Lashley (*J. genet. Psychol.* 1930, 37, 481-484) that the visual acuity of the hooded rat is superior to that of the albino. We also found that hooded rats defecated significantly more in the open field and on the visual cliff than did our albino animals. In sum, there appeared to be at least two significant distinctions that could be made between these two strains: hooded animals had better vision and were more emotional than albino animals.

In the Cross et al (1964) study, it is possible that both greater visual acuity and emotionality hindered the performance of the hooded rat. This animal might take longer to habituate to the maze simply because there are more visual stimuli for it to habituate to than there are for the albino. These stimuli might distract the animal and impair its performance. Emotionality may have also played a part as Cross et al (1964) have

pointed out. They noted that in the Cross & Boyer (*Psychol. Rep.*, 1964, 14, 427-432) study, adapted hooded rats performed in a more predictable fashion, albeit that their performance was inferior to that of the albino animals.

According to the interpretation offered here, the atypical learning curves demonstrated by the hooded animals in the Cross et al (1964) study may have been the result of an interaction between the variables of emotionality and visual acuity, rather than the result of a difference in learning ability. The more typical but still inferior performance of adapted hooded animals (Cross & Boyer, 1964) might have been the result of the visual acuity variable alone, since the emotionality factor was probably reduced by the adaptation procedure.

One possible way to control for the visual acuity variable would be by running peripherally blinded hooded and albino rats.

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Reply

Routtenberg states that Cross, Rankin, & Wilson attribute the maze superiority of albino rats to learning ability. Cross et al. focused on the influence of incentive magnitude on time-independent measures in a complex situation. The "strain comparison" aspect of this study was directed to the problem at hand and was not intended to be a massive assault on possible strain differences. Accordingly, the discussion was guarded. The opening sentence emphasized the restriction imposed by the "emotionality" of the hooded Ss; nowhere was it actually concluded that strain differences in performance were attributable to differences in learning ability, although it was suggested that real strain differences "might" be present even in the absence of the emotionality factor. This was clearly a call for additional research, and the two factors discussed by Routtenberg's proposed research would not necessarily control for the visual factor since it remains a distinct possibility that operation-induced emotionality or disruption might be quite different for two organisms which differed greatly in their initial dependence on the sense modality destroyed.

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