Cost and accessibility of offers as determinants of optional stopping

AMNON RAPOPORT, UNIVERSITY OF NORTH CAROLINA
AMOS TVERSKY, UNIVERSITY OF MICHIGAN

In an optional stopping decision task Ss were presented with a random sequence of offers with a fixed cost per offer. Ss were instructed to maximize their gain by taking the best offer with as few observations as possible. Ss in group H received the highest offer observed by them, while Ss in group L received the last offer observed by them. Group H took more observations and made more errors than group L, though an optimal model prescribes the same behavior.

Many decision-making problems such as renting an apartment, buying a second-hand car, and selling shares of stock can be characterized as follows. The decision-maker makes a sequence of inquiries (observations) for offers and after each one of them decides whether to stop the search and take one of the offers made to him, or to continue the search. Typically, there is a cost associated with each observation which may be expressed in terms of money, effort, time spent in search, or aggravation. The decision-maker's purpose is to maximize his gain, which is determined by the offer he takes and by the cost of the search.

Assuming a fixed cost, c , per observation, and known distribution of offers, $f(z)$, an optimal solution for this class of problems is derived (Radner, 1964; Chow & Robbins, 1961). The solution is given in terms of a fixed cutoff point, z^* , on the distribution of offers, such that the decision-maker should accept the first offer greater than z^* . When $f(z)$ is known, the value of z^* is obtained from

$$(1) \int_{z^*}^{\infty} (z - z^*) f(z) dz = c$$

In the special case when $f(z)$ is a normal distribution, z^* is the value which maximizes

$$(2) \frac{f(z^*) - c}{Pr(z \geq z^*)}$$

where the values of the ordinate, $f(z^*)$, and the area, $Pr(z \geq z^*)$, are readily obtained from tables of the normal distribution. The above solution is optimal in the sense of maximizing the decision-maker's expected gain. Clearly, the value of z^* is a function of c . The higher the cost per observation, the smaller the number of observations that should be made.

In some of the above decision situations the decision-maker is able to accept only the last offer made to him. He has no access to the previous offers he observed. In other decision situations all the offers observed remain available to the decision-maker, who may choose the highest among them. Psychologically, the two kinds of situations seem different. One would expect a decision-maker to make more observa-

tions when all offers remain accessible than when any offer not accepted is immediately lost. The optimal model, however, prescribes the same behavior for both decision situations. Since the optimal solution is to accept the first offer above the cutoff point, the accessibility of past offers is irrelevant.

Method

Ss were 30 male students from the University of Michigan who volunteered to participate in a paid computer-controlled task. Each S served individually for one experimental session.

Each S was seated in front of an LGP-30 computer with a Friden flexowriter connected to it, and given a set of written instructions. S was asked to act as a company representative who wanted to sell some shares of stock. At each trial the computer printed an offer from a different agency and S decided whether to sell the shares, by typing "sell" on the flexowriter or to ask for another offer, by pressing the appropriate key. There was no limit on the number of offers the computer could make.

Ss were divided randomly into two equal groups. Ss in the group H were told that they could sell the shares at any time for the highest offer made to them so far. Ss in group L were told that they could sell the shares at any time for the last offer made to them. All Ss were told that each offer cost a fixed amount, c , which was the fee charged by each agency.

The offers were randomly selected from a normal distribution with mean \$1800 and standard deviation \$300. Each S was presented with 29 separate problems, differing in the cost per offer. There were four cost conditions. The cost was \$5, \$20, \$10, and \$35 for problems 1 through 8, 9 through 15, 16 through 22, and 23 through 29 respectively. The company's pay-off, g , for each problem was given by $g = z' - n c$, where n is the number of offers made, and z' is the highest or last offer made to Ss in group H or L respectively. The value of g was printed at the end of each problem. The first problem was for practice only. Each S observed 20 offers to provide him with initial information about $f(z)$.

A single sequence of offers for all problems was randomly devised and presented to all Ss. Since each S determined the number of observations for each problem, different Ss observed the same offers on different problems (except for problem 1). Ss had to ask for at least one offer for each problem. They were instructed to maximize the company's total gain. Their gain for the experiment, G , was given by

$$(3) G = \frac{\sum g_i}{300} \text{ (in cents), } i = 2, 3, \dots, 29$$

Results and Discussion

The mean gain for both groups under the four cost conditions (5, 10, 20, and 35, in that order) was 2316.4, 2209.2, 2193.7, and 2012.9 for group H, and 2265.5, 2256.6, 2147.5, and 2027.3 for group L. As predicted by the model the mean gain for both groups decreased with an increase in c . The overall difference between the mean gains of the two groups was not significant ($p < .05$).

The mean number of observations, n , for both groups under the four cost conditions was 15.57, 12.22, 8.15, and 5.48 for group H, and 11.78, 9.28, 6.61, and 5.82 for group L. The optimal number of expected

observations for each value of c , n^* , was 25.00, 13.65, 7.40, and 4.75 for the 5, 10, 20, and 35 cost conditions respectively. The value of n^* is obtained from

$$(4) \quad n^* = \frac{1}{P_r(z > z^*)},$$

for a random sequence drawn from a normal distribution with the given mean and variance, rather than for the particular sequence used. Since for any given problem different Ss received different offers, there is no single sequence for which the optimal number of observations can be obtained for all Ss.

The mean number of observations for both groups decreased as the value of c increased. Similar results were obtained by Lanzetta & Kanareff (1962) and by Irwin & Smith (1957), using a similar decision task and two different costs per observation. As hypothesized above, group H observed more than group L. The overall difference between the two groups is significant ($p < .05$).

As c increases, the difference between the two groups with respect to n tends to disappear. Comparing the values of n to n^* for both groups, it is evident that both groups observed less than prescribed by the model for the low cost, and approached the predicted n^* as the cost per observation increased.

The optimal model prescribed taking the first offer greater than a chosen cutoff point. Whenever S terminated his observations at a different offer, he violated the cutoff point model. This is the case whether the cutoff point is chosen optimally or not. Moreover, the above holds even if each S is allowed to have a different cutoff point and to change it from one problem to another, even within a cost condition. This is the weakest form of a cutoff point model which allows learning from one problem to another.

In order to test this weak version of the cutoff point model, the number of observations after the highest offer (errors) was computed for each S for each problem. The mean number of errors, e , under the four cost conditions was 6.30, 3.79, 2.60, and 1.50 for

group H, and 2.54, 1.67, 1.79, and .50 for group L. With the exception of one data point, the values of e decrease as c increases, approaching the predicted value from any cutoff point model, i.e., $e = 0$.

The value of e is higher for group H than for group L for all cost conditions. The difference between the groups decreases as the value of c increases. The overall difference in e between the two groups is significant ($p < .05$). This result indicates that the inaccessibility of previous offers resulted in closer agreement with a cutoff point model. The difference in e between the two groups may be explained by the fact that errors were more costly for group L than for group H. If an S in group H failed to accept a high offer, which turned out to be the highest offer for a particular problem, he could still receive it minus the cost of the additional observations. For an S in group L such a failure would result in a greater loss, since this offer would not remain accessible.

In spite of the significant differences between the two groups with respect to n and e , no significant difference in G was found. This is accounted for by the compensatory effects of the discrepancies of both groups from optimality with respect to n and e . While group H was closer to the model than group L in terms of n , group L was closer to the model than group H in terms of e . Both of these discrepancies decreased as a function of the cost per observation.

References

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