The influence of context and stimulus range on the judgment of numerosness*

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The exponent of the power law describing the attribute of numerosness is a function of methods of judgment, set, and, primarily, the extent of the stimulus range. The latter two variables were investigated by having 36 Os use the method of magnitude estimation to judge three series of stimulus ranges of 12-1,500 dots each. The stimulus range, much larger than any previously investigated, led to a diminished exponent, while varying context by manipulating the size of stimulus intervals within this range had no effect.

Many occasions require the estimation of amount or quantity along a physical dimension under circumstances which do not provide appropriate instruments to give reliable and valid confirmation of initial numerical guesses. The power law (Stevens, 1957) has the advantage of fitting direct estimation judgments, methods similar to what a judge would use outside the laboratory. However, power law exponents vary too consistently for the power law to become an accepted paradigm. Coulton (1968) discussed six psychophysical models to account for such variation, finding the most potent influence to be stimulus range. If the range is increased, the exponent will decrease. His remaining models predict changes in the exponent's value as a function of other "standard" experimental manipulations. Teghtsoonian (1971) argued that stimulus range, or rather the dynamic range of stimulus values, could account for a much larger proportion of exponent variation. He postulated the sensory-judgmental range onto which dynamic stimulus ranges of various extents, from the different sensory modalities, must be mapped.

Numerosness, the judgment of sheer quantity, can be considered as a more pristine representative of the magnitude estimation process or as a concise representative of a psychophysical attribute. Stevens (1957) computed the exponent for numerosness as 1.34, based on half-magnitude judgments observed by Taves (1941). Using this method, experienced judges overestimated what should be half of a standard, or that the standard was underestimated. Doubling the mean half-judgments of 2 to 180 dots leads to an exponent of .95.

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Taves (1941) reported six other studies of numerosness, most using the same stimulus range of 1 to 180 dots spaced logarithmically and most using highly practiced judges. His largely methodological variations all showed two processes operative in the estimation of number. Below 10 or 12 dots, the number of things is clearly apprehended, while above this limit, numbers are estimated with imperfect accuracy. Exponents calculated from these judgments of 12 to 180 dots range from 0.90 to 1.00.

Kaufman, Lord, Reese, & Volkman (1949) studied the effect of the judgmental sets of speed vs accuracy in estimation. Nine naive judges gave magnitude estimations of 35 stimuli ranging from 1 to 210 dots in a logarithmic spacing. As had Taves, these authors noted that when the number to be estimated was less than 7 or so, estimations were perfect, and they distinguished this process by labeling it "subitization." Based on the median estimates of 12 to 210 dots, the exponent under the accuracy set was 0.88 and was 0.86 under the speed set.

Minturn & Reese (1951) used the Kaufman et al stimulus range and a similar procedure to investigate the effect of knowledge of results on subsequent estimations, both immediate and delayed. Ten naive judges were used, and the median percentage error in the first experimental session for the entire group, before any reinforcement had been given, led to an exponent of 0.80 for the range of 12 to 210 dots. This is less than the Kaufman et al exponents for the same stimulus range and is also less than those obtained from Taves (1941), whose range terminated at 180 dots.

More than a decade later, Bevan & Turner (1964) obtained magnitude estimations of 21, 42, 84, 168, and 336 dots under five experimental conditions, with 30 naive Ss in each group. Stimulus duration was 10 times longer here than in previous studies

(2 sec vs 0.2 sec). The size of the frame surrounding the stimulus configuration (large or small) interacted with a set to accept that frame as part of the stimulus configuration figure, or as distinct from it. These manipulations influenced the size of the exponent. "Figure" conditions led to underestimation, with exponents of 0.80 and 0.82, while "ground" conditions led to exponents of 1.04 and 1.18, or overestimation. A control group produced an exponent of 0.93, about the same size as in Taves (1941), with a shorter stimulus range and presentation interval.

Clearly, varying experimental set and stimulus range lead to variation in the exponent for numerosness, so it is reasonable to study such effects more systematically. The present study incorporated a larger stimulus range than previously used. The second variation resulted from questioning whether the logarithmic scale of response assumed by the power law reflected a more basic process than did the logarithmic spacing of stimulus intervals. Thus, stimulus series using logarithmic and arithmetic spacing were chosen, as well as a third series with spacing midway between those of the first two. By holding stimulus range constant, different response systems for the three contexts could emerge.

METHOD AND PROCEDURE

Stimulus Series

The three stimulus series of 12-1,500 dots were determined by considering this range as being divided into 12 intervals. These intervals were equal logarithmically in the log series (L) and equal arithmetically in the arithmetic series (A). The quadratic series (Q) was divided into "quadratically" equivalent intervals so as to be midway between the L and A series. The actual numbers of dots used in each series were as follows: L—12, 19, 29, 45, 69, 108, 167, 259, 402, 623, 990, and 1,500; Q—12, 43, 95, 168, 262, 376, 513, 668, 845, 1,043, 1,261, and 1,500; A—12, 147, 283, 418, 553, 688, 824, 959, 1,094, 1,229, 1,365, and 1,500.

Stimuli

The lower limit of 12 dots was selected as sufficiently above the maximum for subitization, while 1,500 dots represented the maximum number fulfilling the restriction that dots could take no more than 25% of all available positions within the stimulus field. The basic field consisted of the 54 (lines) by 120 (columns) of a computer printout page. The location of each dot in a given array was determined randomly by the IBM 7094 and was indicated by printing a period in the cell (modulus

54 x 120 = 6,480) corresponding to the appropriate line and column. All other cells were represented by blanks. Two different random patterns were generated for each of the 36 stimulus values, and each of these was photographed twice with Kodak high-contrast copy film. The negatives were mounted in glass slides, with the duplicate copy being mounted upside down and backwards, providing four different patterns for each stimulus value. A Wollensak projector with a Haydon timer was used to project the stimulus fields of white dots on black fields onto a lenticularly beaded Dalite screen, located 6 ft in front of S's chair.

Subjects

Thirty-four males and two females were paid volunteers from The Johns Hopkins University S pool. All were naive with respect to the experiment and were assigned randomly to one of six equally-sized groups.

Task

The S was asked to estimate the number of dots in each of many slides he would see on the screen before him. A stimulus was presented for 1 sec, and during the 7-sec interstimulus interval, S recorded his estimate. Within each of the three stimulus series, the 12 stimulus values were presented in four successive and different random orders, with different random orderings for each of the four blocks of 12 stimuli in each series. The same within-series ordering was used for all Ss. Six experimental groups of six Ss each differed only in the order of presentation of the three stimulus series. The first group judged L first, then Q, then A, and was designated as Group LQA. The remaining five groups were LAQ, QAL, QLA, ALQ, and AQL. Each S made 144 judgments in all.

RESULTS

Estimates ranged from 5 to 5,300 dots, the median maximum estimate being 600, with less than one-third of the Ss having maximum estimates of 1,500 or more. The logarithm of each estimate was correlated with the logarithm of the number of stimulus dots presented, producing an exponent and reliability coefficient for each series for each S. The individual exponents ranged from 0.38 to 1.22, with a mean of 0.76, but they did not vary as a function of order of

Fig. 1. Median judgment of numerosity for stimulus dots in log series ($\square$), quadratic series ($\times$), and arithmetic series ($\circ$), over 36 Ss.

presentation ($F = 0.38$), stimulus series ($F = 0.36$), or the interaction of these two factors ($F = 1.45$, $df = 10, 60$, $p > .05$). The majority of the reliability coefficients were greater than 0.90, and analysis of the z-transforms of the coefficients revealed no effect of experimental manipulation [$F(\text{order}) = 1.93$, $df = 5, 30$, $p > .05$; $F(\text{series}) = 0.02$; $F(\text{Order by Series}) = 0.09$].

The variability between Ss suggested using the more conventional calculation of the exponents and reliabilities from the group medians. This produced reliabilities of 0.98 or more for all series, with the mean exponent for each series being 0.76. (Analyses of variance were unnecessary.) A final attempt to discern context effect was the transforming of the estimates of each S such that the sum of his estimates squared equaled the constant 5,000. Exponents and reliabilities based on the group medians of the transformed estimates were identical with those based on the untransformed estimates. That is, Ss, individually or collectively, unretouched or transformed, judged stimuli with great accuracy and with the same amount of over- or underestimation regardless of context or order. Figure 1 displays the psychophysical function for the stimulus values presented, a power law function with an exponent of 0.76, based on the medians of the untransformed estimates over all 36 Ss.

DISCUSSION

When a stimulus range of 12 to 1,500 dots is used to investigate numerosity, the power law exponent is 0.76. Overall, with the exception of Bevan & Turner (1964), the exponent has decreased as the stimulus range has increased. Or, the greater the number of things to be estimated, the more the maximum will be underestimated. Subsequent experimentation increasing the maximum from the 1,500 dots used here could determine that value of the stimulus range beyond which the exponent does not change. While numerosity has the lower limit of subitization, about 10 dots, it has no upper physiological limit. Constructing larger stimulus displays could provide an empirical check of Teghtsoonian's (1971) quantitative hypothesis of sensory resolving power.

The overwhelming lack of any effect of differing stimulus intervals within a given stimulus range also lends indirect support to Teghtsoonian's (1971) postulate of the nearly perfect relationship between the dynamic stimulus range and the power law exponent. Context effects in numerosity exist, but the present procedure was clearly not adequate to the task of discovering them. More adequate was the Bevan & Turner (1964) procedure of coordinating physical configurations with instructional sets.

REFERENCES

- BEVAN, W., & TURNER, E. D. Assimilation and contrast in the estimation of number. *Journal of Experimental Psychology*, 1964, 67, 458-462.
- KAUFMAN, E. L., LORD, M. W., REESE, T. W., & VOLKMAN, J. The discrimination of visual number. *American Journal of Psychology*, 1949, 62, 498-525.
- MINTURN, A. L., & REESE, T. W. The effect of differential reinforcement on the discrimination of visual number. *Journal of Psychology*, 1951, 31, 201-231.
- POULTON, E. C. The new psychophysics: Six models for magnitude estimation. *Psychological Bulletin*, 1968, 69, 1-19.
- STEVENS, S. S. On the psychophysical law. *Psychological Review*, 1957, 64, 153-181.
- TAVES, E. H. Two mechanisms for the perception of visual numerosity. *Archives of Psychology*, 1941, 37, No. 265.
- TEGHTSOONIAN, R. On the exponents in Stevens' law and the constant in Ekman's law. *Psychological Review*, 1971, 78, 71-80.