Periodic probability learning¹

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Nineteen human Ss participated in a two-choice probability learning experiment. The event probabilities were .80/.20 for the first 100 trials, and for the last 270 trials they varied as a sine function of trial number. The prediction that S response probabilities should be periodic (a prediction based on Estes' linear operator model) was tested using periodic regression analysis. The prediction was supported.

This experiment was designed to test a prediction made by Estes' model of probability learning (1957). The experiment reported here is the same as the classical two-choice noncontingent probability learning experiment (Humphreys, 1939), with the exception that the event probabilities vary as a sine function of trial number. Estes (1957) has shown that for this experimental paradigm his model predicts that the response probabilities will also vary periodically. Furthermore, he has shown that the response probabilities will asymptotically be described by a periodic function having the same period as the event probabilities. The importance of this prediction is that it is a parameter-free prediction.

To test this prediction, an experiment was designed, for which the event probabilities cycled every 72 trials. It was predicted that the response probabilities would have a period of 72 trials.

Method

The Ss were 19 introductory psychology students.

The Ss were run in a classroom, at the front of which was an 18 in. x 2 in. x 4 in. board. At the center of the board was a red 20-W light bulb, and at each end a white 10-W light bulb. A 2-in.-high yellow plastic "0" was glued over the left white light, and a "1" over the right white light.

A trial lasted 12 sec. The red light was lit for 1 sec., indicating the start of a trial. The 6-sec. interval after the offset of the red light was terminated by one of the two white lights. The white light remained on for 1 sec. After a 4-sec. intertrial interval, the next trial started. The timing was controlled by a Minarik timer. The E controlled the series of white lights with a microswitch.

Trials 1-8 were practice, with four occurrences of each event. Trials 9-378 were experimental trials. For Trials 9-108 there were 80 "1's." For the remaining 270 trials the Ss received a series of events whose probability varied according to the function

$$\pi_n = .5 + .5 (\sin [2\pi n/72 + \pi/2])$$

where n is the trial number, and π with no subscript refers to the constant 3.14... This is a sine function

centered around .5 varying between 0 and 1, with a period of 72 trials, whose value on Trial 109 is 0.

To determine whether the response probabilities were periodic, a periodic regression analysis (PRA) was performed. PRA is a technique for analyzing data (which are hypothesized to be periodic) into periodic components. A brief description of the technique will be given here. For a complete presentation of the technique see Hayward (1960).

It is well known that a set of numbers $f(x)$ can be approximated by the trigonometric expansion

$$f(x) = a_0 + \sum_{h=1}^H (a_h \cos \frac{2\pi h}{k} + b_h \sin \frac{2\pi h}{k})$$

known as the Fourier function. The coefficients a and b (Fourier coefficients) may be viewed as regression coefficients to be estimated from the data. Least-squares estimates of the coefficients have been developed (Hayward, 1960), and a program is available (Dixon, 1965).

The concepts underlying PRA are similar to those underlying polynomial regression (trend analysis). Whereas in polynomial regression one determines the significance of linear, quadratic, cubic, etc., components, in PRA one determines the significance of periods encompassing k trials, $k/2$ trials, $k/3$ trials, etc. Whereas in polynomial regression one obtains a sum of squares for each component, and a residual sum of squares, in PRA one obtains a sum of squares for each period length, and a residual. Just as in polynomial regression, exact F tests exist for each source of variation.

Results

A PRA was performed on Trials 163-378. These trials, the last 216, were used in the analysis because an even multiple of the hypothesized period length of 72 trials must be analyzed. In this case three periods were studied. The results of this analysis are presented in Table 1. The error term used for the 72 trials and the other periods sources bear some explanation as does the df associated with the error terms. The mean square for 72 trials is the mean square from scatter plus that from Ss x 72 trials, minus that from error. The error term for other

Table 1. Periodic Regression Analysis

	Ss	df	F
72 Trials	341.8	2	109.5
Other Periods	11.7	4	3.7
Scatter	127.7	209	4.8
Ss by 72 Trials	38.7	36	8.5
Error	474.7	3762	--

Table 2. Least Squares Estes' Model

	9-108	109-378
θ	.060	.160

periods is similarly calculated. The df are calculated by squaring the mean square for the source, and dividing by the following quantity: square each mean square, divide each by its corresponding df, and sum the three numbers. For a further explanation of these procedures see Hayward (1960).

In addition to the PRA, the least-squares solution to the prediction equation based on Estes' (1957) model was derived. The model was fitted to Trials 9-108, and also independently fitted to Trials 109-378. On the basis of these two solutions, two estimates of θ were obtained. The results of these two analyses are presented in Table 2.

Discussion

Table 1 indicates that the periodicity matching prediction was clearly supported ($F=109.5$, $df=2/71.76$). Figure 1 presents the data from the last 216 trials, each point representing the mean proportion of "1" predictions over blocks of four trials. The points have been connected to increase the clarity of the drawing. It is easily seen that the response frequencies are strongly periodic. It is important to note that the Ss by period interaction is significant ($F=8.5$, $df=36/3762$). This source indicates that at least two of the least-squares regression curves which were fitted to each S differ from each other. They can differ from each other in one or both of the following ways: The maximum response probability or the lag behind the event probabilities. Either of these differences would indicate that not all individuals have the same learning rate parameter in the model, since both characteristics of the model's prediction function depend on this parameter.

Table 2 indicates that the best fitting value for the learning rate parameter for the first 100 trials is quite different from that for the last 270 periodic trials. This is an unfortunate finding, since it is desirable that separate estimates of the parameter

Fig. 1. Proportion of Ss predicting event 1 every block of four trials (RAW); Periodic Regression Analysis predictions (PRA); and Estes Model's predictions (E).

yield approximately the same value.

Figure 1 presents the two least-squares functions, one based on the PRA, and one based on the model. The overall agreement between these two functions is very good. It is, in fact, a credit to the model that it is as close to the PRA least-squares fit as it is, since the model has only one free parameter, whereas the PRA has seven free parameters.

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Note

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