The polarity coincidence correlator: Significance testing and other issues

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The polarity coincidence correlator (PCC) is a useful nonparametric measure of correlation. In this paper, an error is pointed out in the significance test given by Cochran and Gibson (1977). Some further theoretical and practical issues are mentioned pertaining to the use of PCC with physiological waveforms; in particular, problems relating to autocorrelation and the definition of the sampling unit are discussed.

The polarity coincidence correlator (PCC) is recommended by Cochran and Gibson (1977) as a nonparametric test of correlation. Cochran and Gibson refer to both its ease of computation and its performance with non-normal data. However, there is an error in the significance testing procedure given in their paper. In this paper, we point out this error and discuss some additional issues relevant to the application of PCC.

Briefly, the PCC statistic is a simple count of the number of times two covariates coincide in their polarity (k) less the frequency of their displaying opposite sign (k^1). Provided the two data sets have zero median (Gibson & Melsa, 1975), under the null hypothesis of no correlation, k will have binomial distribution $b(k; N, 1/2)$ and $E(k) = E(k^1) = N/2$ and $E(\text{PCC}) = 0$. The obvious way to test the statistical significance of PCC is through the binomial probability associated with k , using the relationship $k = (\text{PCC} + N)/2$, derived from $\text{PCC} = k - (N - k)$. For the data of their tutorial example (Table 1, p. 276), we have $P(\text{PCC} \geq 6 | H_0) = P(k \geq 8 | H_0) = .055$. Cochran and Gibson (1977) present for significance testing a different judicial procedure, which is defective; it implies the incorrect relationship $k = \text{PCC} + N/2$ and, for their example, yields a p value of .0107, which is seriously low.

To illustrate the performance of PCC, Cochran and Gibson (1977, pp.276-278) apply it to assess the correlation between two physiological (EMG) waveforms. They couch their discussion of the comparative performance of PCC in terms of significance levels. Apart from the error in the calculation of these levels, there are serious theoretical and practical issues in the use of PCC with physiological waveforms which should be considered.

These concern (1) the definition of the sampling unit and (2) the correlation between successive samples within each waveform.

As far as concerns the sampling unit for PCC, there is a conflict between hypothesis testing and estimation. The PCC can be regarded as an estimator of the excess proportion of time that both waveforms lie on the same side of their medial values. The procedure that Cochran and Gibson (1977, p. 276) use, of randomly selecting 100 points on the time axis, can be regarded as giving a Monte Carlo estimator of this proportion. Clearly, the more points (N) that are used, the more precise will be the estimator.

On the other hand, for a given set of data, $E(\text{PCC})$ will increase linearly with N , whereas the standard deviation of PCC will increase linearly with $\sqrt{N}$. Thus, PCC can be made arbitrarily significant as long as $E(\text{PCC}) \neq 0$ (i.e., the waveform pair are such that they do not spend exactly equal times on opposite sides and on same sides).

This phenomenon of making any difference significant by increasing N is closely related to the effect that is produced on chi-square by multiplying all the cell counts in a contingency table by a constant.

The presence of autocorrelations in the waveforms will radically affect the theoretical basis of PCC. As Cochran and Gibson (1977) stress, the null hypothesis is founded in part on the assumption that each series is stationary and independent. In fact, many physiological waveforms exhibit at least some short-term autocorrelations. Thus, the polarities of x, y -pairs belonging to times that are too close together will tend to be correlated. This dependence will affect the distribution of PCC in a manner that depends upon the specific waveforms.

Random selection of points on the time axis will not guarantee the required independence. One option is to require that the minimum intersample time is greater than some time R , where R is chosen so that the autocorrela-

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tion of both waveforms is zero for times greater than R . Alternatively, one may attempt to compensate for dependency by making an adjustment for "lost" degrees of freedom. A useful study of a close relative of this problem is given by Altham (1979). Donchin (1969, p. 208) has also commented on the degrees of freedom problem, specifically in relation to physiological waveforms.

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