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An alternative to Fitts' law

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A new power law of movement time for motor control tasks is proposed as an alternative to the classical Fitts' law. A reanalysis of Fitts' original data indicates that this power law may provide superior fits to those of Fitts' law, which may be interpreted as a special case of the power law. The so-called rate of change of information derived from Fitts' law, which is claimed to be an invalid information-theoretic measure, is an exponentially decreasing function of Fitts' index of difficulty according to the power law rather than a fixed quantity as derived from Fitts' law.

The classical Fitts' law of motor control states that the movement time (MT) is related to the nominal movement amplitude (A) and termination accuracy or target width (W) in terms of the model,

$$MT = a + bI_d; \; I_d = \log (2A/W), \quad (1)$$

where a and b are parameters and I_d is the so-called index of difficulty (Fitts, 1954). Another variant of this law has been proposed by Welford (1968, 1976), and according to it,

$$MT = b\log(A/W + .5), \quad (2)$$

where b is again a parameter. This law has been extended to cover positioning tasks with additional movement constraints (e.g., Kvålseth, 1973, 1974, 1975, 1978) and has been found useful for measuring performance in a wide range of task situations (e.g., Card, English, & Burr, 1978; Drury, 1975; Flowers, 1975; Konz, Jeans, Rathore, 1969; Kvålseth, 1977; Langolf, Chaffin, & Foulke, 1976; Wade, Newell, & Wallace, 1978; Welford Norris, & Shock, 1969).

Following the proposition by Fitts (1954), the logarithms in Equations 1 and 2 are generally taken to base 2, and I_d, as well as Welford's (1968, 1976) logarithmic measure, is typically interpreted as being a measure of the amount of information generated during a movement, with the unit of bits per movement. Thus, the derivative of I_d with respect to MT (i.e., the inverse of b in Equation 1 or in Equation 2) is generally considered to be the generated information rate (in bits per second if the unit of MT is seconds) and the performance measure of primary interest. However, this author claims that the Fitts' measure I_d and also Welford's variant are not valid information-theory measures and that the results of a substantial number of reported experimental studies using these measures are misleading and false (Kvålseth, 1979). The underlying problem is that the analogy suggested by Fitts (1954) between his measure I_d and the Channel-Capacity Theorem 17 by Shannon (1948) is invalid, as pointed out by Kvålseth (1979), who has proposed an alternative information-theoretic performance measure and task paradigm. This alternative measure and paradigm, which do not rely on any assumptions or analogies that are not theoretically valid and cannot be tested experimentally, have been shown empirically to lead to information rates that are substantially different from those resulting from Fitts' law (Kvålseth, 1979).

While the information-theoretic foundation of Fitts' (1954) law is not valid and hence the information-rate performance derived from it is incorrect, numerous empirical studies have shown that the model in Equation 1 provides highly significant fits to empirical data. Nevertheless, even better fits seem to be achieved by a power model. The purpose of this paper is to propose a power law as an alternative to Fitts' law and to demonstrate the superior fit of this power law to the original data collected by Fitts (1954). A more comprehensive study is being planned to establish the general validity of the power law by reanalyzing large quantities of previously published data based on Fitts' paradigm.[1] It will also be shown in this paper that Fitts' law may be considered a special case of the power law. Finally, it is pointed out that the derivative $dI_d/d(MT)$, in addition to being an invalid measure of information rate and rather than being fixed for different values of I_d as indicated by Fitts' law, appears to decrease exponentially with increasing I_d.

POWER MODEL FORMULATION

Formally, the proposed power model is given by

$$MT = aA^bW^c, \quad (3)$$

and for the case of equal exponential weightings with $c = -b$, the model becomes

$$MT = a(A/W)^b, \quad (4)$$

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where A and W denote again the movement amplitude and target size, respectively, and a, b, and c are the model parameters. The parameter a takes into account the units of MT and also those of A and W for Equation 3. The fit of these two forms of the power model to any set of experimental data may be visualized from scatter diagrams on log-log paper, which may also reveal further improvement in the fit by introducing some additional parameter, such as an additive constant term. However, as will be shown below for Fitts' (1954) data, highly adequate fits are achieved by the simple models in Equations 3 and 4 with only three and two parameters, respectively.

The model in Equation 4 may also be expressed in the exponential form,

$$MT = ae^{\beta \log(A/W)}, \quad (5)$$

with $\beta = b$ if the log in Equation 5 is to the natural base e and with $\beta = b/\log_2 e$ if the log in Equation 5 is to base 2. This equivalence between Equations 4 and 5 is immediately apparent by taking logs on each side of the two equations. The model in Equation 3 may also be expressed in a slightly more complex exponential form. In terms of Fitts' index of difficulty, I_d, defined in Equation 1, the model in Equation 4 may be expressed as

$$MT = \alpha e^{\beta I_d}, \quad (6)$$

where $\alpha = a/2^b$ and β is the same as in Equation 5. By expanding the right-hand side of Equation 6 into an infinite power series, it follows that[2]

$$MT = \alpha + \alpha\beta I_d + (\alpha\beta^2/2!)I_d^2 + (\alpha\beta^3/3!)I_d^3 + \ldots, \quad (7)$$

where the symbol ! denotes the factorial. By comparing Equations 1 and 7, it is seen that Fitts' law may be interpreted as a special case of the power law, with Fitts' law incorporating only the first two terms of the power series expansion of the power law. Fitts' law becomes an increasingly good approximation to the power law as the relative contributions to MT by the higher order terms in Equation 7 decrease, with the contribution of the n-order term being less than that of the first-order term if $\beta I_d < (n!)^{1/(n-1)}$ and less than that of the (n − 1)-order term if $\beta I_d < n$.

RESULTS AND DISCUSSION

The parameter estimates derived by fitting the four alternative models in Equations 1-4 to the Fitts' (1954) data using least-squares regression are given in Table 1. For the power models in Equations 3 and 4, logarithms were taken on each side of these equations prior to the regression analyses. However, the standard error of prediction (SEP) and the coefficient of multiple determination (R^2) were determined on the basis of the observed and the predicted (fitted) values of MT and not on the basis of the logarithms of these values, as is often and incorrectly done.

Table 1
Parameter Estimates and Values of Standard Error of Prediction (SEP) and Coefficient of Multiple Determination (R^2) for the Alternative Models Based on Fitts' (1954) Data

Model	E	a	b	c	SEP	R^2
			Tapping (1-oz Stylus)			
Fitts	1	.013	.095		.030	.966
Welford	2		.121		.037	.948
Present	3	.172	.358	−.367	.019	.987
Present	4	.171	.362		.019	.987
			Tapping (1-lb Stylus)			
Fitts	1	−.006	.105		.036	.960
Welford	2		.129		.037	.959
Present	3	.171	.385	−.374	.022	.985
Present	4	.172	.379		.022	.985
			Pin-Transfer Task			
Fitts	1	.022	.086		.058	.890
Welford	2		.104		.065	.860
Present	3	.218	.175	−.288	.033	.963
Present	4	.243	.218		.054	.902
			Disk-Transfer Task			
Fitts	1	.150	.090		.066	.844
Welford	2		.128		.090	.705
Present	3	.368	.227	−.107	.023	.980
Present	4	.383	.167		.064	.851

Note—E = equation number (see text). The unit of movement time used is seconds.

Although each of the four models provides significant ($p \ll .05$) fits to the experimental data, both of the two forms of the power model achieve fits superior to either Fitts' (1954) model or Welford's (1968, 1976) variant of it. Fitts' model tends to be superior to Welford's. It is also seen from Table 1 that for Fitts' "standard" reciprocal tapping task with both a light and a heavy stylus, the parameters b and −c in Equation 3 are nearly identical, and hence the simpler two-parameter model in Equation 4 provides the same fit as the model in Equation 3, both of which explain nearly 99% of the variation in MT for the different values of A and W, compared with about 96% and 95% for Fitts' and Welford's models, respectively. However, for the pin-transfer and disk-transfer tasks, it is apparent from Table 1 that the contributions to MT by A (i.e., movement amplitude) and by W (i.e., tolerance or clearance between pins and holes and between disks and pins) or 1/W were substantially different and in reverse order for the two tasks. The three-parameter power model in Equation 3 is by far the most appropriate one for these two tasks. On the basis of Wilcoxon matched-pairs signed-ranks one-tailed tests, the absolute residuals for the power model in Equation 3 were significantly smaller than those for Fitts' model, with T = 17, n = 20, and $p < .005$ and T = 13, n = 16, and $p < .005$ for the pin-transfer and disk-transfer tasks, respectively. However, for the reciprocal tapping tasks with a light and a heavy stylus, the absolute residuals were not significantly different for the models in Equations 1 and 3, with T = 57, n = 16, and $p > .05$ and T = 38, n = 16, and $p > .05$ for the light and the heavy stylus, respectively.

An important implication from the power law relates to the derivative $dI_d/d(MT)$. It follows from Equation 1 that this derivative is equal to 1/b and, as mentioned in the introduction, that this quantity is typically and incorrectly being interpreted by various researchers as a measure of information rate or rate of change of information with respect to MT. From the parameter estimates given in Table 1, this quantity is seen to range in value from 9.5 to 11.6 for the four data sets of Fitts (1954). However, from the exponential version of the power law given in Equation 6, which provides fit to each data set identical to that of Equation 4 and given in Table 1 for Fitts' data, it follows that the derivative $dI_d/d(MT)$ is an exponentially decreasing function of I_d. Thus, in addition to I_d being an invalid information-theory measure, its fixed rate of change with respect to MT as implied by Fitts' logarithmic law also appears to be of doubtful validity, since the power law, which seems to be superior to that of Fitts, indicates otherwise. While this assertion is true for Fitts' data, additional empirical analyses are needed to establish the general validity of this assertion and of the power law in general (see Footnote 1).

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NOTES

1. A project is presently being planned by this author with the objectives of (1) reanalyzing all previously published data using the Fitts' paradigm to provide a general empirical validation of the power law and (2) producing an up-to-date bibliography on the use of Fitts' paradigm and law. This author would greatly appreciate receiving any information about such applications, and in particular those having been reported in conference proceedings and unpublished reports, theses, and so on.

2. The exponential function e^x has the infinite power series expansion $e^x = 1 + x + x^2/2! + x^3/3! + \ldots$, which converges for any x in the interval $-\infty < x < \infty$.

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