Subjects' ability to use functional rules*

BERNDT BREHMER[†]

University of Umeå, S-901 87, Umeå, Sweden

Ss' ability to assign numbers to lines according to four different rules—positive linear (PL), negative linear (NL), U-shaped (U), and inversely U-shaped (IU)—was studied. The Ss were found to be more skilled in using the PL rule than all other rules and more skilled in using the NL rule than the two nonlinear rules. Implications of these results for functional and correlational learning, as well as for psychophysics, are discussed.

Functional and correlational learning tasks require the S to learn to infer the value of a scaled criterion variable from that of one (or more) scaled cue variables. To perform efficiently in such tasks, the S must (1) discover the rule relating cue to criterion variable, and (2) be able to use the rule correctly when making his predictions of the criterion values from the cue values. Although it seems reasonable to assume that problems in meeting the first requirement, i.e., to discover the rule, are probably responsible for most of the variation among rules with respect to learning difficulty (see Björkman, 1965; Brehmer, 1969; Carroll, 1963; Eisler & Spolander, 1970; Hammond & Summers, 1965; Naylor & Clark, 1968; Summers & Hammond, 1966; Summers, Summers, & Karkau, 1969), some results also suggest that there are differences among rules with respect to Ss' ability to use the rules. Thus, in the studies by Brehmer (1971), Sheets & Miller (1971), and Deane, Hammond, & Summers (1970), the learning of tasks with nonlinear rules was found to be slower than the learning of tasks with linear rules, despite the fact that the Ss were informed of the nature of the rules before the experiment. However, these studies were multiple-cue probability learning studies, and it is therefore not possible to decide from the results whether or not the variation among experimental conditions was due only to differences in Ss' ability to use different kinds of rules. It may also be that the Ss could not use outcome feedback to determine the validity of the various cues as efficiently when the tasks had nonlinear rules as when the rules were linear rules. The purpose of the present experiment, therefore, is to compare Ss' ability to use different kinds of functional rules under nonlearning conditions.

DESIGN

Four different functional rules were chosen for the experiment: a positive

linear function, $Y = X$ (PL); a negative linear function, $Y = A - X$ (NL); a symmetric U-shaped function, $Y = (X - \bar{X})^2$ (U); and a symmetric inversely U-shaped function, $Y = A - (X - \bar{X})^2$ (IU). These four functions cover most of the kinds of rules actually used in studies on functional and correlational learning.

Each S received all four functions. Different orders were used for different Ss. These orders were constructed in such a way that one-fourth of the Ss received each rule in each ordinal position.

This design makes it possible not only to investigate whether there are differences among functions with respect to difficulty, but also whether there are nonspecific transfer effects, i.e., whether the ability to use a given rule improves as a function of experience in using functional rules. It does not permit an analysis of rule-specific learning effects, however, since the Ss received each rule only once. This is no serious limitation, since previous studies show that all of the above functions can be learned.

PROCEDURE

The Ss were informed that the experiment was designed to study the ability of people to follow different kinds of rules. Thus, they would be shown different kinds of functional rules, and their task was to use these rules in assigning numbers to lines. The lines would vary in length from 1 to 20 cm. They were allowed to use any number between 0 and 40. The Ss were then shown a graph illustrating the kind of rule they were to use: positive linear, negative linear, U-shaped, or inversely U-shaped.

The Ss were allowed to study the graph for as long as they considered necessary. When they had done this, they were given a booklet containing cue values in the form of lines. For each function, they were required to assign a number to each of 20 different cue values. These cue values consisted of all 20 lengths possible by varying the length of the line from 1 to 20 cm in steps of 1 cm. The order of the lines was randomized, and different random orders were used for

different Ss and functions. The Ss were allowed to work at their own pace. When they had finished the 20 cue values for one rule, they were shown a graph illustrating the next rule to be used, and they then responded for this rule, and so on, until they had finished all four rules. When assigning the numbers to the cue values, the Ss were not allowed to look at the graphs.

RESULTS

For the linear rules, the product-moment correlation between the cue values and each S's responses was computed. For the nonlinear rules, the correlation between each S's responses and $(X - \bar{X})^2$, i.e., the squared difference between each cue value and the mean cue value, was computed. The results are shown in Fig. 1. Before the computation of the means shown in Fig. 1, the negative signs for the correlation coefficients in the NL and IU conditions were changed to positive signs, and the positive signs in these conditions changed to negative signs, so that all conditions could be compared directly with respect to the magnitude of the correlation.

The correlation coefficients were transformed to Fisher's z scores, and a one-way analysis of variance was performed for each block. The results of these analyses were similar in that all analyses yielded a significant ($p < .01$) effect of rules ($F = 30.86$, $F = 35.06$, $F = 17.46$, and $F = 30.31$, $df = 3/44$, for Blocks 1, 2, 3, and 4, respectively). The results of the individual comparisons among means were also similar for the four blocks. The Newman-Keuls tests indicated that at the .05 level of significance, performance in the PL condition was superior to that in the other three conditions in all four blocks. Performance in the NL condition was superior to that in the IU and U conditions. In no block was the difference between the U and IU conditions reliable.

To test for learning effects, a one-way analysis of variance was performed for each rule condition. Only in that performed on the data from the U condition was the blocks effect significant ($F = 3.66$, $df = 3/44$, $p < .05$).

DISCUSSION

The results of this experiment clearly show that there are differences among rules with respect to how difficult they are to use. Linear rules are easier to use than nonlinear rules and positive linear rules are easier than negative linear rules. There seems to be some nonspecific transfer effects, but these are small and reach significance only for the U rule.

The low correlations for the

*This study was supported by a grant from the Swedish Council for Social Science Research.

Fig. 1. Correlation between Ss' responses and cue values (for linear rules) and $(X - \bar{X})^2$ (for nonlinear rules) as a function of blocks.

nonlinear functions were due to three things. First, the Ss did not make their functions perfectly symmetrical, i.e., they could not find the correct mean of the cue values. Second, and perhaps more important, the Ss were more irregular in the nonlinear conditions than in the linear conditions. That is, the variation around the regression line was greater in the nonlinear conditions than in the linear conditions. Third, there were pronounced end effects: the Ss tended to assign the same number to the lowest three or four cue values and to the highest three or four cue values, thus making their functions look bell-shaped rather than U-shaped or inversely U-shaped. These end effects were present to some extent also in the linear conditions. The difference between the PL and NL conditions was, however, mainly due to the fact that the variation around the regression line was greater in the NL conditions than in the PL conditions.

The fact that the Ss in the NL condition did not perform as well as those in the PL condition is important, since it suggests that the difference between tasks with a positive linear cue-criterion relation and tasks with negative linear cue-criterion relations, found by Björkman (1965) and by Naylor & Clark (1968), is not to be explained in terms of the Ss' ability to discover the correct rule only. The hypotheses, advanced by Naylor and Clark, that the Ss have a "positive set" which causes them to assume a positive linear relation between cue and criterion also when it is, in fact, negative is thus not the only explanation for the differences between the positive and negative linear conditions in their experiment. The Ss' skill in using the two kinds of rules has to be considered as well. Or, to put matters somewhat differently, low achievement in a functional learning task does not necessarily mean that the Ss have not discovered the correct rule. Conversely, just because the Ss have the correct rule, one cannot expect that they will perform perfectly, especially if the rule for the task is a negative linear function or a U-shaped function of some sort.

The present results are important also for other areas of psychology which require the Ss to make numerical responses. Thus, the fact that Ss are less skilled in using NL rules than PL rules may explain why magnitude estimations of an attribute (e.g., darkness) are not perfectly related to estimations of the inverse of the attribute (e.g., lightness) (e.g., Torgerson, 1960).

REFERENCES

BJÖRKMAN, M. Learning of linear

- functions: Comparison between a positive and a negative slope. Reports from the Psychological Laboratories, University of Stockholm, No. 183, 1965.
- BREHMER, B. Cognitive dependence on additive and configural cue-criterion relations. *American Journal of Psychology*, 1969, 82, 490-503.
- BREHMER, B. Effects of cue validity and function form upon interpersonal learning of inductive inferences. *Umeå Psychological Reports*, No. 34, 1971.
- CARROLL, J. D. Functional learning: The learning of continuous functional mappings relating stimulus and response continua. Educational Testing Service, Princeton, Research Bulletin RB-63-26, 1963.
- DEANE, D. H., HAMMOND, K. R., & SUMMERS, D. A. Acquisition and application of knowledge in inference tasks involving nonlinear relations. Institute of Behavioral Science, University of Colorado, Program of Research on Human Judgment and Social Interaction Report No. 131, 1970.
- EISLER, H., & SPOLANDER, K. On the sign of slope in the learning of linear functions. *Scandinavian Journal of Psychology*, 1970, 11, 176-184.
- HAMMOND, K. R., & SUMMERS, D. A. Cognitive dependence on linear and nonlinear cues. *Psychological Review*, 1965, 72, 215-224.
- NAYLOR, J. C., & CLARK, R. D. Intuitive inference strategies in interval learning tasks as a function of validity magnitude and sign. *Organizational Behavior & Human Performance*, 1968, 3, 378-399.
- SHEETS, C., & MILLER, M. J. The effect of cue-criterion function form on multiple cue probability learning. *American Journal of Psychology*, 1971, in press.
- SUMMERS, D. A., & HAMMOND, K. R. Inference behavior in multiple-cue tasks involving both linear and nonlinear relations. *Journal of Experimental Psychology*, 1966, 71, 751-757.
- SUMMERS, S. A., SUMMERS, R. C., & KARKAU, V. T. Judgments based on different functional relationships between interacting cues and a criterion. *American Journal of Psychology*, 1969, 82, 203-211.
- TORGERSON, W. S. Quantitative judgment scales. In H. Gulliksen and S. Messick (Eds.), *Psychological scaling*. New York: Wiley, 1960. Pp. 21-31.