

Intuitive estimation of means¹

LEE ROY BEACH AND RICHARD G. SWENSON

THE UNIVERSITY OF MICHIGAN

Three groups of Ss made intuitive estimates of the means of lists containing 3, 5, and 7 numbers, symmetric and skewed distributions, and low, medium, and high variance. Accuracy was found to be extremely high but it decreased as the number of numbers and their variance increased.

Experiments in which Ss decide whether means of samples of numbers differ from one another, or differ from some specified value, demonstrate that decisions based on intuitive statistical processes are influenced by the same variables that influence decisions based on formal statistical procedures. Increased sample size, decreased variance, and large differences between the means, or between the sample means and the specified value, all increase Ss' confidence in their decisions (Irwin, Smith, & Mayfield, 1956; Irwin & Smith, 1956, 1957; Little & Lintz, 1965). Because these decisions must be based on some intuitive assessment of the means it is reasonable to expect S to be able to make fairly accurate estimates of means. Moreover, the accuracy of these estimates should be influenced by such variables as the number and variance of the data on which the estimates are based.

Spencer (1961, 1963) examined direct estimates using lists of 10 or 20 two-digit numbers that had high or low variance and symmetrical or skewed distributions. The Ss were asked to estimate the averages of the numbers or the numbers that best characterized the lists. The primary finding was that the estimates were extremely accurate. Error apparently was largest for lists with 20 numbers, large variance, and skewed distributions. However, these results must be viewed cautiously because the experimental variables were not independent. Number of numbers and variance were confounded for symmetrical distributions, where they both had statistically significant effects. These two variables were independent for skewed distributions, where only variance significantly influenced accuracy. Because of this it is difficult to know which variable has an effect or if they both do. The finding that skewness influences accuracy of estimation is also difficult to evaluate. The analysis leading to this conclusion used the true means of the lists as the criterion for determining accuracy. But Ss were not specifically instructed to estimate means. And, in the subsequent discussion of the data, evidence was presented that showed only about 36% of the Ss' responses appeared to be estimates of the means. About 55% of the estimates appeared to be of the medians or midranges. Therefore, it is not surprising that estimates for skewed distributions were not similar to the means.

This experiment is similar to Spencer's except that number of numbers, variances, and distribution shapes

were all varied independently and Ss were instructed specifically to estimate means. For three different groups of Ss the mean was defined as (1) a balance point on the number scale, (2) a value that minimizes squared error, and (3) as the sum of the numbers divided by the number of numbers. On the basis of the emphasis on differences between the mean and the numbers and upon the abstract nature of the concepts involved, it was expected that accuracy would be greatest for the first group, next for the second group, and least for the group that received only the computational formula.

Method

Apparatus. Numbers ranging between 0 and 99 were listed in random order on 22 in. x 17 in. sheets of paper. The lists were the 27 possible combinations of 3, 5, or 7 numbers, standard deviations of (approximately) 6.50, 11.00, or 26.50, and distributions that were symmetrical and normal, negatively skewed (-5.71 through -20.10) or positively skewed (5.78 through 19.88). The true means ranged from 40.5 to 75.2. For 17 of the 18 skewed distributions the true mean was not the same as any of the numbers of the lists.

Subjects. Three groups of 14 men university students served as paid Ss.

Procedure. All three groups of Ss were told that they would be shown lists of numbers and that they were to act as intuitive statisticians and attempt to estimate the means but not to try to compute them. The Balance group was to think of the mean as the balance point on a teeter-totter and examples were given. List numbers were to be regarded as unit weights at different places on the number scale and the mean was the fulcrum at which the scale balanced. The Squares group was told to minimize the sum of the squared differences between their estimates and the numbers on the list. Examples were given that emphasized the rapidity with which squared differences increase relative to unsquared differences. The Formula group was told that the mean of the distribution was the sum of the numbers divided by the numbers and computational illustrations were given.

Each list was displayed for 7 sec. Then Ss were given about 2 sec. to write their estimations on their answer sheets. All previous estimates were covered with another sheet of paper.

Results

To see whether Ss had followed instructions and estimated means rather than medians or midranges, the absolute deviations of Ss' estimates from the true means, medians, and midpoints were computed. As can be seen in Table 1, for all three groups the mean deviations from the list medians and midranges consistently were greater than the mean deviations from the list

Table 1. Mean absolute deviations of estimates from true means, medians, and midranges

Group	Mean	Median	Midranges
Balance	3.21	4.93	4.34
Squares	3.33	5.26	4.31
Formula	4.00	5.29	5.37
\bar{X}	3.51	5.16	4.67

means. As a baseline for evaluating these numbers, the mean absolute difference between the true means and the true medians of the lists was 3.38 and between the true means and the true midranges was 2.77. Highly accurate estimates of the means would make the mean deviation from the medians and midpoints approach these minimum values and approach zero for the means.

The first column of means in Table 1 suggests that the instructions had the expected effects on estimation accuracy. Although the trend may exist, an analysis of variance on absolute deviations of estimates from the true list means obtained no significant differences among the groups.

Table 2 contains the mean deviations for each experimental condition for the combined groups. An analysis of variance showed that the number of numbers and standard deviation both had significant effects on the accuracy of Ss' estimates. Skewness had no effect and none of the interactions were significant. Examination of signed deviations showed skewness had no systematic effects on the direction of estimation errors.

Table 2. Mean absolute deviations of estimates from true means for each level of the experimental conditions.

Number of Numbers		Standard Deviation		Skewness
3	2.88	Low 1.75	Neg.	3.43
5	3.50	Med. 3.47	Sym.	3.43
7	4.16	High 5.32	Pos.	3.67

Discussion

As in Spencer's study the most important result of this experiment is the high degree of accuracy evidenced in Ss' estimates. True means ranged from 40.5 to 75.2 and the overall mean absolute deviation of estimates from these values was only 3.51; the overall mean signed deviation was only -.916. These results also agree with Spencer's in that increased number of numbers and high variance both decrease the accuracy of estimation. Apparently, Ss have difficulty assimilating the information on the lists as the number of numbers and their dispersion increased. Contrary to Spencer's conclusions, however, number of numbers and variance did not interact with distribution shape. So, too, the finding that skewness has no effect on accuracy conflicts with the results reported by Spencer, in spite of the fact that skewness was generally greater in this study than in his. This difference probably results from our specifying that Ss estimate means rather than permitting them to select their own descriptive statistic.

References

- Irwin, F. W., & Smith, W. A. S. Further tests of theories of decision in an "expanded judgment" situation. *J. exp. Psychol.*, 1956, 52, 345-348.
Irwin, F. W., & Smith, W. A. S. Value, cost, and information as determiners of decision. *J. exp. Psychol.*, 1957, 54, 229-232.
Irwin, F. W., Smith, W. A. S., & Mayfield, J. F. Tests of two theories of decision in an "expanded judgment" situation. *J. exp. Psychol.*, 1956, 51, 261-268.
Little, K. B., & Lintz, L. M. Information and certainty. *J. exp. Psychol.*, 1965, 70, 428-432.
Spencer, J. Estimating averages. *Ergonomics*, 1961, 4, 317-328.
Spencer, J. A further study of estimating averages. *Ergonomics*, 1963, 6, 255-265.

Note

1. The research reported in this paper was supported by USPHS Fellowship MF-12, 744-01 from the National Institute of Mental Health and by the Air Force Systems Command, Electronic Systems Division, Decision Sciences Laboratory, under Contract No. AF19 (628)-2823 with the University of Michigan. The paper is identified as ESD-TR-66-79. Reproduction in whole or in part is permitted for any purpose of the U. S. Government.