Higher order experimental designs viewed as developments of the one-way analyses of variance

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The relationship of the higher order experimental designs to the one-way analysis of variance becomes apparent upon an examination of the number of levels in which each S appears. In both the simple randomized analysis of variance and the unmixed two-way design, each S is tested under only one level of the factor(s). In a higher order mixed design there is at least one factor in which each S is tested under every level, a characteristic which the mixed design shares with the one-way analysis of variance having repeated measurements.

The two fundamental types of one-way analyses of variance, namely, the simple randomized and the repeated measurements designs can be regarded as foundation stones upon which multi-factorial analyses are built. Unfortunately, the logical connection between these two basic approaches and the more complex designs is frequently lost.

In the simple randomized one-way analysis of variance (anova) there are different groups of Ss in each level. For example, to compare the efficacy of four methods of teaching mathematics, different Ss would be taught by each method as illustrated in Table 1.

In the one-way analysis of variance in which repeated measurements are used, the same Ss are tested in all levels. For example, to evaluate the effect of a drug on auditory thresholds, scores would be determined on each S for every drug level, as shown in Table 2.

In the illustration of a simple randomized design, forty Ss were tested once each; whereas, in the example of a repeated measurements design, four evaluations or scores were determined on each of 10 Ss. The repetition of evaluations on the same Ss is characteristic of the latter analysis, hence the designation "repeated measurements" to distinguish that approach.

In higher order designs the experimenter is interested in the effect of two (or more) variables operating simultaneously. These multi-factorial analyses are logical extensions of the basic types of one-way anovas. Most two-way analyses, for instance, fall into one of the following categories: (1) unmixed designs having two between treatments, or (2) repeated measurements designs having one between treatment and one within treatment. For brevity, this variety will be referred to as a mixed design.

In an unmixed setup containing two between factors, every S is tested under one level of each factor, an approach built upon the simple randomized design. Assume, for example, that the experimenter who is investigating the effectiveness of four methods of teaching mathematics wants to determine whether the various methods are equally efficacious with ninth grade students and with tenth grade students. He now has a two-factor design of Teaching Methods by Grade Levels in which different Ss would be used in every level of each treatment, as indicated in Table 3.

In both the simple randomized one-way analysis of variance and the unmixed two-way design there are different Ss in each treatment level. Viewed from this perspective, the unmixed two-way appears to consist of two independent simple randomized one-way anovas put together (one on 9th grade students and the other on 10th grade students). As a result, 80 Ss are tested once each.

Mixed designs are built upon the repeated measurements approach. In the example of drug effects on auditory thresholds, assume there is reason to believe that Drug X may have a differential effect depending on the age of the person tested. It is decided to use adults who are 20 and 30 years old. Adding this treatment produces a Drugs by Age setup which is a mixed design having the S distribution given in Table 4.

Table 1
Simple Randomized One-Way ANOVA

Methods of Teaching Mathematics			
Method 1	Method 2	Method 3	Method 4
S1*	S11	S21	S31
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
S10	S20	S30	S40

*S=Subject

Table 2
One-Way ANOVA with Repeated Measurements

Drug X			
100 mg	150 mg	Placebo	No Drug
S1	S1	S1	S1
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
S10	S10	S10	S10

Table 3
Unmixed Design with Two Between Treatments

Methods of Teaching Mathematics				
	Method 1	Method 2	Method 3	Method 4
9th	S1	S11	S21	S31
	.	.	.	.
	.	.	.	.
	.	.	.	.
	.	.	.	.
	S10	S20	S30	S40
10th	S41	S51	S61	S71
	.	.	.	.
	.	.	.	.
	.	.	.	.
	.	.	.	.
	S50	S60	S70	S80

Table 4
Two-Way Mixed Design

Drug X				
	100 mg	150 mg	Placebo	No Drug
20 yr old	S1	S1	S1	S1
	.	.	.	.
	.	.	.	.
	.	.	.	.
	.	.	.	.
	S10	S10	S10	S10
30 yr old	S11	S11	S11	S11
	.	.	.	.
	.	.	.	.
	.	.	.	.
	.	.	.	.
	S20	S20	S20	S20

The two-way mixed design contains one between factor and one within factor. Each S is tested under every level of the within factor but only one level of the between factor. In the example cited, therefore, the within factor is Drug X, the between factor is Age. This design can be regarded as an outgrowth of the repeated measurements approach inasmuch as it involves the amalgamation of two independent one-way repeated measurements designs (one on 20-year-old Ss and the other on 30-year-old Ss). As a result, 20 Ss would be tested

four times each.

In the construction of three-way analysis of variance the same logical connection with the two basic types of one-way analyses can be perceived. A three-way unmixed design involves the addition of another between factor with a third set of different Ss in each level. A three-way mixed design contains one of the following structures: (1) one within and two between factors, or (2) one between and two within factors. In either of these instances the same principle of repeated measurements is found.