Reply to Davis and Hart: Unlikely confidence in the size of a sample

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The comments made by Davis & Hart (1968) concerning a study by Pitz (1967) appear to reflect a misunderstanding of the use of likelihoods in statistical decision theory. (See Chernoff & Moses, 1959, Chap. 9.) A *likelihood* is the joint probability of a sequence of observations, $X_1, X_2, \dots, X_n$, conditional upon some population parameter, π . For the case in question, a sequence of red and blue balls is generated according to a Bernoulli process, where the probability of a red ball is π . In a sequence of N events, let there be R red balls and B blue balls; the likelihood of this sequence is

$$\Pr(X_1, X_2, \dots, X_n | \pi) = \pi^R (1 - \pi)^B.$$

If π is completely unknown, a widely used method for estimation is to take the value of π that maximizes the likelihood defined above. This *maximum likelihood estimate* is

$$P = R/(R + B) = R/N,$$

and its precision is a function of N . However, in the original study by Pitz (1967), Ss knew that π was either 0.6 or 0.4. In this event, the estimate P is irrelevant. Instead, a decision concerning π should be made on the basis of the *likelihood ratio*, i.e., the ratio of the two relevant likelihoods. Thus,

$$\frac{\Pr(X_1, X_2, \dots, X_n | \pi = 0.6)}{\Pr(X_1, X_2, \dots, X_n | \pi = 0.4)} = \left(\frac{0.6}{0.4}\right)^R \left(\frac{0.4}{0.6}\right)^B = \left(\frac{0.6}{0.4}\right)^{R-B}$$

The importance of the likelihood ratio is that it summarizes all of the relevant information in a sequence, and may be thought of as the diagnostic impact of that sequence. Thus, for this particular problem, only the difference $R - B$ is relevant to a decision.

If one adopts a Bayesian point of view (Edwards, Lindman, & Savage, 1963; Pitz, in press), one may convert a likelihood ratio into a posterior probability. This conversion is not essential to the

conclusions of the original study, but it helps to make the point clear. When the two values of π are initially equally likely, we have

$$\Pr(\pi = 0.6 | X_1, X_2, \dots, X_n) = \frac{\left(\frac{0.6}{0.4}\right)^{R-B}}{1 + \left(\frac{0.6}{0.4}\right)^{R-B}}.$$

Optimally, then, one's confidence in a decision should be a function of $R - B$ only, not of N or P , except insofar as they are related to the difference. If a S's confidence varies as a function of N for a constant $R - B$, as happened, that S is being nonoptimal. On the other hand, confidence certainly did increase as the absolute difference increased, as it should. At no point in the original paper was the statement made that a person becomes less confident as he gathers more evidence. The point should be made, however, that no reasonable individual—S or statistician—should allow the sample size to affect his judgment independently of the likelihood ratio. Whether or not Ss do behave this way is another question.

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