Some geometric bases for perceived slant¹

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Fifty-seven trapezoids varying along dimensions of length, height, and shape were presented to 16 Ss. Each S estimated the slant of the stimulus in three dimensions which produced each trapezoidal two-dimensional projection. A multiple regression equation was derived from the data showing that the perceived slant around a vertical axis was a function of the difference between the width of the long vertical end and the short vertical end, the length of the stimulus and RMH. Other possible formulations were discussed.

In recent years, studies by Flock and his associates (Flock, 1964; Flock & Moscatelli, 1964) have attempted to quantify the physical attributes of texture which lead to a perception of slant. Very little has been done in the way of such an attempt with stimuli having no texture, such as slanted rectangles. A number of researchers (Clark, Smith, & Rabe, 1955, 1956; Smith, 1956, 1964) have pointed to outline convergence as a monocular depth cue. However, others have denied that outline convergence could act as a cue in and of itself (Gibson, 1951; Beck & Gibson, 1955). In none of these cases has the author clearly indicated in terms of physical dimensions what outline convergence is. A number of combinations of physical dimensions, present on the picture plane when a rectangle is rotated, are correlated with the slant of the stimulus. However, if a series of stimuli are presented to a S, only a single family of equations are perfectly correlated with the actual amount of slant unless several variables are held constant.

Consider, for example, a rectangle rotated about a vertical axis to angle δ . The nearest end has a visual angle of ω , the farthest end a visual angle of ω' . If one now draws a line from the eye perpendicular to the picture planes, somewhere between the near and far end of the projection of the slanted rectangle, there will be an angle, γ , between the intersection of the line and the plane, and a line drawn from the eye to the near edge of the stimulus. There will be a similar angle, α , between the perpendicular and a line to the far edge. These four values can be used to give the equations for δ , the slant of the rectangle away from the picture

plane. These equations are shown in Table 1. Three different assumptions are made about the eye in the equations: I, a flat rectangle; II, a spherical eye with the nodal point in the center of the sphere; and III, a spherical eye with the nodal point 7.20 cm from the cornea and 16.92 cm from the retina as in Gullstrand's schemata eye (Bennett & Francis, 1962). Note that if α , γ , and ω are held constant as in the early studies by Dunn (1966) and Dunn & Thomas (1966) the sole variable is ω' . The equations become:

[1] $\delta = \frac{K_1 - K_2 \omega'}{K_2 + K_4 \omega'}$, where K_1 , K_2 , K_3 , and K_4 are constants. Angles are used instead of tangents, since when very small they are essentially equal. In the case where γ is set at zero, as Dunn has done, the equation becomes:

[2] $\delta = K_1 - K_2 \omega'$, where K_1 and K_2 are each a constant. When all these variables are varied, an important question becomes to what combination of the S attends. One can then perhaps define a convergence cue that has meaning and which can be related in various ways to the amount of slant. In the following study, ω , ω' , γ , and α have been varied. For the present γ has been held constant. An attempt was made to find a combination of these variables which would optimally predict a slant perception. In addition, RMH (Dunn & Thomas, 1966) was varied since it has been shown to independently contribute to the perception of slant.

Subjects

The Ss were 16 Psychology students. All Ss were paid for their services.

Apparatus

The stimuli consisted of 57 trapezoidal transparencies varying in α , ω , ω' , and RMH (see Table 2). All were projections of rectangles rotated about a vertical axis. The stimuli were presented in a tachistoscopic device described elsewhere (Dunn & Thomas, 1966). A rectangular stimulus with $\omega = \omega' = .0417$ and $\alpha = .0833$ was also used.

Procedure

A separate, balanced, incomplete block design was used on each group of eight Ss (Cochran & Cox, 1957,

Table 1. Equations Relating Outline Convergence to Perceived Slant

Assumption about eye	$\tan \gamma \neq \tan \alpha$	$\tan \alpha = \tan \gamma$	$\tan \gamma = 0$	$\tan \alpha = 0$
I	$\frac{\omega - \omega'}{\omega \tan \alpha + \omega' \tan \gamma}$	$\frac{\omega - \omega'}{\tan \alpha (\omega + \omega')}$	$\frac{1}{\tan \alpha} \left(\frac{1 - \omega'}{\omega} \right)$	$\frac{1}{\tan \gamma} \left(\frac{\omega - 1}{\omega'} \right)$
II	$\frac{\omega \sec \gamma - \omega' \sec \alpha}{\omega \sec \gamma \tan \alpha + \omega' \sec \alpha \tan \gamma}$	$\frac{\omega - \omega'}{\tan \alpha (\omega + \omega')}$	$\frac{1}{\tan \alpha} \left(\frac{1 - \omega' \sec \alpha}{\omega} \right)$	$\frac{1}{\tan \gamma} \left(\frac{\omega \sec \gamma - 1}{\omega'} \right)$
III	$\frac{\omega \sqrt{\sec^2 \gamma + .57} - \omega' \sqrt{\sec^2 \alpha + .57}}{\omega \tan \alpha \sqrt{\sec^2 \gamma + .57} + \omega' \tan \gamma \sqrt{\sec^2 \alpha + .57}}$	$\frac{\omega - \omega'}{\tan \alpha (\omega + \omega')}$	$\frac{1}{\tan \alpha} \left(\frac{1 - \omega' \sqrt{\sec^2 \alpha + .57}}{\omega} \right)$	$\frac{1}{\tan \gamma} \left(\frac{\omega' \sqrt{\sec^2 \gamma + .57} - 1}{\omega} \right)$

Table 2. Values of the Physical Dimensions of the Stimulus

ω	ω'	α	RMH
.0417	.0417	.0833	.0137
.0365	.0365	.0729	.0117
.0312	.0339	.0625	.0104
.0260	.0326	.0521	.0098
.0208	.0312	.0517	.0078
	.0299		.0052
			.0039
			.0026

p. 533). Trials substituted for blocks. Each S received all 57 stimuli four times in a counterbalanced order. Between each presentation of the 57 stimuli, rectangular stimuli was presented. Each S served for an hour on each of five days.

Stimulus presentations were made in the following manner: 10-sec. presentation of the trapezoid, 30-sec. response and rest period, 10-sec. presentation rectangle, 30-sec. response (rectangle was responded to, last response was not recorded) and rest period, repeat cycle.

The Ss were instructed to respond to the way the stimuli appeared rather than to their calculated estimate of the slant by use of the rod. If the S felt that there was a discrepancy between the way the stimulus appeared and the way he thought it was actually oriented, he was permitted to make a second response, which did not count. Initial instructions were given by showing S a swinging door opened both ways and indicating to him how slanted it was by use of the rod.

Results

Each group of eight Ss was adjusted separately for confounding. The only significant result was the between stimuli ($p < .01$, $F = 6.58$). Correlations were obtained between the dependent and independent variables. Used as independent variables were: RMH, RMH/α , α , ω , ω' , ω/α , ω'/α , $1/\alpha(1 - \omega'/\omega)$, ω'/ω , and $\omega'/\alpha\omega$. From this a regression equation was derived:

$$[3] \quad \delta = 22.35 \frac{(\omega - \omega')}{\alpha} + 4.7 \frac{\text{RMH}}{\alpha}$$

When the above equation is adjusted for the number of variables, it predicts 85.2% of the variance in the averaged response to the 57 stimuli. Two other equations can be written which predict an equal amount of variance:

$$[4] \quad \delta = 19.9 \frac{(\omega - \omega')}{\alpha} + 4.7 \beta/\alpha$$

$$[5] \quad \delta = 24.6 \frac{(\omega - \omega')}{\alpha} + 4.7 \tau/\alpha$$

Where β/α represents the slope of the lower edge and τ/α the slope of the upper edge of the trapezoid, these two equations are possible due to the relationship:

$$[6] \quad \text{RMH}/\alpha = \frac{\beta + \tau}{2\alpha}$$

Discussion

From the above results it appears clear that the

last predictive equations are not the same as the actual equations relating the physical dimensions to actual slant. In fact, $(\omega - \omega')/\alpha$ by itself predicts 53.8% of the variance, while $1/\alpha(1 - \omega'/\omega)$ (the more complex formula did not give different enough predictions in this experiment to significantly affect the correlations) predicts only 35.4% of the variance by itself. Other combinations of dimensions related to convergence all predict less than 35.4% of the variance.

Notice that if one were to hold ω and α constant, equation 3 becomes:

[7] $\delta = (K_1 - K_2\omega) + K_3 \text{RMH}$, where K_1 , K_2 , and K_3 are constants. The first half of the equation is an equation which may be directly related to the actual slant, depending on the constants.

It is clear from equations 4 and 5 that the empirical equation for perceived slant is not unique. A number of factors might indicate a choice, but perhaps the best approach would be to ask Ss to what they attended. Rather uncontrolled questioning by E has indicated that perhaps equation 4 would be the first choice.

In conclusion, it is apparent that combination of physical variables predicting perceived slant is possible. However, a number of possible variables were not varied in the present study. Certainly γ should be presented at some value other than zero. One also needs to experiment with sufficiently large values of α and γ to distinguish between the assumptions I, II, and III.

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Note

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