Calculational subroutines for target width and movement amplitude corrections and normal distribution parameters

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Fitts (1954) proposed an index of difficulty (ID) metric which reflects the information generated in executing a movement of specified amplitude to a target of specified width. The formula developed by analogy to the information theory of continuous signals states that $ID = \log_2 2A/W$, where A is movement amplitude and W is target width. The validity of this metric rests on several well known assumptions about movement performance. In particular, the spatial distribution of movement endpoints is assumed to be normal, and centered about the middle of the "target" of the movement.

Welford (1968, p. 147) has pointed out that the information content of a normally distributed signal is approximately 4.1333σ (i.e., $\sigma\sqrt{2\pi e}$), where σ is the standard deviation of the distribution. Therefore, if target width is equal to 4.133σ , then target width accurately reflects the information in the distribution of movement endpoints, and ID will be properly calculated. In this case, about 96% of the movements should hit the target, since $\pm 2.067 \sigma$ (i.e., $4.133 = 2 \times 2.067$) includes 96% of the normal distribution. Thus, if more (or less) than about 4% errors are made, nominal target width is an inaccurate reflection of the information in the movement endpoint distribution and a correction is necessary to derive an "effective" target width equal to $\pm 2.067 \sigma$ for use in the ID formula above. For example, if more than 4% misses are made, effective target width would be wider than the nominal target width intended by the experimenter.

Furthermore, a person may consistently overshoot (or undershoot) the intended target, thus producing an effective movement amplitude longer (or shorter) than the amplitude intended by the experimenter. Thus, an additional correction must be applied before computing ID.

Typically, corrections are made by consulting tables of the normal distribution function and determining a z score associated with the probabilities of overshooting and undershooting the target. From these, both effective

target width and effective movement amplitude may be calculated. This is a laborious procedure when many trials and targets are involved. A subroutine, EFTARG, written in FORTRAN IV, automatically computes appropriate corrections. EFTARG is called with the number of on-target hits, the number of overshoots, the number of undershoots, and the measured target width (i.e., the nominal width of the target as specified by the experimenter in whatever length units are appropriate). EFTARG returns the location of the mean of the distribution of movement endpoints and an effective target width covering ± 2.067 of this distribution. The location of the distribution mean is specified as a distance to the nominal target center in the same units used to specify intended target width. If the distance is shifted toward the "overshoot" side of the target, this distance is positive; if it is on the "undershoot" side, this distance is negative. An error condition results if there are no overshoots or undershoots (since corresponding z scores would be plus or minus infinity). If this happens, the subroutine returns $-1 \times$ nominal target width and distribution mean location = 0 (i.e., centered on target midpoint).

The main subroutine, EFTARG, makes use of a second FORTRAN IV subroutine of general utility in other applications. This subroutine, NORMAL, calculates parameters of a normal distribution. It functions in two modes: (1) The user can pass a z score to the subroutine and NORMAL returns with the area under the normal density function to the left of the z score (i.e., the integral from $-\infty$ to the z score); (2) the user can pass an area under the density function (i.e., a probability), and the subroutine will return a z score such that the area to the left of the z score is the given probability. In both modes, the subroutine will also return the ordinate of the density function at the z score specified (or calculated in Mode 2).

To use this subroutine the user simply passes either a z score or a probability and the subroutine returns with the complementary value. This subroutine uses approximation techniques to derive its answers and the results are usually accurate to about five decimal places except beyond $+6$ and -6 standard deviations, where probabilities to the left of the z score are fixed at 1 and 0, respectively.

REFERENCES

- FITTS, P. M. The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, 1954, 47, 381-391.
WELFORD, A. T. *Fundamentals of skill*. London: Methuen, 1968.

Printouts of subroutines EFTARG and NORMAL are available upon request from James Knight, Department of Psychological Sciences, Purdue University, West Lafayette, Indiana 47907. These subroutines were developed under auspices of NIMH Grant MH-26302.

(Accepted for publication March 31, 1977.)

SUBROUTINE EFTARG (HITS, OVERS, UNDERS, SIZE, CENTER)

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C
C THIS SUBROUTINE CALCULATES AN EFFECTIVE TARGET WIDTH
C COVERING +- 2.01 STANDARD DEVIATIONS OF THE DISTRIBUTION
C OF MOVEMENT ENDPOINTS. SUBROUTINE ALSO CALCULATES THE
C LOCATION OF THE MEAN OF THIS DISTRIBUTION AS A DISTANCE
C FROM THE CENTER OF THE PHYSICAL TARGET INTENDED BY
C THE EXPERIMENTER. THESE CALCULATIONS ARE BASED UPON THE
C ASSUMPTION THAT THIS DISTRIBUTION IS APPROXIMATELY NORMAL.
C COMPUTATIONS USE THE PERCENTAGES OF UNDERSHOOTS AND OVERSHOOTS
C TO DETERMINE Z-SCORES CORRESPONDING TO TARGET EDGES.
C CALL WITH NUMBER OF HITS, OVERSHOOTS, AND UNDERSHOOTS, AS
C WELL AS MEASURED WIDTH OF TARGET. IF ANY OF THESE PARAMETERS
C ARE ZERO, AN ERROR CONDITION RESULTS AND ROUTINE WILL RETURN
C -SPECIFIED TARGET WIDTH AND DISTRIBUTION CENTER=0.
C IF DISTRIBUTION MEAN IS ON UNDERSHOOT SIDE OF TARGET,
C DISTANCE TO MEAN IS NEGATIVE: IF ON OVERSHOOT SIDE, POSITIVE.
C
C SUBROUTINE REQUIRES USE OF SUBROUTINE *NORMAL*
C
      CENTER=0.
      IF((OVERS.EQ.0).OR.(UNDERS.EQ.0).OR.(HITS.EQ.0.))
1    SIZE=-SIZE
      IF(SIZE.LE.0.) GOTO 100
C
C CALCULATE Z-SCORE TO OVERSHOOT SIDE OF TARGET
C
      PROB=1.-OVERS/(HITS+OVERS+UNDERS)
      ZS=0.
      CALL NORMAL (ZS,PROB,YORD)
      SDOVER=ZS
C
C CALCULATE Z-SCORE TO UNDERSHOOT SIDE OF TARGET
C
      PROB = UNDERS/(HITS+OVERS+UNDERS)
      ZS=0.
      CALL NORMAL (ZS,PROB,YORD)
      SDUNDER=ZS
C
C CALCULATE PERCENTAGE OF DISTRIBUTION SPANNED
C BY ACTUAL TARGET
C
      SPAN = SDOVER-SDUNDER
C
C CONVERT STANDARD DEVIATIONS TO PHYSICAL UNITS TO GIVE
C, FOR EXAMPLE, STANDARD DEVIATIONS PER INCH
C
      SD=SPAN/SIZE
C
C CALCULATE LOCATION OF END-POINT DISTRIBUTION FROM
C TARGET CENTER. USE TARGET EDGE CLOSEST TO DISTRIBUTION
C CENTER TO COMPUTE THIS VALUE SINCE THIS WILL GIVE
C MOST ACCURATE RESULT.
C
      X=ABS(SDOVER)
      Y=ABS(SDUNDER)
      IF(X.GT.Y) CENTER=-(SIZE/2+SDUNDER/SD)
      IF(X.LE.Y) CENTER=SIZE/2-SDOVER/SD
C
C CALCULATE EFFECTIVE TARGET SIZE COVERING
C +- 2.01 STAND. DEVS. OF END-POINT DISTRIBUTION
C
      SIZE=4.133/SD
100  RETURN
      END

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SUBROUTINE NORMAL (ZS,PROB,YORD)

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C
C ROUTINE TO COMPUTE PARAMETERS OF NORMAL DISTRIBUTION
C CALL WITH Z-SCORE (ZS) OR PROBABILITY TO LEFT OF A
C UNKNOWN Z-SCORE (PROB). OTHER PARAMETER SHOULD BE ZERO
C ROUTINE RETURNS COMPLIMENTARY VALUE AND ALSO ORDINANT
C OF NORMAL DISTRIBUTION AT SPECIFIED (OR CALCULATED) Z-SCORE.
C FOR Z-SCORES LESS THAN -6 ROUTINE RETURNS 0 FOR PROBABILITY
C FOR Z-SCORES GREATER THAN 6, ROUTINE RETURNS 1 FOR PROBABILITY.
C
C
      ILOOP=10
      IF (PROB.EQ.0.) ILOOP=1
      DELT=-1.
      X=0.
      DO 100 I=1,ILOOP
      DELT=-.1*DELT
110    X=X+DELT
      IF (PROB.EQ.0.) X=ZS
      Z=0.
      IF (X.EQ.0.) GO TO 2
      Z=1.
      Y=ABS(X)*.5
      IF (Y.GE.3.) GOTO 2
      IF (Y.GE.1.) GOTO 1
      W=Y*Y
      Z=((((((((0.000124818987*W
1- .001075204047)*W+0.005198775019)*W
2- .019198292004)*W+0.059054035642)*W
3- .151968751364)*W+0.319152932694)*W
4- .531923007300)*W+0.797884560593)*Y*2.
      GOTO 2
1    Y=Y-2.
      Z=(((((((((((-.000045255659*Y
1+ .000152529290)*Y-0.000019538132)*Y
2- .000676904986)*Y+0.001390604284)*Y
3- .000794620820)*Y-0.002034254874)*Y
4+ .006549791214)*Y-0.010557625006)*Y
5+ .011630447319)*Y-0.009279453341)*Y
6+ .005353579108)*Y-0.002141268741)*Y
7+ .000535310849)*Y+0.999936657524
2    PN=(1.0-SIGN(Z,X))*5
      PQ=.5-PN
      IF (PROB.EQ.0.) GOTO 100
      IF (DELT.LT.0.) GOTO 101
      IF (PN.GT.PROB) GOTO 110
      GOTO 100
101    IF (PN.LT.PROB) GOTO 110
100    CONTINUE
      IF (PROB.EQ.0) PROB=1.-PN
      IF (ZS.EQ.0) ZS=-X
      X=-5*(ZS**2)
      X1=2*3.1415927
      YORD=EXP(X)/SQRT(X1)
120    RETURN
      END

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