Visual discrimination thresholds for one- and two-dimensional Markov spatial constraints*

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Forced-choice discrimination thresholds were obtained for visual displays of a large number (1,000-3,600) of binary encoded elements with one- and two-dimensional Markov spatial constraints. Several factors which delimit such thresholds were explored. The primary determinant of sensitivity was the number of constrained elements within the display. Configurational features, such as the shape of the display have little influence on sensitivity. Thresholds for one-dimensional spatial constraints were consistently lower than for two-dimensional spatial constraints.

When faced with large masses of experimental observations, investigators often seek to "see a picture" of the underlying relationships, despite the myriad of techniques which quantitatively summarize the interaction among variables. The term within quotations implies that the understanding and communication of interrelationships is aided—nay, is often required—by representation in terms of an appropriate visual display. The achievement of the three-dimensional representation of the DNA molecule, for example, represented an important synthesis of a vast body of material in microbiology. A less exotic example is that of the city planner,¹ who searches for appropriate interrelations among transportation, communication, sanitation, population density, and zoning factors, etc., by examining overlay maps. Exactly why the large visual display is so effective for achieving "understanding" cannot be answered directly. One feature, undoubtedly, is the conversion of a stream of sequentially presented information over a long time period to a simultaneously presented display which can be successively interrogated with different hypotheses. Under restricted conditions, visual displays may be effective because they lend themselves to a "perceptual" evaluation, in that certain display features appear to be directly available to visual experience. Sensitivity for changes in spatial position, for example, is often dramatically

improved if conditions are introduced for the perception of motion (Pollack, 1972). Stated otherwise, visual displays may permit the "perceptual" appreciation of information which is only poorly appreciated on a "cognitive" level.

The previous remarks imply that people serve—in the language of modern Bayesian theorists—as "intuitive statisticians" in examining large masses of information from visual displays. The present tests were designed to explore some limits for deriving information from visual displays containing large numbers of statistically derived elements. Essentially, I sought to extend a previously crude analysis of the discriminability of one- and two-dimensional Markov spatial constraints, based upon a rating

procedure with a fixed set of displays (Pollack, 1971), to a more rigorously defined set of thresholds, and to explore a wide range of variables underlying such discrimination.

METHOD

Generation of

One-Dimensional Constraints

The production of visual displays with one-dimensional Markov constraints has been described by Pickett (1964) and by Julesz (1962) and was employed previously (Pollack, 1971a, b).² Binary-coded sequences were generated by specifying of the conditional repetition probability, $P(A|A)$, between successive positions. The top line of Fig. 1 represents photographs of 60×60 displays, varying in $P(A|A)$, with two display states: "dot" and "no dot or space." The top row is randomly selected without constraint. Within each column, each item, except the first, is defined by constrained random selection with a fixed level of $P(A|A)$. For example, if $P(A|A) = 1.0$, the entire column consists of dots or of spaces, depending upon the initial row element; if $P(A|A)$ is 0, the entire column consists of dots alternating with spaces or of spaces alternating with dots, depending upon the initial row element; and if $P(A|A)$ is 0.5, each item is independently and randomly selected.

If $0.50 < P(A|A) < 1.0$, there is a tendency to repeat the immediately previous state within each column; if 0

EXAMPLES OF 1-D AND 2-D MARKOV CONSTRAINTS

Fig. 1. Photographs of oscilloscope displays employing one-dimensional constraints and two-dimensional constraints. Differences in photographic density were absent on the original displays.

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Fig. 2. Detection thresholds above a chance reference level for one-dimensional constraints as a function of the number of elements in the width and in the length of the display. The one-dimensional constraints were in the vertical (length) direction. The duration of presentation in Figs. 2 and 5 was S-controlled.

$P(A|A) < 0.50$, there is the tendency to complement the previous state within each column. As suggested in the top line of Fig. 1, high $P(A|A)$ conditions lead to aggregated or "lumpy" textured displays; low $P(A|A)$ conditions lead to nonaggregated or "uniform" textured displays. Note that $P(A|A)$ is applied to the immediately previous element within each column, independently of the sequence leading up to that element ("path independence") and independently of the states of neighboring columns. A two-directional spatial display with one-dimensional constraints is obtained simply by concatenating a series of independently derived one-dimensional strings. In a display of L rows in length and W columns in width, if one row of W positions is generated by independent random selection, the remaining $(L-1)$ positions within each column are generated by constrained random selection with $P(A|A)$. The total number of constrained elements is $W(L-1)$.

Generation of

Two-Dimensional Constraints

The production of displays with two-dimensional Markov constraints has been previously described (Pollack, 1971a, b). The procedure is based upon a parity sum across local overlapping 2×2 groupings of four cells of the display. The method is an adaptation of a procedure of Julesz (1962) based upon a parity sum across local triangular groupings of three cells of the display. The parity sum is considered in the context of binary-coded elements. Let a dot be represented by a 1 and a space by a 0. An even parity sum consists of 0, 2, or

4 dots; an odd parity sum consists of 1 or 3 dots. If we have already specified three of the four positions of a 2×2 grouping, the fourth position is determined by the parity sum. For example, if there is 1 dot in three of the four positions, and the selected parity sum is odd, the fourth position will be a space; if the selected parity sum is even, the fourth position will be a dot. Note that it is unimportant where the initial dot was located within the original three positions. Displays with two-dimensional constraints, therefore, do not also introduce one-dimensional constraints. Note also that whether a given parity sum, e.g., an even sum, leads to a dot

or a space is determined by its neighbors.

The first column and the first row are obtained by unconstrained random selection, except for agreement at their point of overlap—at Matrix Position 1,1. The number of constrained elements is $(W-1)(L-1) + 1$. At the upper left intersection, three of four positions are specified. The fourth position—at Matrix Position 2,2—is determined by the parity sum. We can generate a sequence of parity sums to be applied to the display by specifying the probability of an even parity sum, $P(S_e)$. After the initial fourth position is determined, three of four positions

Fig. 3. Transformation of the data of Fig. 2 in terms of the number of constrained elements.

Fig. 4. Detection thresholds for one-dimensional Markov constraints, relative to a chance $P(A|A)_R = 0.50$. The four sections represent four exposure durations. The shape of the points reflect the distribution of elements within the display.

of the next local 2 x 2 cell are specified and the next fourth position—at Matrix Position 3,2—is determined by the parity sum. This process continues until the second row of the matrix is completed. The process continues at the intersection of the second row and the first column—at Matrix Position 2,3, etc.

In partial review, the spatial microstructure of visual displays can be controlled by variation in the probability of an even parity sum, $P(S_e)$, upon local overlapping 2 x 2 cells. As shown in the second line of Fig. 1, with photographs of 32 x 32 displays, low $P(S_e)$ levels lead to distributed or "lacey" displays; and, as shown in the third line of Fig. 1, high $P(S_e)$ levels lead to aggregated rectangular clumps of "squares" and "holes."

Procedure

Two experimental display systems were employed: a large (18 x 23 cm) display surface (Tektronix 611 with a P4 storage phosphor) driven by a PDP-8I computer for one-dimensional constraints, and a smaller (8 x 10 cm) display surface (Tektronix 602 with a fast P15 phosphor) driven by a PDP-9 computer for two-dimensional

Fig. 6. Increment detection thresholds for two-dimensional Markov constraints as a function of display duration (abscissa). The parameter is the reference level for the two-dimensional constraints.

constraints and for time-controlled tests with one-dimensional constraints. It was assumed, and experimental tests

verify, that the two surfaces were equivalent for long-duration displays with equivalent parameters. Unless specified otherwise, the interelement distance between successive columns was 0.77 mm and between successive rows was 0.83 mm with the 611 display, and was symmetrical at 1.17 mm with the 602 display. The viewing distance was about 75 cm. Binocular natural viewing was employed. The viewing environment was dimly illuminated at about 1 mL. Rapid plotting rates (9 microsec/point) were made possible by plotting the 18-bit word of the PDP-9 computer into simultaneous x- and y-coordinates of 9 bits each (Wojnarowski, Bachman, & Pollack, 1971).

Displays were presented in the format of a four-alternative forced-choice (4AFC) test. Four displays in a 2 x 2 arrangement were painted: three displays with a given reference probability of $P(A|A)_R$ or $P(S_e)_R$, as described above, and one display with a different probability. The task of the O was to indicate whether the northwest, the northeast, the southwest, or the southeast display was the "odd" display. Four response buttons, in the same 2 x 2 arrangement, were employed. An adaptive stimulus programming procedure (Taylor & Creelman, 1967) varied $P(A|A)_v$ or $P(S_e)_v$ to converge upon 50% correct in the 4AFC test. Although unlimited response time was available, rapid response was encouraged by a piece-work incentive. Immediate feedback was provided.

Fig. 5. Effect of reference $P(A|A)$ level on detection thresholds for increments (above the chance reference diagonal) and for decrements (below the chance reference diagonal). The insert graphs show how thresholds vary with the number of constrained elements for the associated reference conditions. The coordinates are of the normal probability grid.

Fig. 7. Decrement detection thresholds for two-dimensional Markov constraints as a function of display duration (abscissa). The parameter is the reference level for the two-dimensional constraints.

Each response triggered the generation of the subsequent display.

At least 15 university students each completed two thresholds under each of 800 experimental conditions for the one-dimensional constraints; at least 15 Ss each completed two thresholds under each of 1,250 experimental conditions for the two-dimensional constraints. The testing program was carried out over a 2-year span and represented approximately 6.2×10^4 thresholds. When the adaptive procedure called for $0 > P > 1.0$, one observation was presented at the limiting condition. An incorrect response at the limiting condition terminated the trial. When $< 10\%$ of the trials were terminated, thresholds represent geometric means across all obtained thresholds, except the terminated trials. With $> 10\%$ terminated trials, median thresholds, including the terminated trials, were obtained. Half-filled points represent 10%-20% terminated trials; filled points represent $> 20\%$ terminated trials.

RESULTS

One-Dimensional Constraints

One-dimensional constraint thresholds for displays consisting of dots and nondots will be expressed as $P(\cdot | \cdot)_v$, relative to a reference constraint of $P(\cdot | \cdot)_R$.

Role of display shape and number of display elements. Is equivalent discrimination obtained for displays

with a large number of short strings vs displays with a small number of long strings? Stated differently, when

one-dimensional constraints are imposed within columns, should we expect better discrimination with displays of many columns and few rows or with displays of few columns and many rows? Or, in the language of decision making, we ask, "Can observers aggregate smaller information contributions from many distributed informational sources as well as aggregating larger informational contributions from few sources?"

Figure 2 shows that discrimination improves—thresholds above the reference $P(A | A) = 0.50$ decrease—as either the number of strings increase or as the length of each string is increased. To a first approximation, Fig. 3 suggests that thresholds are determined by the total number of constrained points. Closer examination reveals only a slight advantage of the narrow-long displays (in the direction of the constraints) over the wide-short displays. Comparison of thresholds for displays of maximum length vs maximum width for a fixed total number of constrained points shows lower thresholds for displays of maximum length in seven out of nine comparisons, with one tie.

The near equivalence between thresholds for displays of maximum length and width, however, fails to reflect the greater subjective difficulty with displays of maximum width. My

Fig. 8. Summary of asymptotic increment (above the dashed reference line) and decrement (below the dashed reference line) thresholds from Figs. 6 and 7. Both coordinates are of the normal probability function.

Fig. 9. Comparison between detection thresholds for one- and two-dimensional Markov constraints. The left section represents threshold increments above an extreme and a chance reference level. The right section represents threshold decrements below an extreme and a chance reference level.

own reaction as an observer was that the task was more "cognitive" in collating the many short contingencies for displays of maximum width, and more "perceptual" in responding to the long string of contingencies for displays of maximum length. Translating the reactions of subjective difficulty into inspection time for the

Fig. 10. Detection thresholds for two-dimensional Markov constraints, relative to a chance $P(S_e)_R = 0.50$, for two forms of constraints. The left curve and left bottom abscissa represent two-dimensional constraints based upon four elements; the right curve and top abscissa represent two-dimensional constraints based upon three elements by the Julesz procedure. The shape of the points reflect the distribution of elements within the display.

Fig. 11. Detection thresholds for two-dimensional Markov constraints. As Fig. 10, except for a near-perfect reference level.

displays, we might anticipate that the wide-short displays require longer inspection times.

Role of exposure duration. Figure 4 considers thresholds for narrow-long and wide-short displays over a range of controlled exposure durations. The main point of Fig. 4 is that thresholds for the wide-short displays ("cognitive" collation of distributed contingencies) are nearly indistinguishable from those for the narrow-long displays ("perceptual" response to nondistributed contingencies) over a wide range of durations. Thus, the anticipated large

Fig. 12. Increment (+) and decrement (-) thresholds for one-dimensional constraints (left section) and two-dimensional constraints (right section) as a function of the conditional repetition probability of the initializing sequence (abscissa). The parameter is the exposure duration.

Fig. 13. Comparison between one- and two-dimensional Markov constraints. As Fig. 12, except that the abscissa is the first-order probability of a dot in the initializing sequences.

differences did not materialize.

Role of reference $P(A|A)$ level. Figure 5 examines the discrimination of increments in $P(A|A)_V$ (points above the dashed reference line) and of decrements in $P(A|A)_V$ (points below the dashed reference line) over a wide range of reference $P(A|A)_R$ levels. The axes are the normal probability grid. The inserts show that the effect upon discriminability of the number of constrained elements is roughly equivalent over a wide range of $P(A|A)_R$ levels. Discrimination of increments is slightly better than the discrimination of decrements. The empirical lines through all, except the extreme, points are almost parallel to the reference line. The departure is in the direction of larger deviations for low $P(A|A)_R$ values. Over the linear portions, the slopes are nearly unity; the distance at $P(A|A)_R = 0.5$ is about 0.16 in normalized, or d' , units for the increment and decrement functions, respectively.

Two-Dimensional Constraints

Two-dimensional constraints will be expressed in the figures as $P(\Sigma_e)_V$, relative to a reference constraint of $P(\Sigma_e)_R$.

Role of reference $P(S_e)$ and exposure duration. Figures 6 and 7

present discrimination thresholds for two-dimensional constraints as a function of viewing time. Figure 6 represents the discrimination of increments in $P(S_e)$; Fig. 7 represents the discrimination of decrements in $P(S_e)$. The abscissa is the duration of presentation. Successive trials were carried out for descending exposure durations within each reference $P(S_e)$. Threshold discrimination improves with longer durations. In many cases, there is little change beyond 90 display cycles of 18 msec (1.6 sec).

Figure 8 summarizes the thresholds of Figs. 6 and 7 for the longer durations. A double normal probability grid is employed, as in Fig. 5, to represent a wide range of conditions.

Substantially greater threshold changes are required with the two-dimensional constraints of Fig. 8 than for the one-dimensional constraints of Fig. 5. A direct comparison should be tempered by the fact that the one-dimensional constraints were imposed upon a 60 x 60 grid upon a storage scope with unlimited viewing time, while the two-dimensional constraints were imposed upon a 32 x 32 grid upon a fast decay scope with limited viewing time. The distance to the chance

dashed reference line in the vicinity of 0.50 is 0.32 d' units for increments and 0.39 d' units for decrements. The slopes of the corresponding functions are 0.90 and 1.05, respectively. Neglecting differences in the number of display elements, the results suggest that about twice the normalized difference is required for the discrimination of two-dimensional constraints as for one-dimensional constraints.

In separate tests, one- and two-dimensional constraints were tested with a 32 x 32 grid on the fast scope. Two different two-dimensional forms of constraint were studied: constraint within groups of four elements on a square, as in the other tests, and constraint within groups of three elements on a triangle, as generated by Julesz (1962). The former are coded by dots within the figure, the latter by tic marks. The results are presented in Fig. 9. The one-dimensional threshold differences are consistently smaller, relative to the reference levels, than the two-dimensional thresholds. Moreover, the effect of restricted duration is not uniform for the two forms of constraint.

Role of number of elements. Two-dimensional constraints were introduced into displays differing in terms of the number of elements. The tests of Fig. 10 parallel those of Fig. 4 with one-dimensional constraints. Thresholds from a chance reference level were obtained for two methods for producing two-dimensional constraints: the left curve represents the squared constraints produced by the present method; the right curve and top abscissa represent the triangular constraints produced by the Julesz method. Detection thresholds fall as either the display length or display width is decreased, being primarily dependent upon the number of constrained elements. Discrimination thresholds rise rapidly with fewer than 50 constrained elements in the present procedure and with fewer than 150 constrained elements with the other procedure.

Figure 11 parallels Fig. 10 in showing that detection thresholds from a near-perfect reference for two-dimensional constraints are determined primarily by the number of constrained elements and are relatively independent of the shape of the display. A smaller number of constrained elements is required for the near-perfect reference level, relative to the chance reference level of Fig. 10, before sharp changes in discriminability are obtained.

DISCUSSION

The consistent superiority of the discrimination of $P(A|A)$ levels for

one-dimensional constraints over the discrimination of $P(S_e)$ levels for two-dimensional constraints is not due to the specific display configuration employed. The tests of Fig. 9, for example, employed the same equipment, display size, etc. Why the difference?

One difference between the two forms of constraint is that the one-dimensional constraints (one row for generation of vertical constraints) are initialized by a smaller number of randomly generated elements than are the two-dimensional constraints (one row and one column). How important is this factor? I have tried to assess this factor in two ways. Most obvious is to compare one- and two-dimensional constraints with an equal number of constrained elements. This comparison is overwhelmingly favorable to one-dimensional constraints. The other varies the uncertainty of the initializing strings. Figure 12 compares one- and two-dimensional thresholds at three exposure durations as a function of the conditional repetition probability of the initializing sequences. There is little effect of this variable, except at the most extreme $P(A|A)_R$ reference conditions.

Another difference is the length of run possible with one- and two-dimensional constraints. With high $P(A|A)$ levels, entire columns are composed of all dots or all spaces with one-dimensional constraints. By contrast, even at high $P(S_e)$ levels, two-dimensional constraints reflect the

abbreviated run structures of the initial randomly generated row and column. Figure 13 compares one- and two-dimensional thresholds at three exposure durations as a function of the absolute probability of a dot, rather than a space, for the initial random sequences. The initializing sequences otherwise were independently generated. Except for the intersection of extreme probabilities, where one- and two-dimensional constraints produce nearly the same displays, there is little effect of such operations.

With the display conditions constant, with the initializing conditions comparable, we are left only with the underlying rules which define one- and two-dimensional constraints to distinguish between the displays. By default, sensitivity differences are assigned to the effects of these rules.³

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NOTES

1. Suggested by eloquent arguments by Professor S. Kaplan, who argues strongly that experimental psychologists can contribute to the solution of important "relevant" complex problems by exercising their own skills over a wider range of environments.
2. The use of one-dimensional, statistically generated, binary-coded "noise" fields found an impetus when information theory was introduced into psychology (Attneave, 1954) and found expression in studies of target detection (Harcum, 1967, and references cited therein).
3. Additional tests have been carried out with constraints displayed in one dimension with a parity sum over four binary elements. These constraints are equivalent to the 2-D constraints of the present tests, except displayed in one direction. Thresholds were even higher than the present 2-D constraint thresholds. This result suggests that 2-D constraint thresholds are higher than 1-D constraint thresholds, not because the 2-D constraints are plotted in two dimensions, but because a parity restriction across four elements is weaker than the comparable restriction across two elements.

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