Estimating hypothesis strengths*

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Violation of the exhaustiveness postulate of Levine's (1965) hypothesis model is shown to produce large errors in the estimation of the proportions of problems mediated by the various hypotheses. In two cases, when stat-subject simulations satisfied the postulates, estimates were reasonably close to actual values. One interpretation proposes that negative values of estimates are an indication that the exhaustiveness assumption is violated. An alternative computational method is described.

Levine (1965) proposes a computational method based on a hypothesis model of discrimination learning behavior which estimates the proportion of problems in a set whose responses are mediated by a given hypothesis. The particular hypotheses were those appropriate to three-trial discriminations in monkeys. A problem in applying Levine's analysis to primate or retarded children data is that negative proportions often appear. The more extreme negative values are most often uninterpretable. Less extreme values are reported by Levine (1965). These may be estimates of zero or they may be an indication of the inadequacy of the postulate set.

This paper shows that violation of the hypothesis set can produce negative proportions and suggests another approach to the estimation of amounts of hypothesis behavior.

ASSUMPTIONS OF THE MODEL

(1) An exhaustive set of H s is available to S at the outset of each three-trial problem. (2) One and only one H is selected by S at the outset of each problem. (3) H determines a specific behavior sequence for a given problem. (4) The H strength remains the same regardless of reward sequence.

Levine has considered the problem of the omitted hypothesis (violations of the first assumption). He has shown that when a hypothesis that is present in the data but omitted from the analysis is orthogonal to each of the hypotheses in the analysis, then its omission has no effect on the estimation of other proportions; it merely goes into an estimate of random responding. It is not, however, always reasonable to assume that the overlooked hypothesis is orthogonal to the rest.

A hypothesis that is dependent rather than orthogonal is that of double alternation with respect to position. Double alternation may not be a particularly prevalent hypothesis in discrimination learning, but it does

illustrate that possible dependent hypotheses are in the realm of the credible; indeed, this response strategy can be learned by children.

To demonstrate the effect of this dependent hypothesis on the estimated values of the others, stat-subject data were generated by computer with the true values of the probabilities known. These data were then analyzed by a least squares method, Method II, developed by Levine, which has since been generalized by Thomson, 1972. The results are shown in Table 1. The true probability of using each hypothesis for the stat-subject data is the column labeled "Actual." The estimated proportion of problems mediated by each hypothesis computed when the data were analyzed by Levine's Method II is in the column labeled "Estimated." In this case, five S s were given 64 problems each. The responses of each S were generated according to the proportions shown in Table 1. As the results show, omission of a nonorthogonal hypothesis, or violation of the exhaustiveness assumption, can have profound effects on the estimation of other hypotheses. It can produce negative probabilities.

ALTERNATIVE METHOD OF ESTIMATION

Although an unspecified hypothesis cannot be estimated, the distorting effect on estimates of other hypotheses can be circumvented by a relaxation of the exhaustiveness postulate. The resulting inequality, that the sum of the strengths of the specified hypotheses is less than or equal to 1, provides a constraint more flexible than the strict equality. The method proposed here reduces the distorting effects of unspecified hypotheses and aids in the identification of hidden hypotheses. Original and final estimates using this method on the same data are listed in Table 1 under the heading, "Linear Programming."

Levine separates 4 ways of presenting stimuli on each three-trial problem. For each of these 4 ways, there are 8 possible response-outcome sequences, yielding 32 possible combinations of stimulus presentation and response-outcome sequences. Every presentation-response pattern can be assigned to one of these groups, and the proportion of patterns falling in

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Table 1
Proportions of Problems Mediated by Each Hypothesis: Actual and Estimated

Hypothesis	Description	Levine Method II			Linear Programming	
		Actual	Estimated	Error	Original	Final
a	Position Preference	.20	-.02	-.22	.10	.20
b	Position Alternation	.10	-.05	-.15	0	.10
c	Stimulus Preference	0	-.02	-.02	0	0
d	Stimulus Alternation	.10	-.05	-.15	.10	.10
e	Win Stay Lose Shift (Position)	0	+.03	+.03	0	0
f	Lose Stay Win Shift (Position)	0	-.02	-.02	0	0
P ₂	Win Stay Lose Shift (Object)	0	0	0	0	0
P ₃	Third Trial Learning	0	-.01	-.01	0	0
R	Residual	.20	1.14	+.94	.60	.20
	Double Alternation (Position)	.40	Not Estimated		Not Estimated	

Group i can be called P_i (for the precise method of computation of this proportion, see Levine, 1965, p. 103). A particular response pattern for each kind of presentation is mediated by the hypothesis under which the S is acting. Let a be the proportion of problems mediated by Hypothesis H_a . If H_a is the only hypothesis which would associate the stimulus sequence and the response sequence of the j^{th} pattern, then the only other contribution to P_i would be from random responding. So: $a + r \leq P_i$. A set of 32 such inequalities can be set up, and reasonable solutions can be obtained through linear programming (e.g., see Hadley, 1962). In the following example, the IBM 360 Mathematical Programming System was used (IBM Reference Library, H20-0290 and H20-0476).

In order to constrain the solution probabilities to nonnegative values, and to eliminate the necessity for the exhaustiveness assumption, the estimation problem can be stated as follows:

Find probabilities which maximize the function:

$$a + b + c + d + e + f + p_2 + p_3 + 4r$$

Subject to the following constraints:

$$a + c + f + r \leq P_1$$

$$p_3 + r \leq P_2$$

$$b + d + r \leq P_3$$

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$$b + f + r \leq P_{31}$$

$$c + p_2 + r \leq P_{32}$$

$$a + b + c + d + e + f + p_2 + p_3 + 4r \leq 1$$

The determining system is very flexible. Alternative objective functions enable the E to maximize or

minimize the proportion of some subset of hypotheses, as in answering the question: What solution will maximize the proportion due to stimulus preference? The objective and constraints simulate mathematically what the investigator does while studying the data sheets to see how much of a particular hypothesis the data will support. The example in Table 1 provides an illustration of the useful features of linear programming. The original solution was obtained by maximizing $a + b + c + d + e + f + p_2 + p_3 + 4r$ under the given restrictions. While these estimates are closer to the actual values than the least squares estimates, there remains an underestimation of a and b and an overestimation of R ($R = 4r$). The estimates are, however, not the only information supplied by this method. With each cell (each P_i), there is associated some "slack activity." This value is a positive proportion representing the amount of P_i not accounted for by the optimal solution. Unlike the least squares method, linear programming does not attempt to distribute this quantity equally through all the P_i . The amount of slack associated with each P_i can provide a hint about the unspecified hypothesis. In the case of the data of Table 1, the original solution revealed that all of the slack activity was concentrated in the cells associated with the response sequences RRL, LLR, RLL, LRR. All of these cells have slack activity of .10, whereas all other cells have zero slack activity. This is, of course, precisely the definition of double alternation used to generate the data. The new hypothesis could be defined and a new (and correct) optimal solution generated. Although there is as yet no specified systematic way of using the slack information, the advantage in this example is obvious. As a demonstration that the actual values of specified hypothesis are recoverable as an optimal solution by this method even when the hidden hypothesis is not identified, the "final estimates" shown in Table 1 were computed. In order to produce this optimal solution, the function $a + b + c + d + 4r$ was maximized subject to the given restrictions and the additional one, $r \leq .05$. Again, in this case, no fixed procedure has been devised that singles out this optimal solution as better than solutions maximizing other functions. The properties illustrated here make linear

Table 2
Proportions of Problems Mediated by Each
Hypothesis: Actual and Estimated

H*	Group A			Group B		
	Actual	Esti- mated	Error	Actual	Esti- mated	Error
a	.27	.19	-.08	.40	.31	-.09
b	.06	.02	-.04	.06	.04	-.02
c	.07	.03	-.04	0	.03	+.03
d	.06	.03	-.03	0	.03	+.03
e	.10	.10	0	.14	.13	-.01
f	.02	.02	0	0	0	0
P ₂	.10	.10	0	.06	.07	+.01
P ₃	.05	.04	-.01	.08	.09	+.01
R	.28	.46	+.18	.25	.37	+.12

*Hypothesis

programming a reasonable approach to hypothesis estimation, whether the exhaustiveness postulate is or is not met. This method does not have the advantage of being a least squares solution of the equations; its particular usefulness is most apparent when the requirements for least squares solution are not met.

ESTIMATION WHEN THE POSTULATES ARE SATISFIED

Some indication of the reliability of the estimates for Method II when the exhaustiveness assumption is not violated comes from the analysis of generated data. Table 2 shows the differences between actual and estimated values for these cases. There are 10 stat-subjects in Group A and 5 in Group B, each of which had a set of 64 three-trial problems. In each group, Ss had about two errors, that is, the S violated his hypothesis with a "mistake" trial about twice in 64 problems. This was incorporated to simulate the approximately 2% mistake trials noted in college students. Otherwise, Ss acted precisely according to the theory. Some of the errors of estimation in Table 2 are large when compared to the magnitude of the actual

value. The relative error is from 0% to more than 50% of the true value in the nonrandom hypotheses. On the whole, the average error per hypothesis is .04 in Group A and .035 in Group B. The range is from -.10 to +.18. The standard error of proportion is at most .03 for these two groups. The largest error appears, however, in the overestimation of random responding, while the other hypotheses seem more fairly represented. The analysis is not grossly misleading.

CONCLUSIONS

Two major conclusions emerge from these results. The first is that the errors of estimation appear to be reasonably small when the postulate set is satisfied. The second is that the violation of the exhaustiveness postulate can lead to gross errors of estimation of the legitimate hypotheses. It does, indeed, produce negative probabilities. The question arises whether this is a likely source of the negative probabilities which are often encountered when this method is applied. Since it is credible that some dependent hypotheses are sometimes controlling behavior, the answer to this question is yes. So, the appearance of negative probabilities is a cue to look for other hypotheses in the data and either incorporate these in the analysis or use the alternative method of estimation, linear programming.

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