Vibrotactile signal detection¹

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The theory of signal detectability was applied to vibrotactile sensitivity in two experiments. The first experiment showed the psychometric function to be satisfactorily linear when a sensitivity index of the d' family was plotted against the signal intensity expressed in decibels. The second experiment yielded receiver-operating-characteristic (ROC) curves of a familiar form for the yes-no and rating response methods. Reasonably consistent estimates of sensitivity were obtained in the second experiment from the yes-no, rating, and forced-choice methods. The sensitivity indices examined were d' and d_e' , based on Gaussian density functions; A , based on Rayleigh density functions, and the distribution-free indices, $P(A)$ and $P(C)$. For each type of index a tendency was observed for the forced-choice value to be lower than the yes-no and rating values.

An experiment was conducted to extend the theory of signal detectability (Green & Swets, 1966) to vibrotactile sensitivity. Receiver-operating-characteristic (ROC) curves were obtained with the yes-no and rating response methods. Indices of sensitivity yielded by these methods were compared with each other and with indices yielded by the forced-choice method. A preliminary experiment established the shape of the psychometric function.

PROCEDURE

Main Experiment

The O rested the tip of an index finger on a disc $\frac{1}{2}$ in. in diam. The signal consisted of a sinusoidal vibration of the disc at 222 Hz for 250 msec.⁴

The signal was presented at two intensities under each of the three response methods. Three Os, female high-school seniors, served 2 h a day 5 days a week. Prior to collection of the data reported here, the Os were given practice with each response method for at least five daily sessions.

Yes-no ROC curves consisting of three points were obtained with a signal-presentation probability of 0.5 and three different pay-off matrices. When the Os were told to adopt a "strict" decision criterion, the values of the matrix were: hit, +1; miss, -1; false alarm, -5; correct rejection, +5. The matrix in force when a "lenient" criterion was desired assigned the larger values, +5 and -5, to hits and misses, and the smaller values, -1 and +1, to false alarms and correct rejections. A "medium" criterion was encouraged with symmetrical values, ± 1 .

Rating ROC curves consisting of three points were obtained, again with a signal-presentation probability of 0.5, by providing four response categories. The categories were: 1, yes-confident, 2, yes-doubtful; 3, no-doubtful; 4, no-confident. Correct and incorrect responses of 1 and 4 were valued at $\pm 1\frac{1}{2}$; responses of 2 and 3 were valued at $\pm \frac{1}{2}$.

Under the forced-choice method the signal was presented in one or the other of two temporal intervals with equal likelihood. The basic datum obtained was the proportion of correct responses, $P(C)$. Correct and incorrect responses were assigned the values ± 1 .

The Os were informed that the various values would be converted to a cash bonus at the end of the experiment, so that the expected bonus for very good performance would amount to \$2.00 per daily session, irrespective of response method or signal

intensity. The bonus was paid in addition to a regular hourly rate.

A daily session contained 10 groups of 100 trials. In each session, a pair of successive groups was devoted to each of the following five conditions: (1) yes-no, strict criterion, (2) yes-no, medium criterion; (3) yes-no, lenient criterion, (4) rating; (5) forced-choice. The order of the five pairs of runs was chosen randomly each day. Three successive daily sessions were devoted to each signal intensity. Therefore, each yes-no ROC point, each rating ROC curve, and each forced-choice $P(C)$, was based on 600 trials.

Preliminary Experiment

Three different Os served in the experiment to determine the shape of the psychometric function. This preliminary experiment differed from the main experiment in using a signal duration of 100 msec and in not using a bonus scheme. After several days of practice with the rating method, the data of 10 days were cumulated. Seven runs of 200 trials per day yielded rating ROC curves based on 2800 trials at each of five signal intensities.

PSYCHOMETRIC FUNCTION

The five rating ROC curves obtained from each O in the first experiment are shown at the top of Fig. 1 (a, b, c). The three points of each curve were determined in the usual manner, by considering each of the boundaries between response categories as a yes-no criterion.

The ordinate is the probability of a hit, $P(S|s)$; the abscissa (labeled along the positive diagonal) is the probability of a false alarm, $P(S|n)$; the abscissa is also labeled (along the top scale) in terms of the normal deviate, $z(S|n)$. These coordinates scale the normal deviate linearly. The trio of points at each signal intensity are fitted by a straight line, in keeping with the assumption commonly made that the density functions underlying the ROC curve are approximately Gaussian. (Only two points are shown on each ROC curve for O 3; she used the first, "yes-confident" response category so infrequently that one point fell off the graph at the left. The two remaining points were arbitrarily fitted by a line of unit slope.)

The index of sensitivity extracted from each ROC curve is d_e' , which is twice the normal-deviate value corresponding to the intersection of the ROC curve and the negative diagonal. At the bottom of Fig. 1 (a, b, c) the values of d_e' are plotted against the signal intensity expressed in decibels re 1.0 g rms. This psychometric function is seen to be reasonably linear throughout its range. The range, from $d_e' \cong 0$ to ≈ 3.0 , is on the order of 10 to 12 dB. (Expressed in terms of displacement amplitudes, the five signal intensities were 2.3, 2.8, 3.6, 4.5, and 5.7 microns, peak to peak.)

ROC CURVES

The yes-no ROC curves obtained from the three Os in the main experiment are shown in Fig. 2 (a, b, c). The points are labeled S (strict), M (medium), and L (lenient), according to the decision criterion the Os were supposed to use. The labels on the curves denote the signal intensities expressed in decibels re 1.0 g rms. (In microns, peak to peak, the signal intensities were: 0 1, 1.1 and 1.8; 0 2, 1.6 and 2.5; 0 3, 2.8 and 4.5.)

It can be seen that the points obtained are ordered

appropriately from left to right, according to the decision criterion intended, for five of the six curves (excepting the upper curve for O 2). In these cases, consistent with the assumption of Gaussian density functions, the points lie nearly along a straight line of positive slope. Somewhat better-looking yes-no ROC curves, based on three Os combined, at seven signal intensities, were recently reported by Gescheider, Wright, and Evans (1968); they fanned out the curves by varying the signal-presentation probability.

The ROC curves obtained with the *rating* method in the main experiment are shown in Fig. 3 (a, b, c). In most instances, again, a linear fit is a reasonable approximation.

COMPARISON OF INDICES ACROSS METHODS

Indices Examined

Previous applications of detection theory in vision and audition have shown that some index of sensitivity derived from the theory is largely independent of the psychophysical method used. In particular, the index for a given O and signal intensity was found to be nearly invariant over the yes-no and rating methods, and over the forced-choice method with various numbers of alternatives (Green & Swets, 1966).

In every case the index considered was one based on the assumption of Gaussian density functions. One such index is d' , which assumes density functions of equal variance, leading to ROC curves of unit slope. Two other indices based on Gaussian density functions, $D(\Delta m, s)$ and d'_c , make no assumption about the relative sizes of the variances, and are consistent with ROC curves of any positive slope.

In this study, values of d'_c obtained from the yes-no and rating ROC curves were compared with values of d' obtained from the forced-choice data. As indicated, the index d'_c is twice the normal-deviate value corresponding to the intersection of the ROC curve and the negative diagonal. Values of d' were obtained from the forced-choice $P(C)$ by means of Elliott's tables (Swets, 1964).

Also considered in this study was the index A , which assumes Rayleigh density functions. These density functions lead to ROC slopes of less than unity. For yes-no and rating data, A can be calculated by adding 0.71 to the value of d'_c , throughout most of the range of d'_c . At the extreme values of d'_c , the number to be added to give A is somewhat less than 0.71 (Jeffress, 1964). For two-interval forced-choice data, $P(C)$ is converted to A by Marill's (1956) equation:

$$A = 2\sqrt{-\ln \{2[1 - P(C)]\}}. \quad (1)$$

Finally, values of the distribution-free indices $P(A)$ and $P(C)$ were compared. The index $P(A)$ is defined for yes-no and rating ROC curves as the proportion of the area of the unit square lying beneath the ROC curve (plotted on linear probability coordinates). In theory, whatever the shape of the density functions, $P(A)$ is equal to the value of $P(C)$ obtained from the two-interval forced-choice method. Green and Swets (1966) have suggested connecting the empirical ROC points by straight lines in order to estimate $P(A)$. This estimate is given by the trapezoidal rule:

$$P(A) = \frac{1}{2} \sum_{i=0}^n (x_{i+1} - x_i) (y_{i+1} + y_i) \quad (2)$$

Fig. 1. Rating ROC curves for three Os at five signal levels (top of each panel), and the psychometric function with d'_c vs signal decibels (bottom of each panel).

where n = number of points,

$$x_0 = y_0 = 0,$$

$$\text{and } x_{n+1} = y_{n+1} = 1.$$

It should be observed immediately that this formula yields a biased estimate of $P(A)$; in fact, it gives a lower bound. The amount of bias of the estimate depends upon the number and the spread of the data points, and upon the value of $P(A)$.

Before turning to the data, let us note that an assessment of the consistency of a sensitivity index over methods can be made with respect to several variables. We can simply observe the range over methods of the sensitivity index itself. Alternatively, if the shape of the psychometric function is known, the range of the sensitivity index can be converted to a range of the physical measure of signal strength. In either case, it is instructive to determine the range over methods relative to the observed range of the psychometric function.

Results— d' and d'_c

The values of d' and d'_c obtained from the three methods are reproduced in Table 1 and in Fig. 4.

We see first, in the right-most column of the table, that the range of the three methods is, on the average, a little less than 0.4 of a d' or d'_c unit. This range, incidentally, is about twice the size of that observed in auditory studies (Green & Swets, 1966, p. 112).

Assuming a linear psychometric function on the grounds of the data presented in Fig. 1, we can use Fig. 4 to convert the range of the sensitivity index to a range in signal decibels. Reading horizontally from the extreme values on the ordinate, for a given O and signal level, to the line representing the psychometric function, and then down to the abscissa, shows that the average range is 1.8 dB for O_s 1 and 2, and 0.8 dB for O_3 . Extrapolating the functions shown in Fig. 4, to estimate the range of the psychometric function for each O (d' or $d'_c = 0$ to 3), leads to the result that the range of the sensitivity index over the three methods is on the order of 1/7 or 1/8 of the range of the psychometric function for each O .

Another impression of the degree of consistency of the three methods is obtained by expressing the values of d' or d'_c in terms of the equivalent $P(C)$ in two-interval forced-choice. Taking a range of ± 0.2 in d' at values of $d' = 0.5, 1.0, 1.5$, and 2.0 results in an average range of $P(C)$ of 0.07. Given the full psychometric range of 0.50 in $P(C)$, we have again the result that the range of the three methods is about 1/7 the range of the psychometric function.

Figure 4 suggests, however, a dependence of these indices in the d' family on the method used to collect data. The forced-choice index is lowest in five of the six cases. (Assuming independence of the six cases, the probability of a deviation from random ordering as great or greater than the one observed is $6[(1/3)^5 \cdot (2/3)] + (1/3)^6$, or approximately 0.02. Partial data not presented here, on O_s 1 and 2 at a third signal level, lend some additional credence to the statistical significance of this result; in both instances the forced-choice index was the lowest of the three indices.) Further calculation based on the data presented here shows that the average difference between the yes-no and rating indices is one-half the average range of the three methods (0.18 vs 0.37).

One possible explanation of this discrepancy is a depression of the forced-choice index caused by a response bias. The column in

Fig. 2. Yes-no ROC curves for three O_s —with strict, medium, and lenient criteria—at two signal levels.

Table 1 headed $d'_{\max}$ indicates that this possible explanation does not hold up. Converting d' to $d'_{\max}$ eliminates the effect of a response bias (Green & Swets, 1966, pp. 408-410), and the largest increment from d' to $d'_{\max}$ is seen to be 0.05. Another possible explanation is the assumption of equal-variance density functions in determining the forced-choice index, d' , while the relative size of the variances is not assumed in determining the yes-no and rating indices, d'_c . Specifically, had we translated the forced-choice $P(C)$ not into d' but rather into Δm —an index based on density functions of unequal variance (Green & Swets, 1966)—and had we assumed the variance of the signal-plus-noise density function to increase with its mean, then higher values of the forced-choice index would have been obtained. A third possible explanation, which refers the discrepancy to the O rather than to the calculation procedure, is that the O does not derive as much additional information from the two-interval procedure—namely, $\sqrt{2}$ in d' (Green & Swets, 1966, p. 68)—as the theory supposes.

Results—The Index A

Values of the index A (assuming Rayleigh density functions), are not shown here, for they give almost precisely the same picture given by d' and d'_c in Fig. 4. Specifically, use of the index A, does not elevate the forced-choice value relative to the yes-no and rating values. As observed in the foregoing, values of A are somewhat larger than values of d' and d'_c , and the average range of A over the three methods is also slightly larger, namely, 0.5.

Results— $P(A)$ and $P(C)$

The values of the distribution-free indices, $P(A)$ and $P(C)$, that were calculated are presented in Table 2 and in Fig. 5. The right-most column of the table shows that the average range of the three methods is about 0.05, or about 1/10 the range of the psychometric function. There is an indication that the agreement is better in the upper half of the range of the function.

No dependence of the index on the method is evident; specifically, as seen most readily in the figure, the forced-choice index is highest in two cases, middling in two cases, and lowest in two cases. However, as mentioned earlier, the (trapezoidal) rule we used to obtain the index values of $P(A)$ gives a lower bound on $P(A)$.

To get some indication of how much our yes-no and rating indices were depressed by use of the trapezoidal rule, we applied Simpson's rule and the Newton-Cotes formula—which give successively better estimates of $P(A)$, still distribution-free (Hildebrand, 1956)—to some simulated data. In particular, we assumed Gaussian density functions of equal variance, took ROC curves with values of $d' = 0.5, 1.0, 1.5$, and 2.0 , and took three points on each of these curves at values of β (ROC slope) = $3/2, 1$, and $2/3$. Simpson's rule yielded estimates higher than the trapezoidal rule by 0.01 at each value of d' . The Newton-Cotes formula yielded estimates higher than the trapezoidal rule by 0.03 at each value of d' . If adjustments of 0.03 are made in the values of $P(A)$ that are given in Table 2 and Fig. 5, we find again that the forced-choice index is lowest in five of the six cases.⁵

CONCLUSIONS

A promise of detection theory, that consistent estimates of sensitivity will be obtained from a variety of methods of data collection, is reasonably well supported by our data on vibrotactile sensitivity. The range of values obtained from the yes-no, rating, and forced-choice methods is on the order of

Fig. 3. Rating ROC curves for three Os at two signal levels.

Table 1
Results, in terms of d' and d_e' , obtained with the yes-no, rating, and forced-choice methods. (The value of d_e' for Observer 2 at -15 dB was estimated from only the middle point of the negatively-sloped ROC curve.)

Observer	Signal Intensity	d_e' Yes-No	d_e' Rating	d' Forced-Choice	$d'_{\max}$ Forced-Choice	d' and d_e' Average	d' and d_e' Range
1	-22	.94	.66	.95	.96	.85	.29
	18	1.70	1.62	1.37	1.39	1.56	.33
2	19	.48	.64	.38	.38	.50	.26
	15	1.73	1.50	1.18	1.23	1.47	.55
3	14	.16	.38	0.00	0.00	.18	.38
	-10	2.26	2.12	1.86	1.86	2.08	.40

1-2 dB, and 10-15% of the range of the psychometric function. These data, however, lead us to question whether values of the various sensitivity indices are strictly independent of the method used. In particular, these data suggest that index values yielded by the forced-choice method may be lower than the index values derived from the yes-no and rating methods.

REFERENCES

- GLISCHIDER, G. A., WRIGHT, J. H., & EVANS, M. B. Reaction time in the detection of vibrotactile signals. *Journal of Experimental Psychology*, 1968, 77, 501-504.
- GREEN, D. M., & SWETS, J. A. *Signal detection theory and psychophysics*. New York: Wiley, 1966.
- HILDEBRAND, F. B. *Introduction to numerical analysis*. New York: McGraw-Hill, 1956.
- JIFFRESS, L. A. Stimulus-oriented approach to detection. *Journal of the Acoustical Society of America*, 1964, 36, 766-774.
- MARILL, T. M. Detection theory and psychophysics. Technical Report No. 319, 1956, Research Laboratories of Electronics, Massachusetts Institute of Technology.
- SWETS, J. A. (Ed.). *Signal detection and recognition by human observers. Contemporary readings*. New York: Wiley, 1964.

NOTES

1. This work was supported by the Human Performance Branch of the NASA-Ames Research Center. Paul F. Coughlin constructed and maintained the experimental apparatus, Barbara Noel collected the data.
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 3. Dr. Franzén was on leave from the Royal Institute of Technology, Stockholm. His present address is Department of Neurology, Karolinska sjukhuset, Stockholm 60, Sweden.
 4. The disc was actually an end surface of a specially machined cylinder that was also $\frac{1}{2}$ in. in length. This end surface buckled in response to the expansion and contraction of a crystal cemented to the inside of the cylinder. The crystal expanded and contracted in response to an electrical waveform. The waveform was gated at a positive zero crossing. A vibratory element of this kind is reasonably independent of mass loading, and has a resonant frequency above 10 kHz.
- The signal wave form was generated by a PDP-8 computer. This computer was also responsible for all timing, information to the Os, pseudo-random implementation of the signal-presentation probabilities, data acquisition, and rudimentary data reduction.
- The Os and the vibratory apparatus were located in a room that was isolated from vibrations in the building. The possibility that the Os could

Fig. 4. Comparison of yes-no, rating, and forced-choice methods in terms of d' or d_e' , for three Os and two signal levels.

Table 2
Results, in terms of P(A) and P(C), obtained with the yes-no, rating and forced-choice methods.

Observer	Signal Intensity	P(A) Yes-No	P(A) Rating	P(C) Forced-Choice	P(A) and P(C) Average	P(A) and P(C) Range
1	-22	.69	.66	.75	.70	.09
	-18	.86	.85	.83	.85	.03
2	-19	.60	.67	.60	.62	.07
	15	.79	.79	.80	.79	.01
3	-14	.54	.59	.50	.54	.09
	-10	.92	.90	.91	.91	.02

make use of a sound generated by the vibratory apparatus was reduced by the presence of noise from an exhaust fan. Readings of sound pressure level next to the apparatus did not depend on whether the vibratory signal was present or absent.

5. At the two lower values of d' , estimates of P(A) obtained by the

Newton-Cotes formula are identical to the value of P(C) corresponding to the given d' ; at the two higher values of d' , estimates of P(A) by this formula are lower than P(C) by 0.03.

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Fig. 5. Comparison of yes-no, rating, and forced-choice methods in terms of P(A) or P(C), for three Os and two signal levels.