Similarity judgments of line length¹

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Nine Ss judged similarity of pairs of lines on a scale from 0 to 100, and 14 Ss made magnitude estimations of similarity of pairs of lines. The two scales of similarity were linearly related. Scale values for subjective length were obtained in one dimension using a nonmetric multidimensional analysis. Subjective length was found to be logarithmically related to objective length, and similarity was found to be an exponential decay function of distance in psychological space.

The use of similarity judgments has been encouraged by the development of techniques for multidimensional analysis of similarities (proximities) data (Shepard, 1962; Kruskal, 1964; Torgerson, 1965). These techniques allow the recovery of a parsimonious stimulus space underlying similarity judgments and of a function relating judgments to interpoint distance in the derived space.

The present research is concerned with the form of the psychophysical function derived from similarity judgments of stimuli varying in one dimension. An advantage of using similarity judgments for constructing subjective scales is that the strong metric assumptions inherent in the traditional scaling procedures may be avoided. For example, in magnitude estimation and category scaling the S's numerical responses are taken at face value. For scales obtained from similarity one need only assume that differences between scale values (for unidimensional scaling) are monotonically related to similarity.

METHOD

Subjects

The Ss were 23 students enrolled in an introductory course in psychology. Participation in psychological experiments met a course requirement.

Apparatus

Nine horizontal lines with projected lengths of 3.4, 5.4, 7.2, 10.3, 14.3, 21.2, 18.6, 40.9, and 57.2 cm were arranged in pairs consisting of all 72 possible permutations. The stimuli were prepared on white cards and presented by an opaque projector. A projection screen was located 6 ft in front of the S.

Procedure

Two groups of Ss judged the similarity of each pair of lines. The groups differed with respect to the procedure used in making similarity judgments. For the first procedure a group of nine Ss made similarity estimates using a scale from 0 to 100. Zero was designated as no similarity, and 100 was designated as identity. This procedure has been referred to as similarity estimation by Ekman, Goude, and Waern (1961). A group of 14 Ss used a magnitude estimation procedure to estimate similarity of the pairs. A standard pair was presented first and assigned the value 10. The Ss were instructed to assign numbers to subsequent pairs that were proportional to the Ss' subjective impression of the similarity of the pairs. For example, a pair that appeared to be twice as similar as the standard was to be assigned 20, and so on. The standard pair consisted of a 10.3-cm line and a 28.6-cm line.

The order of presentation of the 72 pairs was random. Each pair was presented for 10 sec, and presentations were separated by 7 sec.

Fig. 1. Similarity estimation plotted against magnitude estimation of similarity.

RESULTS

Arithmetic means of responses to each of the stimulus pairs were obtained for each of the two judgment procedures. The means were computed over Ss and over the two permutations of each pair. Figure 1 shows similarity estimation plotted against magnitude estimation of similarity. The plot appears linear indicating the two procedures yield approximately the same result.

Scale values for subjective length were obtained by the solution developed by Kruskal (1964) for multidimensional scaling from nonmetric information. Interest in the present study was focused on a solution in one dimension. Computations were performed on the IBM 7040 installation at the University of Alberta Computing Center.

To avoid the problem of local minima three different starting configurations were used with each of the two sets of similarity judgments. The starting configurations were: (a) equal spacing between adjacent points with points arranged in the same order as the lengths of their corresponding stimuli, (b) points spaced from their projections on the first dimension after a two-dimensional solution, and (c) spacing of points relative to magnitude estimations of the length of the lines. The magnitude estimations had been obtained previously from a separate group of Ss.

The three final configurations from similarity estimation were approximately the same. The starting configuration in which adjacent points were equally spaced (a) satisfied the monotonicity requirement immediately with 0% stress. The starting configuration determined from a two-dimensional analysis (b) yielded a solution after five iterations with a stress of 3.5%, and a solution from the magnitude estimation spacing (c) was achieved after six iterations with 2.5% stress. Differences among the three solutions were slight and may be attributed to a criterion used in the computer program to discontinue iterating if stress was less than 5%. Scale values from the solution with 0% stress are plotted against the

Fig. 2. Scale values for length plotted against line length.

physical length of the line stimuli in Fig. 2. The plot indicates a logarithmic relation between scale values and physical length.

The three analyses of S_s ' magnitude estimations of similarity converged to the same solution after 19 to 22 iterations with a stress of 7.3%. Figure 2 presents a plot of the scale values against physical length. The psychophysical relationship again appears logarithmic.

Figure 3 presents a plot of similarity against distance between scale values for each of the two sets of data. The distances presented with similarity estimation were those which fit the data with 0% stress. The curved line through the data represents an exponential decay function.

DISCUSSION

The results of the present study indicate that similarity estimation is linearly related to magnitude estimation of similarity. This finding is in agreement with Ekman and Waern's (1959) finding that similarity estimation of circle size was linearly related to judged ratios of similarity. Although the results from nonmetric scaling were essentially the same for both procedures, the lower stress achieved with data from similarity estimation suggests that responses from this procedure may be more stable.

The extent to which the logarithmic relationship found between subjective length and objective length may be taken as support for Fechner's law rests on the validity of the assumption made in the similarity analysis. The assumption was that similarity is a monotonic decreasing function of distance. With the exception of Ekman's (1963) procedures this is the minimal assumption of all the multidimensional scaling methods currently in use. However, its validity has not been proven.

The equations for the curves presented in Fig. 3 are exponential decay functions, which may be expressed as

$$S_{ij} = a e^{b d_{ij}} \quad (1)$$

where S_{ij} is the similarity between the i th and j th stimuli, d_{ij} is the corresponding distance, a is a positive constant, and b is a negative constant. This function differs from one proposed by Helm, Messick, and Tucker (1961) in which distance was expressed as the following exponential decay function of similarity

$$d_{ij} = g e^{k S_{ij}}, \quad (2)$$

where g is a positive constant and k is a negative constant. By letting $c = 1/k$ and $h = \ln g/k$ and solving for S_{ij} in Eq. 2, similarity may be expressed as the following logarithmic function of distance

$$S_{ij} = c \ln d_{ij} + h. \quad (3)$$

Note that c is negative.

Both Eq. 1 and Eq. 3 are monotone decreasing, positively accelerated functions. Over the range of data expected in research the curves for each are very similar. Figure 4 presents a curve relating similarity predicted from each equation. The constants for the two functions were obtained by fitting each to the data from similarity estimation. As can be seen from Fig. 4 the curve is almost linear over the range of data from the present study. The corresponding curve obtained by estimating constants from the magnitude estimation data is similar.

Helm, Messick, and Tucker (1961) report research on multidimensional scaling in which S_s gave category ratings of similarity among hues. Similarity values were obtained by a successive intervals solution. The multidimensional scaling procedure was similar to that presented by Torgerson (1958). When similarity was assumed to be linearly related to distance 12 dimensions were obtained, whereas when the transformation presented in Eq. 2 was applied to similarity the analysis yielded two dimensions reproducing the color circle. No transformation based on Eq. 1 was tried. It seems reasonable that had it been used essentially the same results would have been obtained. The authors argued that a nonlinear transformation was required because similarity values were obtained by discrimination scaling, and discrimination scales are usually nonlinearly related to direct ratio scales. As evidence that such a relationship holds for similarity they presented data later reported by Helm (1964) on multidimensional ratio scaling of hues. But these data were not relevant since S_s ' responses were in terms of ratios of distances and not similarity. The results of the present study indicate that magnitude estimation of similarity is also nonlinearly related to distance. This finding suggests that similarity is a monotone decreasing, positively accelerated function of distance regardless of whether similarity is obtained by ratio scaling or discrimination scaling. The form of the relationship between similarity and distance in psychological space does not appear to be relevant to arguments on the nature of discrimination scaling and ratio scaling in general.

Equation 3 appears objectionable on theoretical grounds since its curve asymptotes along the Y-axis, indicating that an infinite value of similarity is required for zero distance. Secondly, the curve crosses the X-axis, and hence similarity may be negative for some finite distances. However, Eq. 1 appears to be theoretically meaningful. Since the curve crosses the Y-axis at $S_{ij} = a$, a finite similarity value is defined for zero

Fig. 3. Similarity judgments plotted against derived distances.

Fig. 4. Similarity obtained from a negative logarithmic function of distance as a function of similarity obtained from an exponential decay function of distance.

distance. Secondly, the curve asymptotes along the X-axis; this indicates that for any finite distance similarity is always positive. It seems reasonable that if points for two stimuli are in the same psychological space a positive similarity value

should be defined for the stimuli regardless of how far apart the points may be.

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NOTES

1. This research was conducted while the first author held a postgraduate scholarship from the National Research Council of Canada.
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