A FORTRAN IV function for computing critical values of ordered half-normal scores

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If the standard normal distribution ($M = 0.0$, $SD = 1.0$) is modified by transforming all negative values (those below the mean) to positive values, the half-normal distribution results. Dunlap and Myers (1984) described a FORTRAN program that computes the expected values of ordered half-normal scores, which are useful for half-normal probability plots. Values that may prove to be even more useful are the critical values of ordered half-normal distribution, which can be used directly in tests of significance. These critical values have specific applications for hypothesis testing, two of which are discussed here.

When many correlations in a matrix are tested for significance, repeated use of standard tests such as those based on F or on Fisher's z transformation involves repeated risks of a Type I error; therefore, some correlations may appear significant by chance alone. Hills (1969) and Stavig and Acock (1976) proposed tests of significance for correlations in a matrix that attempt to control overall Type I error rates, unlike repeated F or z tests. The rank adjusted method (Stavig & Acock, 1976) orders correlations in terms of absolute value, transforms these correlations via Fisher's z transformation, and then calculates the corresponding normal deviates by dividing by the standard error of Fisher's z , $1/(N-3)^{1/2}$. Because the process started from the absolute values of the original correlations, the resulting scores should have had the ordered half-normal distribution. Therefore, Stavig and Acock (1976) recommended subtracting the corresponding expected ordered half-normal score and comparing the difference to the critical value of the standard normal distribution. The problem is that comparing these adjusted z scores to the standard normal distribution will result in a conservative test (Silver, 1988).

If instead, each rank ordered z score is compared to the corresponding critical value of the half-normal distribution, a much improved test results. If the i th z score equals or exceeds the i th critical value of the half-normal distribution, it is considered significant. This procedure not only has been shown to control Type I errors well, but exhibits greater power than either the multistage Bonferroni (Larzelere & Mulaik, 1977) or Stavig and Acock (1976) procedures (Silver, 1988).

A second example of the use of ordered half-normal critical values applies to the analysis of a 2^5 factorial design, where each of five experimental variables has two levels. For such a design, there are 31 tests of significance

for the various main effects and interactions. With so many tests of significance, some effects are likely to be quite large by chance alone, and several might be significant if tested in the customary way, by analysis of variance. Because all effects in a 2^5 are linear contrasts of the cell means, and each is based on one degree of freedom, Daniel (1959) and Zahn (1975a, 1975b) have proposed judging significance by plotting the ordered absolute values of the standardized contrasts on half-normal probability paper, where departures from a straight line would indicate an effect larger than that expected by chance alone. Such an approach may be particularly useful in experiments with only one replication, where there is no direct estimate of error. Making such judgments with ordered half-normal critical values, rather than visual inspection of plots, should vastly improve these procedures. Other potential applications for these critical values are mentioned by Dunlap and Myers (1984).

The Program

The half-normal probability function was calculated from the normal probability function, using the algorithm of Divgi (1979). Simpson's rule was used to numerically integrate the function as described by Dunlap and Myers (1984). The program continues to integrate the ordered half-normal distribution until the area exceeds one minus alpha; thus, the critical value is exceeded. Then a process of adjustment ensues, in which the correct critical value is estimated by linear interpolation, and small areas are either added or subtracted from the total area to refine the estimate. This adjustment procedure continues until the integral up to the critical value agrees with one minus alpha to five decimal places (for a fairly complete tabulation of critical values generated please see Appendix A).

Accuracy

To confirm the accuracy of the resulting critical values, a Monte Carlo simulation was performed. Empirical alpha levels for each critical value were obtained by counting the number of times 10,000 randomly generated ordered half-normal scores surpassed these critical values. Empirical alpha values approximated theoretical alpha values well (range = $\pm .0004$) for critical values of small to moderate numbers of ordered half-normal scores. There was, however, a slight conservative trend for larger numbers of ordered half-normal scores (e.g., $N = 100$). The present program is probably accurate to five decimal places, because it uses the same computational scheme as that of Dunlap and Brown (1983) and Dunlap and Myers (1984). The FORTRAN program is given in Appendix B.

Program Availability

These functions are written in standard FORTRAN IV and run on an IBM 3081 GX computer. On this system

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the Unit 9 is assigned by default to the keyboard and CRT. Listings of these functions can be obtained at no cost from William P. Dunlap, Department of Psychology, Tulane University, New Orleans, LA 70118.

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Appendix A
Critical Values for Ordered Half-Normal Scores (.05 Level) *J*th of *N* Ordered Half-Normal Scores

Number of Half- Normal Scores	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	22	24	26	28	30	32	34	36	38	40	
1	1.96																														
2	2.24	1.28																													
3	2.39	1.49	.96																												
4	2.49	1.66	1.15	.78																											
5	2.57	1.77	1.31	.95	.66																										
6	2.63	1.86	1.43	1.10	.81	.57																									
7	2.68	1.93	1.52	1.21	.95	.71	.51																								
8	2.73	1.99	1.59	1.30	1.06	.84	.63	.45																							
9	2.77	2.04	1.66	1.38	1.15	.94	.75	.57	.42																						
10	2.80	2.09	1.71	1.44	1.22	1.03	.85	.69	.52	.38																					
11	2.83	2.13	1.76	1.49	1.28	1.10	.93	.78	.63	.47	.35																				
12	2.86	2.16	1.80	1.54	1.34	1.16	1.00	.86	.72	.58	.44	.33																			
13	2.88	2.20	1.84	1.59	1.39	1.22	1.06	.93	.79	.67	.54	.41	.31																		
14	2.91	2.23	1.87	1.63	1.43	1.26	1.12	.98	.86	.74	.62	.50	.38	.29																	
15	2.93	2.25	1.90	1.66	1.47	1.31	1.17	1.04	.92	.80	.69	.58	.47	.36	.27																
16	2.95	2.28	1.93	1.69	1.51	1.35	1.21	1.08	.97	.86	.75	.65	.55	.45	.34	.26															
17	2.97	2.30	1.96	1.72	1.54	1.38	1.25	1.13	1.01	.91	.81	.71	.61	.52	.42	.32	.25														
18	2.98	2.32	1.99	1.75	1.57	1.42	1.28	1.17	1.06	.96	.86	.76	.67	.58	.49	.40	.30	.23													
19	3.00	2.34	2.01	1.78	1.60	1.45	1.32	1.20	1.09	.99	.90	.81	.72	.64	.55	.47	.38	.29	.22												
20	3.02	2.36	2.03	1.80	1.63	1.48	1.35	1.23	1.13	1.03	.94	.85	.77	.69	.61	.53	.44	.36	.27	.22											
22	3.04	2.40	2.07	1.85	1.67	1.53	1.41	1.29	1.19	1.10	1.01	.93	.85	.77	.70	.63	.55	.48	.41	.33	.22										
24	3.07	2.43	2.11	1.89	1.72	1.58	1.45	1.35	1.25	1.16	1.08	1.00	.92	.85	.78	.71	.64	.57	.51	.44	.31	.19									
26	3.09	2.46	2.14	1.92	1.76	1.62	1.50	1.39	1.30	1.21	1.13	1.05	.98	.91	.84	.78	.72	.65	.59	.53	.41	.28	.17								
28	3.12	2.49	2.17	1.96	1.79	1.66	1.54	1.44	1.34	1.26	1.18	1.11	1.03	.97	.90	.84	.78	.72	.66	.61	.50	.38	.26	.16							
30	3.14	2.51	2.20	1.99	1.82	1.69	1.58	1.48	1.38	1.30	1.22	1.15	1.08	1.02	.96	.90	.84	.78	.73	.67	.57	.46	.36	.25	.15						
32	3.16	2.54	2.23	2.02	1.85	1.72	1.61	1.51	1.42	1.34	1.26	1.19	1.13	1.06	.99	.93	.88	.84	.78	.73	.63	.53	.44	.34	.23	.14					
34	3.17	2.56	2.25	2.04	1.88	1.75	1.64	1.54	1.46	1.38	1.30	1.23	1.17	1.11	1.05	.99	.94	.88	.83	.78	.69	.59	.50	.41	.32	.22	.14				
36	3.19	2.58	2.27	2.07	1.91	1.78	1.67	1.57	1.49	1.41	1.34	1.27	1.20	1.14	1.09	1.03	.98	.93	.88	.83	.74	.65	.56	.48	.39	.30	.21	.12			
38	3.21	2.60	2.29	2.09	1.93	1.81	1.70	1.60	1.52	1.44	1.37	1.30	1.24	1.18	1.12	1.07	1.02	.97	.92	.87	.78	.70	.61	.53	.45	.37	.29	.20	.12		
40	3.22	2.61	2.31	2.11	1.96	1.83	1.72	1.63	1.54	1.47	1.40	1.33	1.27	1.21	1.16	1.11	1.05	.99	.94	.89	.83	.74	.66	.58	.51	.43	.35	.27	.19	.12	
42	3.23	2.63	2.33	2.13	1.98	1.85	1.75	1.65	1.57	1.49	1.42	1.36	1.30	1.24	1.19	1.14	1.09	1.04	.99	.95	.86	.78	.71	.63	.56	.48	.41	.34	.26	.18	
44	3.25	2.65	2.35	2.15	2.00	1.87	1.77	1.68	1.59	1.52	1.45	1.39	1.33	1.27	1.22	1.17	1.12	1.07	1.03	.98	.90	.82	.75	.67	.60	.53	.46	.39	.32	.25	.18
46	3.26	2.66	2.37	2.17	2.02	1.89	1.79	1.70	1.62	1.54	1.47	1.41	1.35	1.30	1.25	1.20	1.15	1.10	1.06	1.02	.93	.86	.78	.71	.64	.57	.51	.44	.37	.31	.25
48	3.27	2.67	2.38	2.18	2.03	1.91	1.81	1.72	1.64	1.56	1.50	1.44	1.38	1.32	1.27	1.22	1.18	1.13	1.09	1.05	.97	.89	.82	.75	.68	.61	.55	.49	.42	.36	.30
50	3.28	2.69	2.40	2.20	2.05	1.93	1.83	1.74	1.66	1.58	1.52	1.46	1.40	1.35	1.30	1.25	1.20	1.16	1.11	1.07	.99	.92	.85	.78	.72	.65	.59	.53	.47	.41	.35
52	3.29	2.70	2.41	2.22	2.07	1.95	1.85	1.76	1.68	1.61	1.54	1.48	1.42	1.37	1.32	1.27	1.23	1.18	1.14	1.10	1.02	.95	.88	.81	.75	.69	.63	.57	.51	.45	.39
54	3.31	2.72	2.42	2.23	2.08	1.96	1.86	1.77	1.70	1.63	1.56	1.50	1.44	1.39	1.34	1.29	1.25	1.21	1.16	1.12	1.05	.98	.91	.84	.78	.72	.66	.60	.55	.49	.43
56	3.32	2.73	2.44	2.25	2.10	1.98	1.88	1.79	1.71	1.64	1.58	1.52	1.46	1.41	1.36	1.32	1.27	1.23	1.19	1.15	1.07	.99	.94	.87	.81	.75	.69	.64	.58	.53	.47
58	3.33	2.74	2.45	2.26	2.11	2.00	1.90	1.81	1.73	1.66	1.60	1.54	1.48	1.43	1.38	1.34	1.29	1.25	1.21	1.17	1.10	1.03	.96	.90	.84	.78	.72	.67	.61	.56	.50
60	3.33	2.75	2.46	2.27	2.13	2.01	1.91	1.82	1.75	1.68	1.61	1.56	1.50	1.45	1.40	1.36	1.31	1.27	1.23	1.19	1.12	1.05	.99	.92	.86	.81	.75	.70	.64	.59	.53
65	3.36	2.78	2.49	2.30	2.16	2.04	1.95	1.86	1.78	1.72	1.66	1.60	1.54	1.49	1.45	1.40	1.36	1.31	1.27	1.23	1.19	1.12	1.05	.99	.92	.86	.81	.75	.70	.64	.59
70	3.38	2.80	2.52	2.33	2.19	2.07	1.98	1.89	1.82	1.75	1.69	1.64	1.58	1.53	1.49	1.44	1.40	1.36	1.32	1.28	1.24	1.17	1.11	1.04	.98	.92	.87	.82	.76	.71	.67
75	3.40	2.82	2.54	2.36	2.22	2.10	2.01	1.93	1.85	1.79	1.73	1.67	1.62	1.57	1.53	1.49	1.44	1.40	1.36	1.32	1.29	1.22	1.15	1.09	1.03	.98	.93	.87	.82	.78	.73
80	3.41	2.84	2.57	2.38	2.24	2.13	2.04	1.95	1.88	1.82	1.76	1.70	1.65	1.60	1.56	1.52	1.48	1.44	1.40	1.37	1.30	1.24	1.18	1.12	1.07	1.02	.97	.93	.88	.84	.79
85	3.43	2.86	2.59	2.40	2.27	2.15	2.06	1.98	1.91	1.84	1.78	1.73	1.68	1.63	1.59	1.55	1.51	1.47	1.43	1.40	1.37	1.30	1.24	1.18	1.12	1.07	1.02	.97	.93	.88	.84
90	3.45	2.88	2.61	2.42	2.29	2.18	2.09	2.00	1.93	1.87	1.81	1.76	1.71	1.66	1.62	1.58	1.54	1.50	1.47	1.43	1.40	1.37	1.30	1.24	1.18	1.12	1.06	1.01	.97	.93	.88
95	3.46	2.90	2.63	2.44	2.31	2.20	2.11	2.03	1.96	1.89	1.84	1.78	1.73	1.69	1.65	1.60	1.57	1.53	1.49	1.46	1.43	1.37	1.31	1.26	1.21	1.15	1.10	1.05	1.01	.97	.93
100	3.47	2.91	2.64	2.46	2.33	2.22	2.13	2.05	1.98	1.92	1.86	1.81	1.76	1.71	1.67	1.63	1.59	1.56	1.52	1.49	1.46	1.43	1.37	1.31	1.26	1.21	1.17	1.12	1.08	1.04	1.00
110	3.50	2.94	2.67	2.50	2.36	2.26	2.17	2.09	2.02	1.96	1.90	1.85	1.80	1.76	1.72	1.68	1.64	1.60	1.57	1.54	1.48	1.42	1.37	1.32	1.27	1.23	1.18	1.14	1.10	1.06	1.02
120	3.52	2.97	2.70	2.53	2.40	2.29	2.20	2.13	2.06	2.00	1.94	1.89	1.84	1.80	1.76	1.72	1.68	1.64	1.60	1.57	1.54	1.48	1.42	1.37	1.32	1.27	1.23	1.18	1.14	1.10	1.06
130	3.54	2.99	2.73	2.56	2.43	2.32	2.23	2.16	2.09	2.03	1.98	1.93	1.88	1.84	1.80	1.76	1.72	1.68	1.65	1.62	1.58	1.52	1.47	1.42	1.37	1.32	1.28	1.24	1.20	1.16	1.12
140	3.56	3.02	2.75	2.58	2.45	2.35	2.26	2.19	2.12	2.06	2.01	1.96	1.91	1.87	1.83	1.79	1.76	1.72	1.												

Appendix A (Continued)

Number of Half- Normal Scores	42	44	46	48	50	52	54	56	58	60	65	70	75	80	85	90	95	100	110	120	130	140	150	160	170	180	190	200	500	1000
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42	.11	.17	.11	.11	.10	.09	.09	.09	.09	.08	.08	.08	.08	.08	.06	.06	.05	.05	.05	.05	.05	.03	.03	.03	.03	.02	.02	.02	.02	.00
44	.24	.16	.11	.11	.10	.09	.09	.09	.09	.08	.08	.08	.08	.08	.06	.06	.05	.05	.05	.05	.05	.03	.03	.03	.03	.02	.02	.02	.02	.00
46	.44	.24	.16	.11	.10	.09	.09	.09	.09	.08	.08	.08	.08	.08	.06	.06	.05	.05	.05	.05	.05	.03	.03	.03	.03	.02	.02	.02	.02	.00
48	.29	.23	.22	.22	.21	.20	.20	.20	.20	.19	.19	.19	.19	.19	.17	.17	.16	.16	.16	.16	.15	.15	.15	.15	.14	.14	.14	.14	.00	
50	.35	.28	.27	.27	.26	.25	.25	.25	.25	.24	.24	.24	.24	.24	.22	.22	.21	.21	.21	.21	.20	.20	.20	.20	.19	.19	.19	.19	.00	
52	.39	.33	.32	.32	.31	.30	.30	.30	.30	.29	.29	.29	.29	.29	.27	.27	.26	.26	.26	.26	.25	.25	.25	.25	.24	.24	.24	.24	.00	
54	.43	.38	.37	.37	.36	.35	.35	.35	.35	.34	.34	.34	.34	.34	.32	.32	.31	.31	.31	.31	.30	.30	.30	.30	.29	.29	.29	.29	.00	
56	.47	.42	.41	.41	.40	.39	.39	.39	.39	.38	.38	.38	.38	.38	.36	.36	.35	.35	.35	.35	.34	.34	.34	.34	.33	.33	.33	.33	.00	
58	.51	.46	.45	.45	.44	.43	.43	.43	.43	.42	.42	.42	.42	.42	.40	.40	.39	.39	.39	.39	.38	.38	.38	.38	.37	.37	.37	.37	.00	
60	.54	.49	.48	.48	.47	.46	.46	.46	.46	.45	.45	.45	.45	.45	.43	.43	.42	.42	.42	.42	.41	.41	.41	.41	.40	.40	.40	.40	.00	
62	.57	.52	.51	.51	.50	.49	.49	.49	.49	.48	.48	.48	.48	.48	.46	.46	.45	.45	.45	.45	.44	.44	.44	.44	.43	.43	.43	.43	.00	
64	.61	.56	.55	.55	.54	.53	.53	.53	.53	.52	.52	.52	.52	.52	.50	.50	.49	.49	.49	.49	.48	.48	.48	.48	.47	.47	.47	.47	.00	
66	.64	.59	.58	.58	.57	.56	.56	.56	.56	.55	.55	.55	.55	.55	.53	.53	.52	.52	.52	.52	.51	.51	.51	.51	.50	.50	.50	.50	.00	
68	.68	.63	.62	.62	.61	.60	.60	.60	.60	.59	.59	.59	.59	.59	.57	.57	.56	.56	.56	.56	.55	.55	.55	.55	.54	.54	.54	.54	.00	
70	.70	.65	.64	.64	.63	.62	.62	.62	.62	.61	.61	.61	.61	.61	.59	.59	.58	.58	.58	.58	.57	.57	.57	.57	.56	.56	.56	.56	.00	
72	.73	.68	.67	.67	.66	.65	.65	.65	.65	.64	.64	.64	.64	.64	.62	.62	.61	.61	.61	.61	.60	.60	.60	.60	.59	.59	.59	.59	.00	
74	.75	.70	.69	.69	.68	.67	.67	.67	.67	.66	.66	.66	.66	.66	.64	.64	.63	.63	.63	.63	.62	.62	.62	.62	.61	.61	.61	.61	.00	
76	.77	.72	.71	.71	.70	.69	.69	.69	.69	.68	.68	.68	.68	.68	.66	.66	.65	.65	.65	.65	.64	.64	.64	.64	.63	.63	.63	.63	.00	
78	.79	.74	.73	.73	.72	.71	.71	.71	.71	.70	.70	.70	.70	.70	.68	.68	.67	.67	.67	.67	.66	.66	.66	.66	.65	.65	.65	.65	.00	
80	.81	.76	.75	.75	.74	.73	.73	.73	.73	.72	.72	.72	.72	.72	.70	.70	.69	.69	.69	.69	.68	.68	.68	.68	.67	.67	.67	.67	.00	
82	.83	.78	.77	.77	.76	.75	.75	.75	.75	.74	.74	.74	.74	.74	.72	.72	.71	.71	.71	.71	.70	.70	.70	.70	.69	.69	.69	.69	.00	
84	.85	.80	.79	.79	.78	.77	.77	.77	.77	.76	.76	.76	.76	.76	.74	.74	.73	.73	.73	.73	.72	.72	.72	.72	.71	.71	.71	.71	.00	
86	.87	.82	.81	.81	.80	.79	.79	.79	.79	.78	.78	.78	.78	.78	.76	.76	.75	.75	.75	.75	.74	.74	.74	.74	.73	.73	.73	.73	.00	
88	.89	.84	.83	.83	.82	.81	.81	.81	.81	.80	.80	.80	.80	.80	.78	.78	.77	.77	.77	.77	.76	.76	.76	.76	.75	.75	.75	.75	.00	
90	.91	.86	.85	.85	.84	.83	.83	.83	.83	.82	.82	.82	.82	.82	.80	.80	.79	.79	.79	.79	.78	.78	.78	.78	.77	.77	.77	.77	.00	
92	.93	.88	.87	.87	.86	.85	.85	.85	.85	.84	.84	.84	.84	.84	.82	.82	.81	.81	.81	.81	.80	.80	.80	.80	.79	.79	.79	.79	.00	
94	.95	.90	.89	.89	.88	.87	.87	.87	.87	.86	.86	.86	.86	.86	.84	.84	.83	.83	.83	.83	.82	.82	.82	.82	.81	.81	.81	.81	.00	
96	.97	.92	.91	.91	.90	.89	.89	.89	.89	.88	.88	.88	.88	.88	.86	.86	.85	.85	.85	.85	.84	.84	.84	.84	.83	.83	.83	.83	.00	
98	.99	.94	.93	.93	.92	.91	.91	.91	.91	.90	.90	.90	.90	.90	.88	.88	.87	.87	.87	.87	.86	.86	.86	.86	.85	.85	.85	.85	.00	
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.00	
102	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	.00	
104	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	.00	
106	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	.00	
108	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	.00	
110	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	.00	
112	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	.00	
114	1.07	1.07																												

APPENDIX B FORTRAN Program

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C  MAIN ROUTINE FOR NUMERICAL CALCULATION OF CRITICAL VALUES
C  OF ORDERED HALF NORMAL SCORES
      DOUBLE PRECISION HNSCR
1     WRITE(9,2)
2     FORMAT('ENTER N, ALPHA (ENTER 0,0 TO QUIT)')
      READ(9,*)N,A
      IF(N.EQ.0)STOP
      DO 4 J=1,N
        CV=HNSCR(J,N,A)
        WRITE(9,3)N,J,CV,A
3     FORMAT(' CV (',2I5,' ) =',F12.5,'  ALPHA =',F8.4)
4     CONTINUE
      GO TO 1
      END

C
      FUNCTION HNSCR(J,N,A)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C  COMPUTES THE CRITICAL VALUE OF THE JTH OF N
C  ORDERED HALF NORMAL SCORES
C  COMPUTE THE CONSTANT OF INTEGRAL IN LOG FORM
      CA=1.-A
      C=FACD(N)-FACD(J-1)-FACD(N-J)
      HNSCR=0.
      STR=0.
      FIN=0.
C  FIND LOWER (STR) AND UPPER (FIN) LIMITS OF INTEGRATION
      DO 1 I=1,50
        XI=I
        XI=XI*.14
        FUNC=FUN(XI,C,J,N)
        IF(STR.EQ.0..AND.FUNC.GT.1.E-7)STR=XI
        IF(STR.GT.0..AND.FUNC.LT.1.E-7)GOTO 2
1     CONTINUE
2     FIN=XI
      STR=STR-.14
      IF(STR.GE.0.)GOTO 3
      STR=0.
      FIN=.28
3     CONTINUE
      W=(FIN-STR)/100.
      XL=STR
      FL=FUN(XL,C,J,N)
C  BEGIN SIMPSON'S RULE INTEGRATION
      DO 4 I=1,101
        XU=XL+W
        XM=(XU+XL)/2.
        FM=FUN(XM,C,J,N)
        FU=FUN(XU,C,J,N)
        XLST=XL
        HLST=HNSCR
        HNSCR=HNSCR+(FU+4.*FM+FL)*W/6.
        IF(HNSCR.GT.CA)GOTO 5
        XL=XU
        FL=FU
4     CONTINUE
C  CONVERGE ON CRITICAL VALUE
C  ESTIMATE CV
5     CV=XLST+(XU-XLST)*(CA-HLST)/(HNSCR-HLST)
      FCV=FUN(CV,C,J,N)
      XM=(CV+XU)/2.
      W=ABS(XM-CV)
      FM=FUN(XM,C,J,N)
      AREA=W/3.*(FCV+4.*FM+FU)
      IF(HNSCR.GT.CA)AREA=-(AREA)
      ANEW=HNSCR+AREA
      IF(ABS(ANEW-CA).LE.0.000001D0)GOTO 7
      IF(ABS(CV-XLST).GT.ABS(XU-CV))GOTO 6
      XU=CV
      HNSCR=ANEW
      FU=FCV
      GOTO 5
6     XLST=CV
      HLST=ANEW
      GOTO 5

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APPENDIX B (Continued)

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7      HNSCR=CV
      RETURN
      END

C
      FUNCTION FUN(X,C,J,N)
C COMPUTES VALUE OF FUNCTION TO BE INTEGRATED - IN LOG FORM
C THEN IN DECIMAL FORM IF LARGE ENOUGH
C C=N!/ (J-1)! / (N-J)! THE CONSTANT OF INTEGRATION
C FUN=C*P** (N-J)*Q** (J-1)**Z
C WHERE P IS PROPORTION BELOW & Q IS PROPORTION ABOVE X
C ON THE HALF NORMAL CURVE; AND Z IS THE ORDINATE OF THE
C HALF NORMAL CURVE AT X.
C
      DOUBLE PRECISION X,C,POFZ,P,Q,XJ,XN,FUN,FL
      IF (J.LT.N.OR.X.GT.0.0) GOTO 1
      XN=N
      FUN=XN*.7978846D0
      RETURN
1      XJ=X
      XN=N
      P=POFZ(X)*2.D0-1.0D0
      Q=1.D0-P
      FUN=0.D0
      IF (P.LE.0.D0.OR.Q.LE.0.D0) RETURN
      FL=C+DLOG(P)*(XN-XJ)+DLOG(Q)*(XJ-1.D0)-X*X/2.D0-.2257914D0
      IF (FL.LT.-40.D0) RETURN
      FUN=DEXP(FL)
      RETURN
      END

C
      FUNCTION FACL(K)
C COMPUTE THE NATURAL LOG OF K!
C ABOVE 36 USES STIRLINGS APPROXIMATION
      IF (K.GT.36) GOTO 2
      FACL=0.
      IF (K.LE.1) RETURN
      DO 1 I=2,K
      X=I
1      FACL=FACL+ALOG(X)
      RETURN
2      X=K
      FACL=.918938533+ALOG(X)*(X+.5)-X+1./(12.*X)
      RETURN
      END

C
      FUNCTION POFZ(Z)
C -- DOUBLE PRECISION FUNCTION FOR THE NORMAL CURVE INTEGRAL
C -- FROM -INFINITY TO Z. BASED ON THE ALGORITHM OF
C -- DIVGI (1979) ANNALS OF STAT, 7, 903-910.
      DOUBLEPRECISION ZZ,POFZ,H,P,Z
      ZZ=DABS(Z)
      IF (ZZ.LE.18D0) GOTO 1
      POFZ=0.0D0
      GOTO 2
1      H=0.3989422804D0*DEXP(-ZZ*ZZ/2.0D0)
      P=3.534244658D-6*ZZ
      P=(P-7.830823677D-5)*ZZ
      P=(P+7.771088713D-4)*ZZ
      P=(P-4.644960579D-3)*ZZ
      P=(P+1.915649350D-2)*ZZ
      P=(P-5.982834993D-2)*ZZ
      P=(P+0.1522723563D0)*ZZ
      P=(P-0.3316218430D0)*ZZ
      P=(P+0.6262972801D0)*ZZ
      P=(P-0.9999673043D0)*ZZ
      P=P+1.253313402D0
      POFZ=H*P
2      IF (Z.GT.0.0D0) POFZ=1.0D0-POFZ
      RETURN
      END

```