Similarity of schematic faces: A test of interdimensional additivity¹

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In this study we obtained direct and comparative judgments of the dissimilarity between schematic faces varying on three binary attributes. These data were used to test the hypothesis that the overall dissimilarity between faces can be decomposed (in an ordinal sense) into three additive components, one for each attribute. The hypothesis was strongly confirmed by both the direct and the comparative judgments. The study illustrates the possible usefulness of the measurement-theoretic analysis of simple combination rules for psychological dimensions.

The present study investigates the empirical adequacy of interdimensional additivity for dissimilarity between schematic faces. Each face was composed of three features: overall shape, eyes, and mouth. The overall shape was an ellipse which was either wide or long. The eyes were two circles which were either empty or black. The mouth was a single line which was either straight or smiling. The combinations of these three binary attributes formed a product set of eight schematic faces, shown in Fig. 1.

To formulate the hypotheses tested, some notation is introduced. Let x_i, y_i, z_i denote, respectively, the values of the i th stimulus on the dimensions of shape, eyes, and mouth. We let $x_i = o$ for a long face, $x_i = a$ for a wide face; $y_i = o$ for empty eyes, $y_i = b$ for black eyes; and $z_i = o$ for straight mouth, $z_i = c$ for curved mouth. Thus, each face can be described as a three-component vector as shown in Fig. 1. For example, (abo) denotes wide face, filled eyes, and straight mouth. Let $\delta(i,j)$ be the dissimilarity of Faces i and j . Then the hypothesis of interdimensional additivity (Beals, Krantz, & Tversky, 1968) states that

$$\delta(i,j) = F[\phi_1(x_i, x_j) + \phi_2(y_i, y_j) + \phi_3(z_i, z_j)] \quad (1)$$

where F is a strictly increasing function of one variable, each of the ϕ s is a symmetric function of two (nominal scale) variables, and the value of ϕ equals 0 if the two variables are equal and is positive otherwise.

It should be noted that Eq. 1 includes as special cases the Euclidean, city-block, and other metric-space models used in multidimensional scaling. Thus, rejection of Eq. 1, in a

Fig. 1. The eight schematic faces used in the experiment, with three-component vector designations. The first component refers to shape of face, the second to empty or filled eyes, and the third to type of mouth.

Fig. 2. Representation of the eight schematic faces as vertices of a rectangular solid.

particular stimulus domain, rules out the applicability of any of these special models, at least for the specified dimensions. (It is, of course, possible that Eq. 1 is valid with some quite different dimensional representation of the same stimuli, hence the present test depends on the particular characterization of the stimuli.)

In order to follow the detailed predictions of Eq. 1, it is helpful to visualize the eight stimuli in Fig. 1 as the vertices of a rectangular solid in three-dimensional space (Fig. 2). Pairs of stimuli that differ on only one feature are represented as edges in Fig. 2. There are four x -edges parallel to the x -axis, including pairs [(ooo),(aoo)], [(obo),(abc)], etc.; four y -edges; and four z -edges. Pairs of stimuli that differ on two features can be visualized as secondary diagonals (diagonals lying in one face in Fig. 2). There are four (x,y) -diagonals, including pairs [(ooo),(abo)], [(aoo),(obo)], etc.; four (y,z) -diagonals; and four (x,z) -diagonals. Finally, there are four pairs of stimuli that form main diagonals, or (x,y,z) -diagonals, differing on all three features.

The predictions derivable from Eq. 1 are as follows.

I. *Equality:* Dissimilarity is constant over any four edges or any four diagonals within one class. Thus, $\delta[(ooo),(ooc)] = \delta[(abo),(abc)]$, since both pairs are z -edges; $\delta[(ooo),(abo)] = \delta[(aoc),(obc)]$, since both pairs are (x,y) -diagonals.

II. *Ordering:*

A. *The dissimilarity increases if new differences are added.* Thus, the dissimilarity of an x -edge or a y -edge is less than that of an (x,y) -diagonal, which is in turn less than that of a main diagonal.

B. *The ordering of secondary diagonals is determined by the ordering of edges.* Thus, if the dissimilarity of x -edges is smallest and of z -edges is largest, then the dissimilarity of (x,y) -diagonals is smallest and that of (y,z) diagonals largest (among secondary diagonals).

We shall illustrate the use of Eq. 1 to derive the above

predictions by deriving two special cases, one of I, the other of IIB.

(1) Proof that $\delta[(ooo),(abo)] = \delta[(aoc),(obc)]$: By symmetry of the ϕ s, $\phi_1(a,o) = \phi_1(o,a)$. In addition, $\phi_3(o,o) = \phi_3(c,c) = 0$. Thus, $\phi_1(o,a) + \phi_2(o,b) + \phi_3(o,o) = \phi_1(a,o) + \phi_2(o,b) + \phi_3(c,c)$. Computing F of both sides yields the desired result, since $\delta[(ooo),(abo)] = F[\phi_1(o,a) + \phi_2(o,b) + \phi_3(o,o)]$, $\delta[(aoc),(obc)] = F[\phi_1(a,o) + \phi_2(o,b) + \phi_3(c,c)]$.

(2) Proof that if $\delta[(ooo),(aoo)] \leq \delta[(ooo),(obo)]$, then $\delta[(ooc),(aoo)] \leq \delta[(aoo),(abc)]$: Since F is strictly increasing, the hypothesis that $F[\phi_1(o,a) + \phi_2(o,o) + \phi_3(o,o)] \leq F[\phi_1(o,o) + \phi_2(o,b) + \phi_3(o,o)]$ yields the conclusion that $\phi_1(o,a) \leq \phi_2(o,b)$. But this, with $\phi_2(o,o) = \phi_1(a,a)$ and $\phi_3(c,o) = \phi_3(o,c)$, yields $\phi_1(o,a) + \phi_2(o,o) + \phi_3(c,o) \leq \phi_1(a,a) + \phi_2(o,b) + \phi_3(o,c)$. The conclusion $\delta[(ooc),(aoo)] \leq \delta[(aoo),(abc)]$ follows from the fact that F is strictly increasing.

From a psychological viewpoint, Eq. 1 asserts that the "perceived distances" along each of the dimensions contribute additively to the overall dissimilarity between the faces. The contribution of each dimension, therefore, is independent of the values on any of the other dimensions. This is a strong form of the independence axiom that entails the lack of interactions between attributes.

The principle of interdimensional additivity, embodied in Eq. 1, has been axiomatized by Beals, Krantz, and Tversky (1968). It can be shown that the predictions I, IIA, IIB above can be derived using only axioms (A1) and (A3) of their paper. Since these two axioms, plus the "continuity" axiom (A2), are sufficient to guarantee the validity of Eq. 1, the test of the equality and ordering predictions provides a complete test of the applicability of Eq. 1.

METHOD

Two types of dissimilarity judgments were obtained. In the *single stimulus* judgments, a single pair of faces was presented to the S, who rated its dissimilarity by marking a point on a scale from 1 (minimal dissimilarity) to 20 (extremely dissimilar). In *pair comparison* judgments, two pairs were presented simultaneously, and the S indicated which pair was more dissimilar.

In the single stimulus design, each of the 28 pairs of nonidentical faces was presented 10 times to each S (except that by mistake, one pair was presented 9 times and another 11 times). These 280 single stimulus trials were divided into four blocks of 70, and one block was presented per session for the first four sessions of the experiment. In the fifth and sixth sessions (which occurred three weeks later) each of the 24 edge or secondary diagonal pairs was replicated once per session to check for stability.

For pair comparisons, 48 pairs of pairs were constructed as follows. Each vertex in Fig. 2 lies on three edges and on three secondary diagonals. These three edges with a common vertex were compared with one another in pairs (three pairs of edges) and so were the three secondary diagonals (three pairs of secondary diagonals). Thus, there were six pair comparisons per vertex, times eight vertices. (Note that the pair comparison method reduces to a modified method of triads, since any two pairs compared have a common element, the vertex.)

These 48 pairs were replicated twice in each of the fifth and sixth sessions, for a total of four replications per pair. If we identify the eight different comparisons involving the same two classes of edges (e.g., x edges vs y edges) or the same two classes of diagonals, at the eight vertices, then we obtain a total of 32 replications per comparison, per S.

Clearly, all the predictions can be tested using the single stimulus design, since all the nonidentical pairs were judged. The pair comparisons are designed to test the most critical prediction, IIB, that the order among classes of edges predicts that among classes of diagonals.

The Ss were six paid volunteers, undergraduates at the Hebrew University of Jerusalem. During the first four sessions, Ss were run in pairs (to save time and provide social facilitation), while in the last two sessions, five Ss were run as a group and the sixth alone.

Stimuli were projected on a white surface using a Victorlite Model 1502 "Visualcast" projector and 2 x 2 in. slides. Each pair appeared on a separate slide, with the two faces side-by-side. The major axes of the faces subtended approximately 0.4 deg visual angle (the same as a normal human face at about 35 m). The centers of the faces were separated by about 1.4 deg. For pair comparisons, two slides were inserted simultaneously on the projection surface and they appeared side-by-side, with centers separated by about 6 deg, and with the black borders of both slides separating the two pairs.

RESULTS

The single stimulus results of Sessions 1 through 4 are summarized in Table 1, which gives the mean of the ratings (N = 10 except where indicated) for each of the 28 pairs of faces and six Ss. The 28 stimulus pairs are grouped into the three classes of edges, three classes of secondary diagonals, and one class of main diagonals.

It is evident from Table 1 that predictions I and IIA are strongly confirmed. The differences among the four means within any one class for any one S are so small as to be of no importance, while there is practically no overlap between the means for an edge-class and for a secondary diagonal-class that includes it, or between the secondary and main diagonals. A similar picture is obtained from the medians.

Table 1
Single Stimulus Results:
Mean Ratings of Dissimilarity (N = 10 unless indicated)

Pair	Subject					
	1	2	3	4	5	6
x-edges (shape)						
(ooo),(aoo)	4.7	11.5	3.5	6.2	9.1	7.7
(ooc),(aoc)	4.6	11.9	3.0	8.1	8.7	8.3
(obo),(abo)	4.9	10.5	3.3	7.2	8.7	7.9
(obc),(abc)	5.3	11.1	3.2	6.2	9.1	8.6
y-edges (eyes)						
(ooo),(obo)	7.3	7.4	3.3	7.1	5.7	5.7
(ooc),(obc)	7.4	4.2	3.4	7.6	5.1	6.2
(aoo),(abo)	6.4	3.6	3.2	6.6	5.4	5.6
(aoc),(abc) (N = 9)	7.5	5.8	3.3	7.1	6.2	8.0
z-edges (mouth)						
(ooo),(ooc)	4.7	8.3	3.1	6.4	5.8	5.5
(obo),(obc)	3.8	8.8	3.4	7.1	5.4	5.5
(aoo),(aoc)	4.0	10.0	3.8	6.2	5.6	5.5
(abo),(abc)	4.1	6.7	3.5	7.1	6.2	6.1
(x,y)-diagonals						
(ooo),(abo) (N = 11)	12.3	13.2	4.9	14.2	11.5	12.2
(ooc),(abc)	12.8	10.6	4.5	14.0	13.2	13.3
(aoo),(obo)	10.6	13.0	4.9	14.8	13.3	11.7
(aoc),(obc)	12.7	11.9	4.9	14.3	12.0	12.2
(y,z)-diagonals						
(ooo),(obc)	10.6	9.5	4.6	14.2	10.6	9.5
(aoo),(abc)	10.9	8.3	4.7	12.3	8.1	10.4
(obo),(ooc)	10.6	9.6	4.8	14.2	10.2	10.9
(abo),(aoc)	11.3	8.1	4.6	13.5	9.5	10.9
(x,z)-diagonals						
(ooo),(aoc)	8.9	14.3	4.6	13.4	11.3	9.8
(obo),(abc)	11.5	13.0	4.6	14.3	11.2	11.3
(aoo),(ooc)	9.9	12.7	4.6	13.1	11.1	12.2
(abo),(obc)	10.9	14.6	4.1	13.0	12.4	13.3
(x,y,z)-diagonals						
(ooo),(abc)	14.7	14.4	5.5	19.3	16.7	16.5
(ooc),(abo)	15.2	14.5	5.3	20.0	15.6	16.8
(aoo),(obc)	13.1	14.4	5.4	20.0	17.2	17.8
(aoc),(obo)	13.5	18.0	5.4	19.5	16.5	17.7

Table 2

Test of Prediction IIB from Single Stimulus Results

Entries are proportions that the dissimilarity judgment of pair in class to the left of dash is larger than judgment of pair to the right of dash.

Subject	Pairs of edge-classes			Pairs of diagonal-classes		
	x - y	y - z	x - z	(x,z)-(y,z)	(x,y)-(x,z)	(x,y)-(y,z)
1	.142†	.927	.668	.463†	.716	.717
2	.910	.201	.757	.828	.332	.753
3	.483	.398*	.398*	.421	.613*	.536*
4	.480	.533	.505	.461	.582	.545
5	.820	.519	.800	.752	.519	.768
6	.696	.605	.695	.689	.550	.935

The asterisks indicate apparent violations of IIB (S 3). A suggestion of a possible violation in S 1 is indicated by †. If the true populations are identical, the expected value of the proportion in the table is .500 and the standard deviation is about .066. Thus, proportions between .368 and .632 are not significantly different from .500, at the .05 level. Thus, the apparent violations by S 3 may be due to sampling error.

In order to test prediction IIB in an ordinal-invariant manner, we calculated values of the Mann-Whitney U statistic for each pair of edge-classes and each pair of secondary diagonal-classes. For example, each S made 40 judgments of x-edges and 39 judgments of y-edges. Forming all possible pairs of an x-judgment and a y-judgment yields $1560 = 39 \times 40$ pairs. The U statistic is the number of these 1560 pairs for which the x-judgment is higher (counting $\frac{1}{2}$ for a pair where the judgments are tied). This statistic is obviously unchanged by any strictly increasing transformation of the response scale. Since the number of pairs varies slightly, we present, in Table 2, the values of $V = U/N_1 N_2$, where N_1, N_2 are the numbers of judgments in the two groups involved. One value of V is calculated for each S for each comparison between two edge-classes or two secondary diagonal-classes.

Under the null hypothesis that the two classes being compared by means of V are drawn from identical continuous distributions, the V statistic is normally distributed (approximately) with mean .500 and standard deviation about .066. Since the distributions are in fact discrete (many ties occurred), the standard deviation under the null hypothesis is actually less than .066.

The values of V in Table 2 define a definite ordering of the three edge-classes, and an ordering of the three diagonal-classes, for each S. That is, the V proportions satisfy weak stochastic transitivity (WST), and even (except for three tiny deviations) strong stochastic transitivity (SST).³ For Ss 1, 2, and 6, the ordering is very clearcut for the edge-classes, admitting no possibility of error due to sampling. For S 5, the x-class is clearly largest, with y and z about equal. For Ss 3 and 4, none of the values of V are significantly different from .500, so the single-stimulus method fails to yield a convincing ordering. The reasons for this are apparent in the raw data: S 3 confined her responses almost entirely to the range 2-6 on the 20-point scale (because, she claimed, none of the differences were really large), while S 4, after the first experimental session, merely counted the number of dimensions on which the faces were different, responding 7, 14, or 20 according to whether there were differences on one, two, or three dimensions. The other Ss did not adopt such stereotyped response patterns.

An apparent violation of IIB occurs whenever a proportion in one of the first three columns of Table 2 is on the opposite side of .500 from the corresponding proportion in the last three columns. For convenience, we have placed in Columns 4, 5, 6 proportions that correspond to Columns 1, 2, 3

respectively. The only apparent violations occur, therefore, in comparing Columns 2,5 and Columns 3,6 for S 3. The 2,5 violation approaches statistical significance.

For the other Ss, the patterns of probabilities are quite similar on the left and right sides of the table. There is a tendency for diagonal-class probabilities to be closer to .500 than corresponding edge-class probabilities. For S 1 (Columns 1,4) this is so marked as to suggest a violation of IIB, of form $y > x$ but $(y,z) = (x,z)$.

The results for the partial replication of the single-stimulus design in Sessions 5 and 6 are not incorporated; they show no systematic tendency to depart from previous patterns of responding. Ss 3 and 4 maintained their stereotyped response patterns.

The results for pair comparisons are summarized in Table 3. Table 3 is like Table 2, except that the proportions are based on 32 actual experimental comparisons of the pairs, rather than comparisons of the single-stimulus ratings.

We remark first that the choice probabilities in every case define an ordering of the edge or the diagonal-classes (WST always holds, and there are only three violations of SST, only one of them—S 6, Columns 4,6—approaching significance). Second, the ordering of the three edge-classes is now very clear-cut, even for Ss 3 and 4. For Ss 1, 2, 5, and 6 the pair comparison ordering of edge-classes is identical to the Mann-Whitney U ordering, except that the $y > z$ finding, which is represented only by a slight tendency in the single-stimulus data of S 5, is very pronounced in the pair comparisons. For Ss 3 and 4, the pair comparison method leads to sharp orderings of the edge-classes, which differ from the ill-defined orderings suggested by single-stimulus data. Sixteen out of 18 of the pair comparison probabilities for edge-classes are more extreme than the corresponding values of V in Table 2.

Violations of IIB are defined in Table 3 in the same way as in Table 2. Only one such violation occurs in Table 3 (see Columns 1,4 for S 1) and it is, clearly, not statistically significant. However, when taken together with earlier independent evidence (Table 2), it suggests the existence of a violation.

To summarize: For Ss 2, 5, and 6, the pair comparison and single-stimulus orderings are essentially the same (possibly slightly different for S 5) and show no indication of a violation of IIB. For S 1, the pair comparison and single-stimulus orderings are essentially the same and suggest the same violation of IIB. For Ss 3 and 4, only the pair comparison orderings are clearly defined, and they do not suggest any violation of IIB.

Table 3

Test of Prediction IIB from Pair Comparison Results

Entries are proportions (out of 32 trials) that pairs in class to the left of dash were chosen as more dissimilar than the pair in class to the right of dash.

Subject	Pairs of edge-classes			Pairs of diagonal-classes		
	x - y	y - z	x - z	(x,z)-(y,z)	(x,y)-(x,z)	(x,y)-(y,z)
1	.281†	.938	.938	.531†	.875	.938
2	.938	.281	1.000	.938	.469	.938
3	.062	.812	.031	.125	.875	.281
4	.938	.906	1.000	.906	1.000	1.000
5	.938	.906	1.000	.906	.969	1.000
6	.625	.844	.782	.594	.875	.782

An apparent violation in S 1 is indicated by †. If the true choice probability is .500, then the standard deviation of the proportion in the table is about .089. Thus, the apparent violation could be due to sampling error; however, compare Table 2.

DISCUSSION

The present approach to the testing of geometrical models of proximity (or distance) has the following methodological characteristics: (1) It is based on the data analysis of individual Ss; (2) it treats the responses as establishing only an ordering, rather than an interval or ratio scale; (3) it is aimed at testing qualitative predictions, made by a family (Eq. 1) of models, rather than at testing the goodness of fit of any one model. Each of these aspects is now discussed in turn.

(1) The six Ss yielded four out of the six possible orderings of edge-classes (Table 3). We shudder to think what the results would look like if we pooled data across Ss.

(2) The six Ss used the rating scale in widely different ways. From the mean ratings, in Table 1, it is clear that a city-block metric (Attneave, 1950) would not give a good fit to the numerical values of mean ratings, notwithstanding the "analyzability" of these stimuli (Hyman & Well, 1967, 1968; Shepard, 1964). To us, it seems more satisfactory to treat the ratings as ordinal, in the spirit of ordinal multidimensional scaling (cf. Beals et al, 1968; Coombs, 1964; Guttman, 1966; Shepard, 1962, 1966). The exact conditions under which rating scales act like an interval scale remain to be clarified (see Anderson, 1962, for a good example where rating data are well-behaved.) Similarly, pair comparison data are used in this study only to establish an ordering. The attempt to extract ratio or interval scale measures from pair comparison data faces grave difficulties (see Krantz, 1967).

In Table 1, we depart from ordinal methods, by presenting means, to dramatize the way in which Predictions I and IIA were supported. The same picture is obtained from medians or by scanning the raw data, however. It did not seem worthwhile to report a large number of Mann-Whitney U statistics just for the sake of ordinal purity.

On the other hand, it is important to emphasize that the Predictions I, IIA, IIB do follow from Eq. 1, for ordinal data, and that the test of IIB, involving more delicate comparisons, is best done by ordinal-invariant statistics.

(3) Rejection of the predictions of Eq. 1 would rule out a wide class of models, including the Euclidean and all the Minkowski power metrics. The common feature of all these models is an additive combination rule for combining the contributions of different dimensions. Much more delicate qualitative tests are needed to distinguish among the different models satisfying Eq. 1, using ordinal methods. In any case, qualitative predictions focus on the critical features of the data for accepting or rejecting a class of models, while the usual goodness-of-fit measures combine critical and noncritical data.

The above results offer no convincing evidence against the validity of Eq. 1 for similarity of schematic faces varying on the three dimensions studied here. Indeed, the similarity of the pattern of single-stimulus results, on the left and right sides of Table 2, for Ss 2, 5, and 6, and of pair comparison results, on the left and right sides of Table 3, for Ss 2, 3, 4, 5, and 6, suggests that for those five Ss, an additive (ordinal) model is likely to produce excellent predictions. Clearly, the eight faces employed are a systematic sample from a "continuous" space of such faces, which are themselves a systematic sample from a larger space of possible attributes. Hence, the conclusions are subject to the reservation that the fuller space has not been thoroughly explored, and the introduction of additional levels and/or dimensions may reveal some interactive patterns.

Assuming that Eq. 1 gives a good account of the way in which differences in shape, eyes, and mouth combine to determine similarity of schematic faces, what is the psychological significance of this finding? We think it lends some support to the thesis that formation of overall impressions can be analyzed into the formation of a priori specifiable subimpressions and the combination of these

independent subimpressions. This two-stage analysis refers primarily to mental processing and not to covert behavior. S 4, indeed, used a covert counting process to generate her stereotyped single-stimulus reactions. However, a much more complicated rule for covert calculation would be needed to account for the full panoply of data here presented, which include the differentiation of dissimilarity of the edge-classes and the preservation of this differentiation in the diagonal-classes (Prediction IIB), for both single-stimulus and pair comparisons. From the speed of response, the variability of the data, and the lack of subjective reports of such rules, we are very doubtful that covert calculation was involved.

We believe nevertheless that interactive or Gestalt effects are important in impression formation. Such effects were reported for schematic faces by Harrison (1964), who reviews a huge literature on judgments of facial expressions and pictures, in which interactive contributions of physical features were considered. Nevertheless, it is possible that with the right choice of physically-specified dimensions—taking into account interactive effects—a simple rule of combination can still be used to generate overall impressions from subimpressions. One research strategy, not different in spirit from earlier formulations of multidimensional scaling, is to adopt as a working assumption the hypothesis that a simple combination rule such as Eq. 1 is operative, and to search for the characterization of stimulus dimensions which satisfies that rule. Clearly, it is advantageous to use as general a rule as still imposes real constraints. From this viewpoint there is a great advantage to Eq. 1 over the more restrictive Euclidean model.

If the above strategy is to work at all, then the simple combination rule should be verifiable in a case where the stimuli are chosen so as to make possible a priori choice of noninteracting dimensions. Such is the case with the present stimuli, which were chosen in the hope they would be noninteractive. Equation 1 has still to be tested in contexts where additivity or independence are less likely, at least on a priori grounds.

The significance of the present confirmation of interdimensional additivity, therefore, is that it supports the hypothesis that a simple combination rule can describe mental processing of independent impressions, and it encourages attempts to use this simple rule as a criterion for isolating the dimensions on which subimpressions are formed.

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NOTES

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3. Let $P(\alpha, \beta) = 1 - P(\beta, \alpha)$ denote the probability of choosing α over β , estimated by relative frequency of direct comparisons or by the V statistic based on single-stimulus evaluations. Suppose further that $P(\alpha, \beta) \geq \frac{1}{2}$ and $P(\beta, \gamma) \geq \frac{1}{2}$; then WST is satisfied if $P(\alpha, \gamma)$ exceeds one-half, and SST is satisfied if $P(\alpha, \gamma)$ exceeds both $P(\alpha, \beta)$ and $P(\beta, \gamma)$.

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