The effects of figural shape on the perception of area¹

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It was hypothesized that figures of equal area and different shapes are perceived as differing in area and that these apparent differences in area are due to the interaction of certain shape variables. Thirty figures were presented to 45 Ss for paired comparisons. Forty-five sets of 30 Choice scores were calculated. The performance of each S was tested for consistency by the Method of Circular Triads ($p > .01$). A Coefficient of Agreement (u) indicated that the Ss agreed among themselves ($p > .01$). Fifteen Ss were tested twice for test-retest reliability ($p > .01$). The 45 sets of Choice scores were then converted to rank values and averaged over all Ss. The mean rank values of the 30 figures were treated with the two-way analysis of variance by ranks to determine the significance of the differences among them ($p > .01$). A Coefficient of Concordance (W) was calculated on the 45 sets of rank values to support the results of the analysis of variance ($p > .01$). Subsequently, the 30 mean rank values were treated as the dependent variable in a multiple correlation (R) with the shape variables as the independent variables ($p > .01$). The apparent area of the figures was found to vary inversely with perimeter, number of turns, breadth, and external area.

In the past few decades only the studies of Anastasi (1936) and Smith (1958) have directed attention to area as a pattern variable comparable to color, brightness, orientation, and, particularly, shape. Both investigators employed stimuli of standard geometric patterns. Anastasi found that outline patterns, or unfilled figures, of equal area and different shapes were perceived as differing in area. A positive correlation was found between estimated area and greatest linear dimension. Smith found also that unfilled figures of equal area and different shapes were perceived as differing in area. Although the results tended to show a positive correlation between estimated area and perimeter, no definite cause was determined for the phenomenon.

In these two studies no attempt was made to sample a population of experimental stimuli as is often done with experimental Ss. Consequently, the findings were limited in application to the stimuli which had been used.

This same limitation had until recently been common to all pattern perception experiments. However, Attneave and Arnoult (1956) have proposed a number of systems, or methods, of pattern construction which permit the generalization of results to stimuli beyond those used in the experiment. These methods employ the concept of a stimulus population (the total possible number of stimuli which may be generated by a given method). If a sample of stimuli is drawn from such a population, the results of the experiment can be generalized to other comparable samples from that stimulus population. This concept of a stimulus population has since become prevalent in pattern perception studies (Anderson, 1957; Coules, Duva, & Ganem, 1960; Crook, 1957).

Although Anastasi and Smith had demonstrated that patterns of equal area may be perceived as differing in area when shape is varied, studies continue to be performed in which area is not controlled or even considered (Coules et al, 1960; Goldstein, 1961). The tacit assumption has been made that, while experimental findings concerning the recognition, discrimination, or recall of patterns differing in form may be a function of some aspect of shape such as perimeter or number of turns, the findings are never affected by pattern area.

The primary problem of this experiment was to demonstrate with relatively wide generalization of results that the perception of the areas of certain filled figures varies with the shape of those figures. It was hypothesized that figures from the defined population, although of equal area, are perceived as differing in area. It was further hypothesized that these differences in perceived area result from the combined effect of certain shape variables. In the investigation of these hypotheses an attempt was made to isolate and measure those aspects of shape which may cause this phenomenon and to generalize the results to other figures.

METHOD

The methods of random stimulus generation suggested by Attneave and Arnoult do not readily permit the control of area and do not lend themselves sufficiently to the measurement of a substantial number of shape variables. While Brown, Hitchcock, and Michels (1962) have created a method of stimulus generation which does permit the control of area, the method constrains the random distribution of shape variables, providing a serious limitation to the testing of our hypotheses. Consequently, a new method of figure construction was devised and a sample of 30 figures subsequently was developed. Each figure was composed of 36 square units randomly cohered according to certain rules of construction. The following seven shape variables were then measured for each figure so that their possible effect on the apparent areas of the figures could be calculated: perimeter (Deese, 1956), number of turns (Attneave, 1957), width to height ratio (Brown et al, 1962), length, breadth, external area, and angle of tilt. Length, breadth, external area, and angle of tilt were used in this study for the first time. *Length* is defined as the distance between the two points on the contour of a figure that are most widely separated. *Breadth* is defined as the narrowest separation between two parallel lines through which a figure can be passed just touching both lines. *External area* may be described as follows: If a rubber band were drawn tightly around the perimeter of a figure, the external area would include that space between the rubber band and the perimeter. *Angle of tilt* refers to the inherent orientation of a figure relative to an imaginary base line. This term does *not* refer to the rotation of a figure in a plane perpendicular to the line of sight nor to the slant of a figure away from this plane (Hake, 1957). Rather, the tilt of a figure is understood in relation to its potential erectness on the imaginary base line. For example, if the figure, as it is drawn, is perfectly erect on this imaginary base line, the angle of tilt is 0 deg. The formula for determining the angle of tilt (AT) in degrees and decimals for any of the figures in this population is

$$AT = \frac{\frac{L_a}{L_a} \cdot \frac{M_a}{M_a} \cdot -A_a + \frac{L_c}{L_a} \cdot \frac{M_c}{M_a} \cdot -A_c}{2} + \frac{\frac{L_b}{L_a} \cdot \frac{M_b}{M_a} \cdot A_b + \frac{L_d}{L_a} \cdot \frac{M_d}{M_a} \cdot A_d}{2}$$

where L = length of one of four imaginary lines (quadrant lines) extending from the center of the figure to

the center of that quadrant
M = area of one of four quadrants
A = angle formed by the quadrant line and the imaginary line that divides the figure vertically
a = the upper left-hand quadrant
b = the upper right-hand quadrant
c = the lower right-hand quadrant
d = the lower left-hand quadrant

and where the longest quadrant line length and the largest quadrant area appear as denominators in the formula. (Quadrant line length a and quadrant area a are inserted here for purposes of demonstration.)

Experimental Design

The 30 figures were presented to each S for the judgment of area according to the Method of Paired Comparisons. A choice, or C, score (Woodworth & Schlosberg, 1954) was calculated for each figure as a measure of the area of that figure as judged by that S. Due to the liability of C scores to change in relative value from one S to another, the 30 C scores were converted to rank values (Guilford, 1954) for each S. The rank values of each figure were then averaged over all Ss to provide a mean rank value (MRV) for every figure. The 30 MRVs were treated with Friedman's two-way analysis of variance by ranks (Siegel, 1956) to determine the significance of the differences among them. The MRVs were also treated as the dependent variable in a multiple correlation (R) in which the shape variables were the independent variables.

Apparatus

Figures were constructed of black paper and centered on white backgrounds. The experimental stimuli consisted of all possible pairs of figures, or 435. Each pair was photographed with the figures side by side. A Nikon F camera with 35 mm, ASA-10, daylight Kodachrome film was used to produce 435 slides. Slides were projected on a 30 in. x 40 in. screen by a Kodak Cavalcade projector.

Subjects

Forty-five male high school seniors were obtained as Ss. Ss who ordinarily wore corrective lenses were requested to wear them. All were paid for their time.

Procedure

Ss were tested in groups of three. The screen was located 9 ft in front of the projector. Ss rotated seating positions on either side of the projector at three set intervals in order to minimize position effect. Ambient lighting was sufficient for the use of answer booklets.

Table 1

Mean Rank Values of 30 Figures Based on Judgments of 45 Ss

Figure Numbers	Mean Rank Values	Figure Numbers	Mean Rank Values
1	13.878	16	14.211
2	21.222	17	22.600
3	9.511	18	16.689
4	17.711	19	13.233
5	19.311	20	18.744
6	21.778	21	10.900
7	19.989	22	15.878
8	10.756	23	18.722
9	10.578	24	17.167
10	16.056	25	19.467
11	16.500	26	17.822
12	9.711	27	14.900
13	6.578	28	19.444
14	8.567	29	13.844
15	20.233	30	9.000

Fig. 1. Stimulus figures from smallest-appearing in upper left-hand corner to largest-appearing in lower right-hand corner as perceived by 45 Ss.

Ss were instructed to choose from each pair of figures that figure which appeared larger in area and to mark their responses in the answer booklets. Ss were then given a practice session with 10 slides, the first eight of which had been constructed specially for this purpose. For the experimental session proper, the order of the 23 slide trays was randomized for each of the 15 S groups.

The last 15 Ss were tested twice at an interval of one week to determine the reliability of judgments over a period of time.

RESULTS

The performance of each S was tested for consistency by the Method of Circular Triads (Kendall, 1962). A consistent S was one whose judgments tended to locate each figure in a constant position on a continuum of apparent area. The obtained Coefficients of Consistence (zeta) ranged from .260 to .874 and were significant at the .01 level. This finding indicated that each S showed a certain consistency in judging and had been able to, in effect, rank the figures.

The extent to which the Ss agreed with one another was determined by calculating Kendall's (1962) Coefficient of Agreement (u) which was found to be .095 ($p < .01$). This finding indicated that the 45 Ss did not disagree with one another to a significant extent on the ranking of the figures.

The MRV of each figure appears in Table 1. The range is from 6.578 for the 13th, and smallest-appearing, figure to 22.600 for the 17th, and largest-appearing, figure. These MRVs represent locations of the 30 figures on a continuum of apparent area as perceived by the Ss. The figures are presented in order of apparent area in Fig. 1. The two-way analysis of variance by ranks on these MRVs resulted in a chi square of 329.52 ($p < .01$) which confirmed the first hypothesis, viz., that figures from the defined population, although of equal area, are perceived as differing in area. Kendall's (1962) Coefficient of Concordance (W) was calculated as .253

($p < .01$), indicating significant agreement among the 45 sets of rank values and further confirming the first hypothesis.

For each of the 15 Ss tested twice, the rank-order correlation (ρ) was calculated between the 30 rank values of the figures obtained from the first test session and the 30 rank values of the figures obtained from the second test session. The 15 coefficients ranged from .552 to .955 and were significant at the .01 level. These results confirmed the recurrence of the perception of apparent differences in area in the same order on a second occasion.

Before R was calculated between the MRVs of the figures and the figural shape variables, all these independent variables were tested for the assumptions of the Pearson r . *Width to height ratio*, which showed a curvilinear relationship with certain of the other independent variables, was excluded from further consideration. The computed R was found to be .753. By the use of Wherry's formula (DuBois, 1957) a corrected R of .674 ($p < .01$) confirmed the second hypothesis, viz., that differences in the perceived areas of the figures result from the combined effect of certain shape variables.

In calculating R , the Pearson r s between the dependent variable and each of the six independent variables were found and appear in Table 2. These results indicated that for figures employed in this study *perimeter*, *number of turns*, *breadth*, and *external area* vary inversely with apparent area.

Pearson r s were also calculated between each pair of independent variables and appear in Table 3. Perimeter, number of turns, length, and external area were found to be significantly ($p < .01$) intercorrelated.

The following regression equation was obtained in the calculation of R :

$$X'_0 = 42.553 - .898X_1 + .480X_2 + .378X_3 - .952X_4 + .257X_5 - .086X_6$$

This equation permits the prediction of the dependent variable, i.e., apparent area, in other samples of 30 drawn from the same population of figures.

Descriptive data on the dependent variable and each of the six independent variables appear in Table 4. Seen first are the sum, the mean, and the standard deviation of the MRVs of the 30 figures in the generated sample as perceived by these 45 Ss. Following this first row of statistics are the sum, the mean, and the standard deviation of each of the six shape variables as measured in these same 30 figures.

DISCUSSION

The most interesting finding of this study was the fact that those shape variables which were individually correlated with apparent area were correlated in the negative direction. In other words, figures which were the most "compact," or the least "stretched-out," were perceived as relatively large. Since compact figures have the most "goodness" according to Gestalt theory, this finding might very well have been predicted by the *Law of Pragnanz*. This law states essentially that relatively good figures tend to be perceived more readily than figures of less goodness (Vernon, 1954).

However, this finding is directly opposed to those of Anastasi and Smith who found the opposite effect, namely, that "stretched-out" figures were perceived as relatively large. A partial explanation for this divergence in results might be found in the nature of the patterns used in these studies. In the present experiment, filled figures were used. In the studies of Anastasi and Smith, outline patterns were used. In the latter cases the Ss may have unknowingly directed their attention to the shapes of the outlines rather than to the areas which they encompassed. This attention may have led to the estimation of the lengths of the perimeters rather than the magnitudes of the areas.

Table 2
Pearson Correlation Coefficients (r) Between Mean Rank Values and Six Shape Variables

Variable	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆
X ₀	-.572**	-.388*	-.279	-.429*	-.504**	-.143
Note—	X ₁ = Perimeter			X ₄ = Breadth		
	X ₂ = Number of Turns			X ₅ = External Area		
	X ₃ = Length			X ₆ = Angle of Tilt		
	X ₀ = Mean Rank Value					
	*p < .05					
	**p < .01					

Table 3
Pearson Correlation Coefficients (r) Among Six Shape Variables

Variable	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆
X ₁		.897**	.778**	.259	.946**	.212
X ₂			.689**	.155	.804**	.200
X ₃				-.329	.788**	.487**
X ₄					.257	-.337
X ₅						.297
X ₆						
Note—	X ₁ = Perimeter			X ₄ = Breadth		
	X ₂ = Number of Turns			X ₅ = External Area		
	X ₃ = Length			X ₆ = Angle of Tilt		
*p < .01						

Table 4
Sums, Means, and Standard Deviations of Mean Rank Values and Six Shape Variables

Variable	Sum	Mean	SD
X_0	465.00	15.5	4.4
X_1	1426.00	47.5	10.6
X_2	752.00	25.1	7.1
X_3	405.40	13.5	3.1
X_4	197.98	6.6	1.0
X_5	806.50	26.9	14.7
X_6	752.80	25.1	15.2
Note—	X_1 = Perimeter		X_4 = Breadth
	X_2 = Number of Turns		X_5 = External Area
	X_3 = Length		X_6 = Angle of Tilt
	X_0 = Mean Rank Value		

Note—In the calculation of these statistics for the Angle of Tilt, negative signs were disregarded.

Note—In the calculation of these statistics for Perimeter, Length, and Breadth, linear units were used, while calculations for External Area involved square units.

A possible explanation for the opposite effect found in the present study can be found in McCormick (1957) where the relative apparent sizes of black numbers and letters on a white background and white numbers and letters on a black background are discussed. It had been found that the black numbers and letters appeared smaller. Presumably this finding had occurred because the white background "irradiated" on to the black numbers and letters while in the case of the white numbers and letters the irradiation was outward or simply not effective. In the present study essentially the same effect may have taken place, namely, those figures which were the least compact, or which had the greatest perimeters, numbers of turns, lengths, and external areas, were the most effected by the irradiation of their white backgrounds and consequently appeared the smallest.

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NOTES

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