The effects of duration, area, and background intensity on the visual intensity difference threshold given by the forced-choice procedure: Derivations from a statistical decision model for sensory discrimination

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A model for visual discrimination is outlined. This assumes that detection is based on separating signal and noise by means of a statistical decision procedure, and that the noise arises from physical and biological sources. When the model is applied to such traditional detection procedures as the method of constant stimuli, it successfully predicts the laws of spatial and temporal summation and the form of the Weber function. The model is extended to the forced-choice procedures. It is found that it predicts departures from the classical laws for spatial and temporal summation and an increase in the slope of the Weber function when the pulsed forced-choice procedure is used, and data are presented that agree with the predictions. It is shown that data obtained with forced choice against a continuous background are consistent with the assumption that the S either covertly detects or fails to detect an increment at each of the two alternative locations and this determines his final overt response, rather than that he directly compares the inputs at the two locations. A comparison between results obtained with the forced-choice procedure with a pedestal, or with the continuous forced-choice procedure, provides further evidence that Ss employ a double detection strategy in the latter case.

I. INTRODUCTION

At an early stage, the systematic study of sensory discrimination revealed that marked effects on the threshold were produced by such stimulus parameters as background intensity and the duration or area of the stimulus increment. These findings were enshrined as eponymous laws (Weber's law, Bloch's law, Ricco's law, Piper's law, etc.), and although similar effects have been found not only in vision but in other modalities too, the provision of a coherent theoretical treatment to explain and relate the various findings in a single modality, let alone over the range of modalities, has proved a difficult problem.

The way to a possible solution to this problem was opened by the suggestion that repeated presentations of a stimulus of nominally constant intensity will produce effects in the S that vary from trial to trial, and that the quantal variability of light (in the case of visual intensity discrimination to which we shall limit the present discussion) was a source of variation acting to limit discrimination (Thurstone, 1927; Hecht, Schlaer, & Pirenne, 1942; Rose, 1948; Aguilar & Stiles, 1954). The problem faced by the sensory mechanisms in making a discrimination was then essentially that of distinguishing a signal from noise, a statistical decision (Tanner & Swets, 1954; Barlow, 1956; Gregory, 1956; Swets, Tanner, & Birdsall, 1961).

However, "fluctuation theory" (the name sometimes given to the hypothesis that the threshold is limited by the quantum variability of light) was not wholly successful in explaining the relation between the threshold increment, ΔI , and the background intensity against which it is measured, I (the relation between ΔI and I will be referred to as the "Weber function"). If a stimulus background of intensity I is repeatedly presented, the

numbers of quanta absorbed by the eye on successive trials will be described by a Poisson distribution, with a mean proportional to I and an SD proportional to $I^{1/2}$, which suggests that ΔI should be proportional to $I^{1/2}$. But Aguilar and Stiles (1954) found that the Weber function approximated a square root law only for a limited range of values above the absolute threshold; as the magnitude of I increased beyond this level the data tended towards the linear relation between ΔI and I traditionally known as Weber's law. Barlow (1957) confirmed this finding and asked "can reasons be found for the departures from the theoretical predictions which occur when the test stimulus is of long duration or large area, and when the background intensity is high?"

The relative but not complete failure of the theory suggests that it must be inadequate in some important respect, and what this may be is indicated by the observation that the theory's proponents have almost completely discounted or failed to consider sources of variation other than the physical variability of the stimulus. But if we allow that variability may arise in the nervous pathways as well as in the physical stimulus, it becomes evident that the prediction of a square root law from the quantum variability of light must rest on one of the following assumptions: (1) The discriminatory response is determined at the receptor level, immediately on the absorption of quanta—but there is ample evidence, reviewed by Treisman (1966), that the behavioral threshold is more centrally determined. (2) The absorption of a quantum or quanta triggers a sequence of neural events that result in "messages" being transmitted to the central locus at which the discriminatory response is determined, the magnitude of the total message indicating the size of the stimulus; the variability or "noise" in this central transmission is negligible, so that the variation of the central message from trial to trial reflects only the Poisson variability of the stimulus. But it is almost inconceivable that the processes of impulse generation and transmission across synapses should contribute very much less noise than the limiting quantum variability of light. (3) A message is transmitted centrally, and biological noise does arise during the course of this central transmission, but its characteristics are such as not to alter the form of the law predicted from a consideration of the variability of the stimulus alone (McGill, 1967). This third assumption is tenable but, as we have seen, the square root law that results from it is not consistent with the whole range of experimental findings.

It is apparent that if we wish to determine whether or not a signal-noise theory of sensory discrimination can explain the effects of stimulus parameters on the size of the threshold, we must take into account all the major sources of noise arising in the path from the physical stimulus to the central discriminatory mechanisms. Unfortunately our knowledge of the visual pathways is insufficient for us to use physiological observations to characterize definitively all sources of biological noise. However, Treisman (1964a, 1966) has considered a simplified schema of

Fig. 1. The relation between the physical stimulus intensity, I , and its central effect, E , that underlies measurement of the threshold by detection procedures, such as the method of constant stimuli. The psychometric function given by the MCS relates $P(Y)$, the probability of the response "Yes," to the physical magnitude of the stimuli presented. The three distributions on the central decision axis, E , correspond to three stimuli: I_N , the background stimulus; $I_{0.50}$, the background + increment whose mean central effect coincides with the criterion E_c ; and $I_{0.84}$, the stimulus whose mean central effect lies 1 SD above E_c and which therefore gives $P(Y) = 0.84$. The increment threshold for the method of constant stimuli is given by $\Delta I = I_{0.50} - I_N$. When $I_N = 0$ this procedure gives the absolute threshold, which is then $I_0 = I_{0.50}$.

the visual system that makes minimal assumptions about the physical and biological sources of noise. This model was applied to the increment threshold, the threshold for the detection of a transient increase in the intensity of an area of a uniformly illuminated background field, as determined by such classical procedures as the method of constant stimuli (MCS). It was found that the model successfully predicted the form of the Weber function, giving a square root section at low background intensities with a transition to a linear section as I increased, and it also predicted the form of the spatial summation and the temporal summation curves. Thus, despite the simplification necessarily involved in framing the model, it appeared able to explain a number of different phenomena and to relate them to one another; moreover, the essential ideas are applicable to other modalities, suggesting an explanation for the ubiquity of Weber's law.

The present paper is an attempt to test and develop the model further by applying it to a new area. Treisman (1966) applied it to what may be called the "detection procedures," i.e., those procedures in which the opportunity for an increment occurs once in each trial and the S may detect or fail to detect it. Here we shall apply the model to the "forced-choice procedures" (FCP) in which the increment may occur at two or more

alternative locations and the S must decide which one contained it. As we shall see, in this case the predictions differ in some respects from those for the classical procedures, and data are available against which some of these predictions may be tested. But before we consider the forced-choice procedures it will be useful if we first summarize the main assumptions of the model, which is described more fully elsewhere (Treisman, 1964a, 1966), and illustrate how it leads to predictions for the detection procedures.

II. DETECTION PROCEDURES

Figure 1 illustrates the assumptions made about the overall strategy of discrimination, when the absolute or increment threshold is determined by such a procedure as the Method of Constant Stimuli. We use I to represent stimulus intensity (e.g., in quanta/sec deg²). The constant background field is at intensity I_N (when the absolute threshold is to be determined, $I_N = 0$), and one of a set of intensity increments, ΔI_v , is added to this on each trial. In the MCS several values of ΔI_v are used, presented in a random sequence on successive trials. Each increment is presented for t_i sec over an area A_i deg². The S is asked to respond "Yes" or "No" on each trial, and for each value of $I_v = I_N + \Delta I_v$ that is used the frequency of the response "Yes," $P(Y)$, is found. A plot of $P(Y)$ against I is called the psychometric function and usually approximates a normal ogive; this is shown in the lower half of Fig. 1. The increment threshold, ΔI , is often taken as $\Delta I = I_{0.50} - I_N$, where $I_{0.50}$ is the stimulus such that $P(Y | I_{0.84}) = 0.84$. The SD of the threshold, σ_I , can be taken as $\sigma_I = I_{0.84} - I_{0.50}$.

We suppose that on each trial the stimulus gives rise to a central message, the magnitude of this central effect being represented on a scale or decision axis, E . When the background intensity I_N is presented without an added increment, the values of E that result are represented by the "noise" distribution centered on E_N , the latter being the mean effect of I_N . Similarly, a series of presentations of any other value of I , such as $I_{0.50}$, will give a similar distribution of central effects centered on $E_{0.50}$. As a first approximation we suppose that for the small range of values of I used in determining a threshold these distributions can be taken to be normal with equal variance, σ_E^2 . A basic assumption is that the discriminatory mechanisms accumulate statistical information about the values of E that are produced when the background alone is presented, and a positive detection response is given whenever an input departs far enough from the expectations thus built up. This is done by choosing a criterion E_c at a value such that the false positive rate [$FPR = P(Y | I_N)$] does not exceed an acceptable limiting value, ϵ . [This is the Neyman-Pearson criterion, which seems the most appropriate for the traditional psychophysical procedures (Neyman & Pearson, 1933; Treisman, 1964b).] Then $E_c = E_N + z\sigma_E$, where z is the standardized normal deviate corresponding to the limiting false positive rate (thus $\epsilon = 0.0227$ would give $z = 2$). If on a given trial the input E exceeds E_c the S will respond "Yes"; if it does not, "No." It follows that the mean central effect of $I_{0.50}$, which is $E_{0.50}$, will coincide with E_c , and thus $E_c - E_N = z\sigma_E$ will correspond to ΔI . Similarly, the mean central effect of $I_{0.84}$ will be $E_{0.84}$ and thus σ_I will correspond to σ_E . The classical procedures have usually been used with well-trained Ss giving very small false positive rates. In such cases we might expect z to be of the order 2-6 (Treisman, 1965).

It appears that in order to make predictions about ΔI we need to know the determinants of σ_E , i.e., the sources of noise, and the form of $E = f(I)$, the "transducer function" relating values on the decision axis, E , to I . This information can be derived from a schema of the visual pathway. For simplicity the pathway is considered to have three stages: (1) the path from the physical

stimulus source to the retinal receptors; (2) transmission from the receptors across the retina to the ganglion cells; (3) transmission from the ganglion cell to the central locus at which the threshold decision is made.

We shall now describe the schema as it applies to the determination of the threshold by the method of constant stimuli, and we start by considering how the "noise" distribution is built up when the background alone is presented.

(1) The first stage is essentially that of "fluctuation theory." If a stimulus of intensity I and area A is exposed to the eye for duration t , the number of quanta absorbed by the receptors is given by $fAtI$, where f is the fraction of the quanta incident at the cornea that is eventually absorbed by visual pigment. This number is a Poisson variable.

(2) From the receptors to the ganglion cells. The "noise" distribution is built up from samples of the background, which may be of large area and may be exposed for some time. We suppose that the sensory mechanisms take each sample of the retinal input over an area A_r and for a duration t_r , so that the mean number of quantal absorptions falling in a sample of the background is given by $fA_r t_r I_N$. The absorption of quanta of light causes the decomposition of visual pigment molecules, and the transmission of corresponding messages across the retina. Biological noise could arise in two ways: the spontaneous generation of messages that are confusable with those arising from the light stimulus, and variability in the transmission of the visual message along the pathway. It is convenient to consider the former source as though it arose wholly at this stage, as would be the case if it were all caused by the spontaneous decomposition of molecules of visual pigment. It would then be equivalent to the effect of an additional constant background illumination or "dark light," I_n . Then m , the mean number of visual pigment decompositions falling within a sample of the background, is given by $m = fA_r t_r (I_N + I_n)$, and its variance is also equal to m .

The intraretinal effects that are triggered by the molecular decompositions are transmitted across the retina and converge on the ganglion cells. We can expect that the mechanisms underlying light adaptation will greatly reduce the volume of messages transmitted from the receptors across the retina, especially when stimulus values are high. Various models for this reduction have been studied by computer simulation (Treisman, 1966); it appears that, for the predictions we are concerned with here, the effects of light adaptation need not be explicitly included. Since there is considerable evidence that transmission through the retina to the ganglion cells is linear (Treisman, 1966), we can take m to represent the convergent effect acting on the ganglion cells to initiate activity in the latter.

(3) Transmission from the ganglion cell to the locus of response determination. It is convenient to consider the output from the ganglion cell as consisting of a number, m' , of "sensory messages," each of mean magnitude s (which might correspond, for example, to a constant contribution to the firing of the cell), any nonlinearity in the visual pathway then being contained in the transformation from m to m' . Then E_k , the central effect on the decision axis of a given sample k , would be the sum of the corresponding m'_k sensory messages,

$$E_k = \sum_{j=1}^{m'_k} s_j,$$

where s_j are the individual sensory messages. In this schema the "transducer function," $E = f(I)$, is determined by $m' = f(m)$, where f might, for example, be a logarithmic or power function. The computer simulation study provided predictions for various assumptions about the parameters and mechanisms of the model.

When these predictions were compared with data they indicated that the logarithmic function was extremely implausible, but that the results were compatible with existing data provided the transducer function was taken to be a power function with an exponent somewhere in a range of approximately 0.5-1.2 (Treisman, 1966). It is then simplest for the present purpose to consider the case where $m' = m$, which gives us, for the expected value of E_N ,

$$E_N = s f A_r t_r (I_N + I_n). \quad (1)$$

This gives the overall functional relation between values on the E and I scales. We now need to derive the variance of distributions on the decision axis, σ_E^2 . This arises from the Poisson variability of m (i.e., from the quantum variability of light, and the spontaneous generation of messages or "dark light"), and from the variability attaching to s . This last will arise from variation in the transmission of neural messages along the visual pathway, which will make the magnitude of the "sensory messages" vary about the mean s , so that it has a variance σ_s^2 ; we take it that this variation is normal.

Before proceeding to derive σ_E^2 we must consider another relevant parameter, which may be called the intrasample correlation between sensory messages, r_s . To illustrate this, consider a sequence of N samples, giving central effects $E_1, E_2, \dots, E_i, \dots, E_N$, each being the sum of m' sensory messages, and N being large. Then $E_i = s_{i1} + s_{i2} + \dots + s_{ij} + \dots + s_{ik} + \dots + s_{im'}$, where of course the second subscripts are assigned to s in an essentially random manner. It follows that the product-moment correlation, r_{jk} , between any pair of sensory messages s_j and s_k , $j \neq k$, over the N samples should be the same for all pairs s_j and s_k and can therefore be represented as a single value, r_s . Now, whereas the correlation between two random variables can have any value between -1 and $+1$, this is not true of r_s when $m' > 2$, the lower limit then being greater than -1 . It is obvious that when we have three random variables giving $r_{12} = r_{13} = r_{23} = r_s$, each variable cannot be correlated at $r_s = -1$ with both the others, and in general $r_0 \leq r_s \leq 1$, where the lower limit is given by $r_0 = -1/(m' - 1)$. When m' is at all large, as will almost always be the case with visual stimuli, r_0 will be negligibly different from zero. We may also note that there are factors that would tend to make r_s substantially greater than this necessary lower limit. For example, variation in the level of oxygenation at different phases of the pulse cycle or other effects of the cardiac and respiratory cycles, or moment-to-moment variation in the level of reticular activation might make the values of the component "sensory messages" summed in a single sample vary for the most part in the same direction from the mean, s . It has been shown that the reaction time is affected by the phase of the pulse cycle (Birren, Cardon, & Phillips, 1963), as is the amplitude of the EEG (Cohn, 1962), and cortical evoked responses to a visual stimulus are more similar when determined at the same phase of the cardiac or respiratory cycle (Callaway & Buchsbaum, 1965).

With these assumptions and Eq. 1 we can now write the variance of E_N as

$$\sigma_E^2 = r_s \sigma_s^2 f^2 A_r^2 t_r^2 (I_N + I_n)^2 + (s^2 + \sigma_s^2) f A_r t_r (I_N + I_n). \quad (2)$$

We saw from Fig. 1 that the increment threshold determined by the MCS is $\Delta I = I_{0.50} - I_N$. $I_{0.50}$ would be the intensity of background plus increment giving a central distribution whose mean coincides with E_c . In considering how the samples that constitute this distribution are made up we must take account of the parameters of sampling for both the background and the increment. We saw that a sample of the background is taken over

an area A_r and duration t_r . It is simplest to assume that in a trained S these will usually approximate to the area A_i and duration t_i for which the increment is given. An exception will arise when the physical stimulus area or duration is less than the physiological lower limits on the fineness of the sampling that the visual system is capable of: we suppose that it is not possible for the visual mechanisms to take samples of an area less than α (perhaps corresponding to the size of the receptive field) or duration less than τ . Then in general $A_r = \max(A_i, \alpha)$ and $t_r = \max(t_i, \tau)$, so that α and τ are minima for A_r and t_r . We can now use the model to derive predictions.

Since $E_c = E_{0.50}$, we can write $E_c - E_N = z\sigma_E = sf[A_r t_r(I_N + I_n) + A_i t_i \Delta I] - sf A_r t_r(I_N + I_n) = sf A_i t_i \Delta I$, so that

$$\Delta I = \frac{z\sigma_E}{sf A_i t_i} = \frac{z}{sf A_i t_i} [r_s v^2 f^2 A_r^2 t_r^2 (I_N + I_n)^2 + (1 + v^2) f A_r t_r (I_N + I_n)]^{1/2} \quad (3)$$

where $v = \sigma_s/s$, the coefficient of variation of the sensory message.

Since Eq. 3 expresses the threshold as a function of background intensity and the duration and area of the increment, it predicts the form of the Weber function and of spatial and temporal summation. The predictions are shown more clearly if we simplify the equation. Since it is very likely that $v^2 \ll 1$, when $f A_r t_r (I_N + I_n)$ is small the second expression in the square brackets in Eq. 3 will dominate the relation which will then approximate to

$$\Delta I = \frac{z A_r^{1/2} t_r^{1/2} (I_N + I_n)^{1/2}}{f^{1/2} A_i t_i} \quad (4)$$

but when $f A_r t_r (I_N + I_n)$ is large the first expression in the square brackets will be the most important and the equation will approximate to

$$\Delta I = \frac{z r_s^{1/2} v A_r t_r (I_N + I_n)}{A_i t_i} \quad (5)$$

The Effect of Background Intensity

Let us suppose that I_N alone is varied, being increased from zero. When $I_N = 0$, Eq. 3 gives I_0 , the absolute threshold. As I_N is increased above this level, it will at first be negligible as compared with I_n so that the threshold will be substantially constant at the absolute threshold level. If we plot $\log \Delta I$ against $\log I_N$ we will have a straight line with a slope of zero. At higher values of I_N the relation will be described by Eq. 4, giving a square root law represented by a line with a slope of 0.5 on the log-log plot. At still higher background intensities, Eq. 5 will apply, giving a transition to a linear relation (Weber's law) represented by a straight line with a slope of 1. These predictions agree, of course, with the experimental result.

It is of interest to consider the effect of the parameter A_i , the area of the increment, on the Weber function. When Eq. 4 applies, the larger $A_r = A_i$ is, the smaller will the threshold be. But the larger A_r is the larger will $f A_r t_r (I_N + I_n)$ be, and so the transition to Eq. 5 will occur at lower values of I_N if A_i is increased. Thus when A_i is large Weber's law will be approximated sooner, and the Weber functions for different values of $A_i \geq \alpha$ will converge at high values of I_N . Corresponding predictions can be made for the effect of t_i as a parameter of the Weber function. These effects are shown by the experimental data (Barlow, 1957).

The Effect of Increment Duration

If t_i alone is varied, then when $t_i \leq \tau$, $t_r = \tau$ and so t_i is the only variable on the right-hand side of Eq. 3, so that the equation approximates to $\Delta I = k/t_i$, where k is a constant. This is Bloch's law, and if we plot $\log \Delta I$ against $\log t_i$ it will give a straight line with a slope of -1 .

If $t_i > \tau$ and $f A_r t_r (I_N + I_n)$ is small, Eq. 4 will apply; this can be written as $\Delta I = k'/t_i^{1/2}$, where k' is a constant, giving a line with a slope of -0.5 . As t_i increases further Eq. 5 will come to apply, making ΔI constant, so that there will be a transition to a line of zero slope. It is evident that when the parameters I_N or A_i are large then for $t_i > \tau$ we will get a more rapid or immediate transition to the relation described by Eq. 5.

The Effect of Increment Area

Exactly similar predictions can be derived for variation in A_i .

The predictions above, and other predictions, have been found to match the data surprisingly well (Treisman, 1966), so that although the model that has been outlined here is clearly oversimple as compared with the redoubtable complexities of the system it attempts to explain, it nevertheless appears to have sufficient explanatory force to justify examining and testing it further.

III. FORCED-CHOICE PROCEDURES

The forced-choice procedure (FCP) has recently become popular as a method for determining the threshold, and as the relations between the threshold determined in this way and the stimulus parameters may differ in some respects from those obtained using classical procedures, it is of interest to examine whether or not the model described above can be applied successfully to these procedures.

We shall consider here only the forced choice between two alternatives. In this procedure two stimuli are presented on each trial, I_N and $I_v = I_N + \Delta I_v$. These might be presented simultaneously in two different spatial locations, positions a and b ; or they might be presented in succession, at two indicated times of occurrence, a and b . In each case the S is asked to indicate which position or time of occurrence contained the increment by responding "a" or "b," "Left" or "Right," "First" or "Second," etc.

In this procedure there is a further important experimental variable. The background intensity, I_N , may be continually present during the trial, and the two stimuli are then produced by adding the increment, ΔI_v or 0, to this background at the indicated positions or times of occurrence. We shall call this "forced choice with continuous background," or "continuous forced choice." Or the background field may be dark (i.e., I_N alone is present), and I_N is presented as a single pulse covering an area A_i for a duration t_i at one location and $(I_N + \Delta I_v)$ is given with the same area and duration, at the other location. Thus a temporal forced-choice trial might consist of: an initial dark interval—presentation of a stimulus of area $A_i = 1 \text{ deg}^2$ illuminated at intensity I_N for a duration $t_i = 20 \text{ msec}$ —an interstimulus interval (ISI) of 1 sec—presentation of a stimulus of area $A_i = 1 \text{ deg}^2$ illuminated at intensity I_v for a duration $t_i = 20 \text{ msec}$. The S would then indicate which stimulus was the more intense. We shall call this procedure "forced choice with pulsed stimulation," or "pulsed forced choice."

A third procedure, the "pedestal FCP," has features of both the others. A background of intensity I_N is continually present during each trial, and the increment ΔI_v , of area A_i and duration t_i , is added at one or the other of the locations a and b . But at both locations on each trial a constant "pedestal" is always added. This is an increment I_p of area A_i and duration t_i . Thus on any trial the increment at one location will be I_p and at the other

Fig. 2. This illustrates distributions of differences between the central effects of I_v and I_N , when both are presented on a forced-choice trial. (We assume a differencing strategy.) The differences axis is scaled in units of $\sigma_D = \sigma_{E_v - E_N}$, the SD of the distribution of differences, with the origin at $D_N = E_v - E_N = 0$, the mean of the distribution of differences given when a zero increment $\Delta I_v = 0$ is used. The criterion $D_c = D_N = 0$. $D_{0.75} = E_{0.75} - E_N = D_N + 0.6745\sigma_D = 0.6745\sigma_D$, where $I_{0.75} = I_N + \Delta I_{pfc}$ is the stimulus giving $P(C) = 0.75$. $D_{0.977} = E_{0.977} - E_N = 2\sigma_D$; $I_{0.977}$ then gives $P(C) = 0.977$. The psychometric function obtained when $P(C)$ is plotted against I is shown in the lower half of the figure.

($I_p + \Delta I_v$). The S must then indicate at which location ΔI_v was given.

We shall analyze the three procedures separately.

IV. THE PULSED FORCED-CHOICE PROCEDURE

It is difficult to apply the model of Fig. 1 to this procedure. When an increment is added to a continuous background as in the detection procedures, the latter provides a continuing opportunity to take numerous samples that serve to define the E_N distribution with little ambiguity. We do not know the extent to which the S's knowledge of the form of the E_N distribution is dependent on previous perceptual experience (Treisman, 1964b) and on concurrent sampling of the background during the experimental trials; in either case he needs to know E_N to locate E_c , the criterion, and the continuous presence of the background will provide this information. But when the stimuli are given as isolated pulses only, the S is not in a position to accumulate and utilize statistical information about the effects of I_N in the same way. It seems likely that he will then be forced to use a different procedure for selecting a response, and a common model for this is given below (Tanner & Swets, 1954).

We suppose that on each trial the S is presented with a pair of stimuli, I_N and I_v , given at locations a and b (the assignment being randomized over trials), and he must report which stimulus is the more intense. The E records the proportion of "correct" responses, $P(C)$, for each value of ΔI_v that he uses, and takes the threshold for this condition, ΔI_{pfc} , as that value of ΔI_v that gives $P(C) = 0.75$.

On each trial the S records two central effects, E_a and E_b . He takes the difference between these effects, examines whether it is positive or negative and chooses his response accordingly, i.e., if

($E_a - E_b \geq 0$) he responds "a," otherwise "b." If E_a is in fact E_v , the effect of I_v , and he responds "a," or if E_b is E_v and he responds "b," the response will be correct. So that $P(C)$ will correspond to the proportion of trials on which the difference $D = E_v - E_N \geq 0$. (If there is bias, i.e., the criterion for selecting between the responses "a" and "b" is not 0, this will add some extra variance, but as long as the bias is not large we can ignore it.)

Figure 2 illustrates the distribution of $D = E_v - E_N$. When $I_v = I_{0.75} = I_N + \Delta I_{pfc}$, the stimulus giving $P(C) = 0.75$, then 0.75 of the area of the distribution of differences will lie to the right of the criterion, $D_c = 0$. To derive the relation between ΔI_{pfc} and I_N we need to take account of the variance of this distribution and the relation between E and I .

As before, we suppose that $A_r = \max(A_i, \alpha)$ and $t_r = \max(t_i, \tau)$ but we note that I_N is presented only over the area A_i and for the duration t_i , so that

$$E_N = sf(A_i t_i I_N + A_r t_r I_N) \quad (6)$$

and

$$E_{0.75} = sf[A_i t_i (I_N + \Delta I_{pfc}) + A_r t_r I_N]. \quad (7)$$

Then

$$E_{0.75} - E_N = sf A_i t_i \Delta I_{pfc}. \quad (8)$$

The variance of the E_N distribution would be

$$\begin{aligned} \sigma_{E(pfc)}^2 &= r_s^2 \sigma_s^2 f^2 (A_i t_i I_N + A_r t_r I_N)^2 \\ &\quad + (s^2 + \sigma_s^2) f(A_i t_i I_N + A_r t_r I_N) \end{aligned} \quad (9)$$

(we will again take σ_E as effectively constant over the range of values involved in determining a threshold).

It is evident from Fig. 2 that

$$E_{0.75} - E_N = 0.6745\sigma_D = 0.6745\sigma_{E_{0.75} - E_N}. \quad (10)$$

From Eqs. 8 and 10 we have

$$\begin{aligned} \Delta I_{pfc} &= \frac{0.6745\sigma_D}{sf A_i t_i} \\ &= \frac{0.6745 \cdot 2^{1/2} \cdot (1 - r_E)^{1/2}}{f A_i t_i} [r_s^2 f^2 (A_i t_i I_N + A_r t_r I_N)^2 \\ &\quad + (1 + v^2) f(A_i t_i I_N + A_r t_r I_N)]^{1/2} \end{aligned} \quad (11)$$

where r_E , the intersample product-moment correlation, must not be confused with r_s , the intrasample correlation. r_E is the correlation between the two values of E across trials, and will lie between -1 and $+1$. With temporal forced-choice and long interstimulus intervals, r_E might be close to 0. At shorter interstimulus intervals and in spatial forced choice, factors such as spontaneous slow changes in the level of "amplification" of the sensory messages (Wertheimer, 1953) might produce positive values, and effects of the cardiac and respiratory cycles might produce positive or negative values, depending on the length of the ISI.

When $f(A_i t_i I_N + A_r t_r I_N)$ is small, Eq. 11 will reduce to

$$\Delta I_{pfc} = \frac{0.9539(1 - r_E)^{1/2}}{r_E^{1/2} A_{t_i}} (A_{t_i} I_N + A_{t_i} I_n)^{1/2} \quad (12)$$

and when it is large the equation will reduce to

$$\Delta I_{pfc} = \frac{0.9539(1 - r_E)^{1/2}}{A_{t_i}} r_E^{1/2} v (A_{t_i} I_N + A_{t_i} I_n). \quad (13)$$

We can now derive predictions for the effects of the stimulus parameters when the pulsed forced-choice procedure is employed. We consider variation in each parameter, with the others held constant.

Variation in t_i . This will have different effects, depending on the magnitude of I_N . (a) Consider I_N large, so that Eq. 13 applies and $A_{t_i} I_n$ is negligible as compared with $A_{t_i} I_N$. Then Eq. 13 reduces to

$$\Delta I_{pfc} = 0.9539(1 - r_E)^{1/2} r_E^{1/2} v I_N \quad (14)$$

which implies that the threshold is constant and independent of t_i however small this is: Bloch's law will not be shown even at stimulus durations well below the critical duration, τ (as determined by a classical procedure). If $\log \Delta I_{pfc}$ is plotted against $\log t_i$ this will give a straight line with zero slope. (b) Let $f(A_{t_i} I_N + A_{t_i} I_n)$ be small and $A_{t_i} I_n$ negligible as compared with $A_{t_i} I_N$. Then Eq. 12 will apply, and it can be written as $\Delta I_{pfc} = k/t_i^{1/2}$, where k is constant, for the whole range of values of t_i . Thus as I_N decreases the zero slope given by Eq. 14 (when $\log \Delta I_{pfc}$ is plotted against $\log t_i$) will change to a slope of -0.5 , and this will apply for values of t_i both above and below τ . Again, Bloch's law is not shown. (c) When I_N is small enough for $A_{t_i} I_N$ to be negligible as compared with $A_{t_i} I_n$, i.e., at or near the absolute threshold, Eq. 12 implies that ΔI_{pfc} will be inversely related to $t_i^{1/2}$ when $t_i \geq \tau$, and Bloch's law, a slope of -1 on the log-log plot, will be shown when $t_i < \tau$.

Figure 3 shows data obtained by Leshowitz, Taub, and Raab (1968), using the two-alternative temporal forced-choice procedure for a number of values of I_N (the parameters of the curves give $\log I_N$, in log mL). The open circles give values of the pulsed forced-choice threshold. We see that the temporal summation curves for $\log I_N = 2.3$ log mL and 1.3 log mL are flat or almost flat over a range of values of t_i from 0.07 msec to 320 msec. As the background intensity is reduced there is a tendency for the slope of the curves to get steeper, the curves appearing similar both above and below τ (which here is in the region of 32 msec when the background intensity is 1.3 log mL). The approximate overall slopes as $\log I_N$ decreases from 2.3 to 2.3 are: 0, -0.04 , -0.14 , -0.25 , and -0.25 . These results are in accordance with predictions (a) and (b) above: both above and below τ there is a zero slope for I_N high, with a change to a negative slope as I_N is decreased. Presumably a slope of -0.5 is not shown because a sufficiently low value of I_N has not been used. [In the computer simulation study it was found that the slope of partial summation curves was affected by n , the exponent of the transducer function $E = I^n$, when this was taken to be a power function (Treisman, 1966). We have here taken n to be 1; it remains to be seen whether with other acceptable values of n the terminal slope as I_N is decreased (when the conditions for prediction (b) apply) may not be greater or less than -0.5].

Variation in A_i . The predictions obtained are similar to those for variation in t_i . As yet these have not been experimentally tested.

Variation in I_N . Inspection of Eqs. 12 and 13 suggests that the

Fig. 3. The relation between the forced-choice threshold and the duration of the increment: data obtained by Leshowitz, Taub, and Raab (1968) by a temporal forced-choice procedure. Each trial began with a warning click; two additional clicks spaced 0.8 sec apart occurred 0.8 sec after the start of the trial and accompanied the two flashes. ○: pulsed forced-choice procedure; ●: continuous forced-choice procedure. The parameters of the curves are the intensities of I_N in log mL. The results of two Os were averaged. The threshold is the value of the increment giving $P(C) = 0.75$.

Weber function will show a square root section at low values of I_N , and become linear at high values, as in Eqs. 4 and 5. However, any prediction about the form of the Weber function is subject to two further considerations. First it should be noted that predictions derived from Eq. 11 will depend on the extent to which r_E is constant, and should take into account factors that might cause r_E to vary. One such factor would be the length of the interstimulus interval. It is conceivable that r_E will decrease as ISI is increased, being maximal in the spatial forced-choice case and falling to 0 at long interstimulus intervals, and it will be of interest to examine this experimentally. It is also conceivable that r_E might decrease as I_N increases. Barlow, FitzHugh and Kuffler (1957) noted that changes in the neural organization of the cat's retina occur during the process of dark adaptation, the area of the receptive field increasing, and the inhibitory periphery disappearing when the eye has adapted sufficiently to the dark. If at the higher values of I_N the receptive fields are more tightly organized and lateral inhibitory effects more marked, this might tend to reduce the similarity of response of neighboring areas of retina and so reduce r_E . If this effect occurs to a significant extent then as I_N is increased r_E will decline, and consequently ΔI_{pfc} will increase as a function of I_N more rapidly than Eq. 11 suggests.

A second factor that must be considered is the effect of light adaptation, or its absence. In Sections I and II, the statistical decision model was applied to procedures concerned with sensitivity to small departures from an intensity level to which the eye is adapted. The transducer function, $E = f(I)$, was assumed to describe the response of the visual pathway at and near such an adaptation level, and on this assumption was found to be a power function, $E = I^n$, with n ranging about 1.0. Consequently we have developed the model for the case where $n = 1$. But in the pulsed FCP, I_N and $(I_N + \Delta I_v)$ are both flashed

onto a dark-adapted retina, not a retina adapted to I_N ; thus, especially when we are considering variation in I_N , the effects of the level of adaptation cannot be ignored. At low levels of I_N the eye will be more or less appropriately adapted; at high levels it will be inadequately adapted, so that the volume of messages transmitted along the visual pathways will be excessive, and the fibers involved will be increasingly forced towards their upper limiting response rates. This should make them less responsive to further stimulation, so that the greater I_N is, the less the response to a further increase in stimulus intensity will be. This negatively accelerated response to increase in I corresponds to a transducer function with n less than one. Thus we might expect an increase in I_N effectively to reduce the value of n in the transducer function, $E = I^n$. In a computer simulation of the application of the statistical decision model in which the effects of varying the parameters were studied, it was found that either decrease in the level of adaptation, or decrease in the exponent of the transducer function would cause a rise in threshold, the rise being greater when the two factors are combined (see Figs. 5 and 6 in Treisman, 1966). As I_N is increased in the pulsed FCP the level of adaptation will be increasingly inadequate, and the exponent of the transducer function may fall too; thus ΔI_{pfc} should rise more rapidly as I_N is increased than is suggested by Eq. 11 since the latter assumes adequate adaptation and a constant transducer function.

Thus both the possible decline in r_E as I_N increases, and the absence of adaptation to I_N , suggest that the Weber function will be steeper than indicated by Eq. 11. Some data bearing on this prediction will be discussed after the continuous FCP has been considered.

V. THE CONTINUOUS FORCED-CHOICE PROCEDURE

In a second variant of the FCP, the "continuous forced-choice procedure," I_N is a constant background that is continuously present, not restricted in its presentation to the same area and duration as the increment, and ΔI_v is added to one or the other of two spatial or temporal locations. We shall assume in considering this procedure that the two possible sites or times of occurrence of the increment are sufficiently clearly indicated to the S for any uncertainty about the times or positions of occurrence of the stimuli to be negligible (Treisman, 1964b).

In relation to this procedure we shall consider two main strategies that the S might adopt:

The Differencing Strategy

On each trial the S may record two inputs, E_a from location a and E_b from location b, and use the difference between these inputs to determine his response. Thus if $(E_a - E_b) \geq 0$ he may respond "a," otherwise "b." If E_a is in fact E_v , the effect of I_v , and he responds "a," or if E_b is E_v and he responds "b," the response will be correct. Thus $P(C)$ will correspond to the proportion of trials on which the difference $D = E_v - E_N \geq 0$ (see Fig. 2). This is the identical strategy that was assumed in developing a model for the pulsed forced-choice procedure, and we derive predictions from it in the same way. As before, we let $A_r = \max(A_i, \alpha)$ and $t_r = \max(t_i, \tau)$; $\Delta I_{cfc}'$ is the continuous forced-choice threshold given when the differencing strategy is applied. When $I_v = I_{0.75} = I_N + \Delta I_{cfc}'$, then $P(C) = 0.75$ and 0.75 of the area of the distribution of differences, $D = E_v - E_N$, will lie to the right of the criterion $D_c = E_N - E_N = 0$. Then we have

$$E_N = sf A_r t_r (I_N + I_n) \quad (15)$$

and

$$E_{0.75} = sf [A_r t_r (I_N + I_n) + A_i t_i \Delta I_{cfc}'] \quad (16)$$

Then

$$E_{0.75} - E_N = sf A_i t_i \Delta I_{cfc}' \quad (17)$$

The variance of the E_N distribution will be

$$\sigma_E^2 = r_s \sigma_s^2 f^2 A_r^2 t_r^2 (I_N + I_n)^2 + (s^2 + \sigma_s^2) f A_r t_r (I_N + I_n). \quad (18)$$

We see from Fig. 2 that $E_{0.75} - E_N = 0.6745 \sigma_D = 0.6745 \sigma_{E_{0.75} - E_N}$. Then

$$\begin{aligned} \Delta I_{cfc}' &= 0.6745 \sigma_D / sf A_i t_i \\ &= \frac{0.6745 \cdot 2^{1/2} (1 - r_E)^{1/2}}{f A_i t_i} [r_s v^2 f^2 A_r^2 t_r^2 (I_N + I_n)^2 \\ &\quad + (1 + v^2) f A_r t_r (I_N + I_n)]^{1/2}. \end{aligned} \quad (19)$$

When $f A_r t_r (I_N + I_n)$ is small this reduces to

$$\Delta I_{cfc}' = \frac{0.9539 (1 - r_E)^{1/2}}{f^{1/2} A_i t_i} A_r^{1/2} t_r^{1/2} (I_N + I_n)^{1/2} \quad (20)$$

and when this is large Eq. 19 reduces to

$$\Delta I_{cfc}' = \frac{0.9539 (1 - r_E)^{1/2}}{A_i t_i} r_s^{1/2} v A_r t_r (I_N + I_n). \quad (21)$$

It is evident that Eq. 19 resembles Eq. 3 with z replaced by $0.9539 (1 - r_E)^{1/2}$, and it leads to similar predictions for spatial and temporal summation.

Variation in t_i . (Since I_N and ISI are both constant, r_E may be taken to be constant.) When $t_i > \tau$, variation in t_i will give $\Delta I_{cfc}' = k$ when Eq. 21 holds and $\Delta I_{cfc}' = k'/t_i^{1/2}$ when Eq. 20 holds. For $t_i \leq \tau$, Bloch's law is given in each case.

Variation in A_i . Similar predictions are given.

Variation in I_N . The presence of the term $(1 - r_E)^{1/2}$ in Eq. 19 will have the same implications as its presence in Eq. 11. Thus if variation in r_E affects the form of the pulsed FCP Weber function, it will have the same effect on the continuous FCP Weber function, assuming that the differencing strategy is employed in the latter procedure. But any effect of inadequate adaptation to I_N on the pulsed FCP Weber function will not be shown for the continuous procedure, since in the latter I_N is constantly present. This will be discussed further below.

We see that if in the continuous forced-choice procedure the S applies the differencing strategy, we should expect the effects of the increment duration and area on the threshold to resemble those found when the detection procedures are employed, rather than those that result from applying the differencing strategy in the pulsed FCP. The continuous FCP data in Fig. 3 are in agreement with the prediction for temporal summation: a curve is shown that gives values of ΔI_{cfc} determined against a constant background of 1.3 log mL. In this case the data approximate closely to Bloch's law when t_i is below a value in the region of 32 msec (the slope on the log-log plot is approximately -0.9) and the curve flattens out above this value. The curve is notably different from those given with the pulsed FCP.

The Double Detection Strategy

It has commonly been assumed that the differencing strategy is always applied in the FCP, although, in fact, other strategies are

possible. Thus Tanner and Swets (1954) assume what is essentially a differencing strategy for a visual intensity discrimination using the continuous forced-choice procedure. However, Blackwell (1963) argued that their data were not fully consistent with this assumption and proposed instead a "high threshold" model similar to the alternative strategy that will be described below.

The "double detection strategy" is the separate application, to each of the two locations that the S must choose between, of the strategy for the detection procedures that was described in Section II and illustrated in Fig. 1. As the background is continuously present, the S is in the position of having a "noise" distribution to which each input sample can be related, as with the detection procedures. Since on each trial there are two locations at which the increment might occur, the response could be determined in two stages. Thus on a given trial the discriminatory mechanisms might compare the input E_a with a criterion $E_{c(a)}$ (corresponding to E_c in Fig. 1), and if $E_a \geq E_{c(a)}$ make and store a "covert positive judgment." The input E_b might be less than the corresponding criterion, $E_{c(b)}$, giving rise to a covert negative judgment, and the S would then proceed to make an overt response indicating that the increment had occurred at location a. If a covert detection occurred at both a and b, or no detection occurred at either location, we might suppose that the S would then respond at random.

Obviously, other strategies could also be suggested. For example, the S might apply a hybrid procedure in which he employs the double detection strategy, but when covert detections occur both at a and b, or at neither a nor b, he reverts to considering the sign of the difference ($E_a - E_b$). However, we shall here limit our consideration to the double detection strategy as described above.

This strategy may seem less elegant than the differencing procedure, but under some experimental conditions its application might have advantages. If very long ISIs are used in temporal forced choice, the possibility of decay in memory would become important. With the double detection strategy the information to be retained from the first stimulus presentation on each trial is not more than one bit—whether the first covert judgment was positive or negative. With the differencing strategy, the actual value of the first input, E_a , must be retained in memory, and this may constitute a considerably greater load. Experiments in which an accessory stimulus followed a critical stimulus increment at various intervals, and the ability of the accessory stimulus to affect the criterion, E_c , used in detecting the critical increment was studied, have provided evidence that sensory inputs can be retained in memory in uncategorized form for periods only up to 0.5-1.0 sec (Treisman, 1964b). This makes it seem unlikely that differencing could be applied at long ISIs. If double detection is applied at long interstimulus intervals, then as the ISI decreases there might be a transition to the differencing strategy, but it is not obvious that this must occur. Since it is likely that in daily life the detection of isolated stimulus increments is a far more common and important necessity for the senses than the making of forced choices, it is quite possible that there may be a strong bias for the sensory mechanisms to operate in the detection mode whenever the presence of a continuous background makes this possible.

If the double detection strategy is applied, two stimuli, I_N and $I_V = I_N + \Delta I_V$, being presented on each trial, then the probability of a "correct" response on any trial, $P(C)$ will depend on $P(PJ | I_V)$, the probability of a positive covert judgment for the location at which I_V is presented, which corresponds exactly to $P(Y | I_V)$ for the method of constant stimuli, and on $P(PJ | I_N)$ the probability of a positive judgment at the location that does not have the increment, which corresponds to the false positive rate $FPR = P(Y | I_N) = \&$. For a given value of I_V

$$P(C) = P(PJ | I_V)[1 - P(PJ | I_N)]$$

$$= \frac{1}{2} \{ P(PJ | I_V)P(PJ | I_N) + [1 - P(PJ | I_V)][1 - P(PJ | I_N)] \}$$

$$= 0.5 [P(PJ | I_V) + 1 - \&]. \quad (22)$$

The value of I_V that gives $P(C) = 0.75$ is often used to define the continuous forced-choice threshold, ΔI_{cfc} . (We shall use $\Delta I_{cfc}'$ to represent the continuous forced-choice threshold given when the double detection strategy is applied.) In a trained S who maintains his false positive rate, $\&$, at a low level, it will be approximately true that the value of I_V that gives $P(C) = 0.75$ will be the value that gives $P(PJ) = 0.5$; i.e., it will be the value represented as $I_{0.50}$ in Fig. 1, which gives the threshold $\Delta I = I_{0.5} - I_N$ when the method of constant stimuli is used. Thus if the S employs the double detection strategy, then $\Delta I_{cfc}'$ will be a good approximation to ΔI as obtained by the detection procedures, and so Eq. 3 will predict all the effects of stimulus parameters on $\Delta I_{cfc}'$; i.e., Bloch's law will hold below τ , and Ricco's law will hold below α , and the Weber function will be the same as that given by the detection procedures.

We can now compare the claims of the differencing and double detection strategies as possible descriptions of the decision processes underlying the determination of the continuous forced-choice threshold. We have seen that both strategies predict the same spatial and temporal summation curves as does the model applied to the detection procedures in Section II, and that, for temporal summation, this accords with the data for ΔI_{cfc} shown in Fig. 3. Thus either strategy would be adequate to explain the effects of A_i or t_i on ΔI_{cfc} . However, if I_N is varied, the predictions given by the two strategies may differ.

Variation in I_N . The Weber function for the detection procedures, which derives from Eq. 3, has been described. The Weber function for the pulsed FCP is given by Eq. 11. It contains the term $(1 - r_E)^{1/2}$. We have suggested that r_E might decrease as I_N increases, and we have noted that in the pulsed FCP the retina is not adapted to I_N ; either or both factors would cause ΔI_{pfc} to rise more rapidly as I_N increases than is the case in the detection procedures. It is of interest, then, to see whether the pulsed FCP Weber function is in fact steeper than that obtained from the detection procedures. If it is, this would support our analysis of the pulsed FCP, and it would then be of interest to examine the form of the Weber function for the continuous FCP. The expression for $\Delta I_{cfc}'$, Eq. 19, contains the term $(1 - r_E)^{1/2}$, but the expression for ΔI_{cfc} does not. So if the S applies the double detection strategy, the Weber function that results will be similar to that for the detection procedures. But if he applies the differencing strategy, and r_E decreases to a significant extent as I_N increases, he will give a steepened Weber function.

Figure 4 shows Weber functions obtained using the pulsed and continuous forced-choice procedures by Cornsweet and Pinsker (1965), and Leshowitz, Taub, and Raab (1968). Cornsweet and Pinsker's data were obtained using spatial forced choice with $t_i = 4.5$ msec for pulsed forced choice and 3.6 msec for continuous forced choice [the data shown in Fig. 4 are for S DO; results for two pulsed FCP experiments (I and III) have been averaged together]. To find the threshold Cornsweet and Pinsker used a staircase procedure. Their rule for varying the stimulus was: after one wrong response increase I_V ; decrease I_V only after two correct responses. This procedure estimates the 71% point (Wetherill & Levitt, 1965), thus both ΔI_{pfc} and ΔI_{cfc} are defined by $P(C) = 0.71$ for their results. Leshowitz, Taub, and Raab (1968) used temporal forced choice; the data in Fig. 4 (the results of two Ss averaged together) were obtained with $t_i = 32$ msec. Each trial began with a warning click. Two additional clicks spaced 0.8 sec apart occurred 0.8 sec after the start of the trial and accompanied the two flashes. $P(C)$ was determined for a

Fig. 4. Log ΔI plotted against log I (both in log mL): pulsed and continuous forced-choice thresholds obtained by Cornsweet & Pinsker (1965) [spatial forced choice: S DO, the results for two pulsed forced-choice experiments (I and III) have been averaged together] and Leshowitz, Taub, and Raab (1968) (temporal forced choice, the results for two Ss have been averaged together). In the latter case the threshold is the value of the increment giving $P(C) = 0.75$. The data obtained by Cornsweet and Pinsker have been shifted upwards 3 log units. The straight lines, fitted by eye, have the following slopes (from left to right): for the continuous forced-choice data—0.5 and 0.9; for the pulsed forced-choice thresholds—0 (for one curve), 0.67, and 1.15.

number of values of I_N and the threshold, the value of ΔI_N giving $P(C) = 0.75$, was obtained from the resulting psychometric function.

Straight lines have been fitted by eye to the Weber functions. It can be seen that, to a first approximation, both sets of continuous forced-choice data can be described as consisting of two segments, the first with a slope of 0.5 on the log-log plot, the second with a slope of 0.9. Cornsweet and Pinsker's pulsed forced-choice data show a segment at absolute threshold (zero slope); beyond this both sets of ΔI_{pfc} values can be fitted with slopes of 0.67 (when I_N is low) increasing to 1.15 (for I_N high).

It was predicted that the pulsed FCP Weber function would be steeper than is conventionally found; and this is borne out by both of the sets of pulsed FCP data in Fig. 4. There is a similar increase in slope with both spatial and temporal forced choice, lines of the same slope fitting reasonably well in both cases. Two mechanisms were discussed above that might give rise to an increase in slope: a decrease in r_E with increase in I_N , and the effect of inadequate adaptation to I_N . Two features of the data argue against the importance of the first factor: (1) The slope for the pulsed FCP functions is about 1.3 times as great as that for ΔI_{cfc} at both low values of I_N and high; this is a greater increase than could readily be explained by an increase in the term $(1 - r_E)^{1/2}$ alone. (2) If a decrease in r_E made an important contribution to the steepness of the curve, we would expect the slope of the spatial forced-choice curve to be greater than that for temporal forced choice with a long ISI, since the former

condition would be optimal for insuring a high initial value of r_E , but when the two flashes are separated by a long interval r_E might well be low even for small values of I_N , leaving little scope for increase in the term $(1 - r_E)^{1/2}$. But the spatial and temporal forced-choice data appear to have identical slopes. These results suggest that variation in r_E as a function of I_N is not important and that the steep slopes of the pulsed FCP Weber functions are mainly or wholly attributable to the effect of inadequate light adaptation, as discussed above.

The Cornsweet and Pinsker (1965) pulsed FCP data in Fig. 4 are the means of two experiments that gave virtually identical results. In the first the S was initially dark adapted, and the field was dark except when the flashes were given. In the second an adapting field at intensity I_N was present for part of each trial: "After the observer fixated the adapting field for 15 sec, the field was darkened for 5 sec . . . The observer then pressed the key to deliver the flash . . . After a brief dark period . . . the cycle was repeated [Cornsweet & Pinsker, 1965]." Thus in this experiment some photochemical adaptation to I_N was maintained, but the rapid neural component of light adaptation was absent at the moment when the flashes were given (Crawford, 1947; Baker, 1963). Thus the absence of this component of light adaptation seems mainly responsible for steepening the Weber function.

Two possible strategies for the continuous FCP have been discussed. We have seen that both predict the conventional findings for spatial and temporal summation, and to the extent that data are available (see Fig. 3) this has been supported. It was also shown that the double detection strategy would lead to the conventional Weber function. The differencing strategy, however, would give a steepened function if an increase in the term $(1 - r_E)^{1/2}$ occurs as I_N gets bigger, but not otherwise. (Inadequate light adaptation would, of course, not be a factor in the continuous procedure.) In fact, Fig. 4 shows that the continuous procedure gives a conventional Weber function. Thus the Ss either employed the double detection strategy, or they employed the differencing strategy, and there is no significant decrease in r_E as I_N increases. As we have seen, this last would be quite consistent with the pulsed FCP results.

It appears that the statistical decision model can be extended to apply to both pulsed and continuous forced-choice procedures. It predicts or accounts for all the main findings. Two strategies can be suggested for the continuous FCP, and the results we have so far considered are not able to distinguish between them.

A remaining point of interest is the size of the thresholds given by the two procedures. We can see from Fig. 4 that at low values of I_N , ΔI_{pfc} is less than ΔI_{cfc} ; this superiority is greater for Cornsweet and Pinsker's data than for those obtained by Leshowitz, Taub, and Raab (1968). As I_N increases, ΔI_{pfc} increases more rapidly than ΔI_{cfc} and eventually exceeds it, the crossover occurring earlier in Leshowitz, Taub, and Raab's data. These relations follow fairly simply from the model:

(a) If we assume that the double detection strategy is employed, then ΔI_{cfc} as given by Eq. 4 (for I_N small) can be compared with ΔI_{pfc} as given by Eq. 12. (We shall ignore I_n , and A_r and A_i ; for the two experiments being considered here we can assume $A_r = A_i$.) Then the former equation differs from the latter in two ways: it contains z (of the order 2-6), corresponding to $0.9539(1 - r_E)^{1/2}$ (maximum possible value 1.35); clearly this will tend to make ΔI_{cfc} the larger. Equation 4 also contains $t_i^{1/2}/t_r$, corresponding to $1/t_i^{1/2}$ in Eq. 12. When $t_i = t_r$ the two expressions will be identical; when $t_i < t_r = \tau$, the first expression will be greater than the second, again tending to make ΔI_{cfc} exceed ΔI_{pfc} . Thus we expect $\Delta I_{cfc} > \Delta I_{pfc}$, as shown for small values of I_N in Fig. 4, and we expect the difference to be greater the smaller $t_i < \tau$ is. In Cornsweet and Pinsker's experiment, t_i is of the order of 4 msec, in the Leshowitz, Taub, and Raab data it is

32 msec; as we would expect from the argument above, ΔI_{cfc} exceeds ΔI_{pfc} by a bigger margin in the first case.

When I_N is large we compare Eq. 5 with Eq. 13. As before the former contains z , corresponding to the latter's $0.9539(1 - r_E)^{1/2}$; and t_r/t_i corresponds to $t_i/t_i = 1$. ΔI_{cfc} will be greater than ΔI_{pfc} , and especially so when $t_i < \tau$.

Equations 12 and 13 will make the pulsed threshold always less than the continuous threshold. But they do not include the effect of inadequate light adaptation, which, as discussed above, will make the pulsed Weber function increase more rapidly than these equations suggest, and thus result in the crossing over of the pulsed and continuous functions.

(b) If we assume that the differencing strategy is employed, then, for I_N small, $\Delta I_{cfc}'$, as given by Eq. 20, is compared with ΔI_{pfc} , given by Eq. 12. Ignoring I_N , they differ only in that the former includes $t_r^{1/2}/t_i$ and the latter $1/t_i^{1/2}$. Similarly, for I_N large, Eq. 21 has t_r/t_i where Eq. 13 has t_i/t_i . It follows that for $t_i \geq \tau$ the equations will give identical thresholds; taking the effects of adaptation into account ΔI_{pfc} would then be always equal to or greater than $\Delta I_{cfc}'$. For $t_i < \tau$, the equations will give $\Delta I_{pfc} < \Delta I_{cfc}'$, and the greater slope of the pulsed FCP Weber function will reverse this relation when I_N is sufficiently large. As t_i is less in Cornsweet and Pinsker's data than in those of Leshowitz, Taub, and Raab, the greater excess of ΔI_{cfc} over ΔI_{pfc} for low values of I_N in the former data is to be expected. In the latter data, $t_i = 32$ msec, which is of the order of the value of τ for the ΔI_{cfc} data in Fig. 3; but the latter were determined at $I_N = 1.3$ log mL, and since τ tends to increase as I_N falls it would be larger at low values of I_N (Keller, 1941).

Further Comparison of the Double Detection and Differencing Strategies

We have described two strategies that might be employed in the continuous FCP, but the data discussed above have not enabled us to distinguish between them. It seems worth considering a number of points that might help to indicate which strategy is in fact employed.

(1) From the discussion above of the relative sizes of pulsed and continuous thresholds, we see that if t_i is made clearly greater than τ , then with the differencing strategy we will have $\Delta I_{pfc} \geq \Delta I_{cfc}$ for all values of I_N , but with the double detection strategy ΔI_{pfc} may be less than ΔI_{cfc} at low values of I_N . If the latter were found we would then have evidence that the double detection strategy is employed. However, comparable data for both procedures with t_i known to be sufficiently large are not yet available.

(2) The differencing and double detection strategies predict different forms of psychometric function. If we refer to Fig. 2, we see that the psychometric function given by the differencing strategy is a half-ogive; it is the normal distribution function taken from the 50% to the 100% points. But the psychometric function will have a different form if the double detection strategy is applied (Blackwell, 1963). We saw from Eq. 22 that when the latter strategy is employed $P(C)$ is a linear function of $P(PJ | I_v)$. But $P(PJ | I_v)$ can be assumed to equal $P(Y | I_v)$, and a plot of $P(Y)$ against I , as in Fig. 1, gives a normal ogive psychometric function. Thus, if the double detection strategy is applied, the psychometric function will be an ogive linearly transposed to lie between the asymptotes $P(C) = 0.5(1 - \epsilon)$ and $P(C) = 0.5(2 - \epsilon)$ or, when ϵ is small enough to be ignored, 0.5 and 1.0. Although it follows that the form of the psychometric function obtained from the continuous FCP should indicate which strategy was employed, in practice it may be difficult to determine the form of the psychometric function with sufficient precision. In Fig. 5 the expected form of the psychometric function for each strategy is shown. In the two lower graphs $P(C)$

Fig. 5. Theoretical psychometric functions for the differencing and double detection strategies. The lower left curve shows the differencing psychometric function, obtained as in Fig. 2. Above is shown the graph given by plotting the values of $z(C)$, the standardized normal deviates corresponding to values of $P(C)$ below. The lower right curve is the psychometric function for the double detection strategy; values of $P(C)$ are obtained from values of $P(Y) = P(PJ)$ (compare Fig. 1) by Eq. 22, assuming $\epsilon = 0.04$. The flattened sigmoid curve given when $z(C)$ is plotted against ΔI in this case is shown above. A straight line could be fitted to it reasonably well except at the extremes; the dashed line is an example. For both curves the abscissa is scaled in units of σ_1 (as in Fig. 1); the origin corresponds to I_N alone. The SD for the differencing psychometric function, $\sigma_{I(\text{diff})}$, is taken to be $2^{1/2}\sigma_1$.

is plotted against ΔI . For the double detection strategy, $P(C)$ is calculated, in accordance with Eq. 22, from the values of $P(Y) = P(PJ)$ that would be obtained by the method of constant stimuli (as in Fig. 1), assuming that the limiting acceptable false positive rate, $\epsilon = 0.04$. The abscissa is plotted in units of σ_1 , the SD of the MCS psychometric function. The resulting sigmoid function passes through the point (0,0.5) and its asymptotes are $P(C) = 0.48$ and $P(C) = 0.98$. The psychometric function for the differencing strategy, derived as in Fig. 2, is a half-ogive; it has been taken that its SD, $\sigma_{I(\text{diff})}$ is equal to $2^{1/2}\sigma_1$, and the abscissa is marked in units of σ_1 . The upper graphs show plots of the corresponding standardized normal deviates, $z(C)$. For the differencing strategy this will be a straight line passing through the origin. For double detection it will be a somewhat flattened sigmoid curve. A dashed straight line, such as might be fitted by probit analysis, is also shown. A fairly good fit to a straight line might be obtained, except for extreme values of the data. The straight line intersects the abscissa at a positive value, not at the origin.

It follows from this that if probit analysis (Finney, 1952) is used to fit continuous forced-choice data, two effects of the strategy chosen may be expected: (a) If the S employs differencing, the straight line should be a good fit, if double detection, a poor fit. (b) For differencing, the straight line should pass through the origin, for double detection it should have a positive intercept with the abscissa.

Figures 6 and 7 show data obtained by Leshowitz, Taub, and Raab (1968) for two Ss. In each case probit analysis has been applied to continuous and pulsed forced-choice data and the

Fig. 6. Data obtained by Leshowitz, Taub, and Raab (1968) for the pulsed and continuous FCPs (S BL). Each value of $P(C)$, the probability of a correct response, has been converted to the corresponding standardized normal deviate, $z(C)$, and a straight line has been fitted to each set of data by probit analysis. The values of $z(C)$ are plotted against $I_v = I_N + \Delta I_v$ (in mL). The parameters of the curves are I_N (in log mL) and t_i (in msec). The filled circles and triangles represent the continuous FCP data, and the unfilled symbols represent the pulsed FCP data. For each curve the axes $z(C) = 0$ and $I_v = I_N$ are shown as dotted lines.

Fig. 7. As Fig. 6: Data for S HT.

Fig. 8. The continuous forced-choice data shown in Figs. 6 and 7 are shown on a single plot. In each case $z(C)$ is plotted against ΔI_v standardized by division by the SD given by the probit analysis of the data for that curve. The parameters of the curves, $\log I_N$ (log mL) and t_i (msec) are shown.

results are presented as plots of $z(C)$ against $I_v = I_N + \Delta I$, with the best-fitting straight line.

Does probit analysis give a good fit to the continuous FCP data, as differencing would imply, or a poor fit, as we would expect if double detection were the strategy employed? To test this, a value of chi square for goodness of fit was obtained for each line (Finney, 1952), and these values, and their degrees of freedom, were summed for eight curves (the remaining two curves had insufficient data points) to give:

Subject	χ^2	df	Significance
BL	16.10	7	$p < 0.025$
HT	12.35	5	$p < 0.05$
Both	28.45	12	$p < 0.005$

This is significant evidence of departure from goodness of fit, which is evidence against the differencing strategy having been employed. It can also be seen that for all 10 curves the intercept on the abscissa is positive; the (one-tailed) probability of this occurring by chance is 1 in 2^{10} . We saw from Fig. 5 that a deviation in this direction was to be expected if a double detection strategy were employed. To examine the departure from good fit to the probit line indicated by the high value of chi square, all the continuous FCP curves were replotted in Fig. 8; in each case $z(C)$ was plotted against ΔI_v , normalized by division by the SD given by the probit analysis of the given set of data. When the data are put together in this way, the general shape, indicated roughly by the two line segments that were fitted by eye, is compatible with the flattened sigmoid curve passing through the point (0,0), which would be expected if the double detection strategy were applied.

It appears that the results are most consistent with the strategy applied in the continuous FCP being that of double detection rather than differencing, at least in this experiment.

The pulsed forced-choice data obtained in the same experiment are also shown, for comparison. Here the double detection strategy would not be capable of being applied, and we must

assume that the Ss will have been obliged to employ the differencing strategy. In this case we would expect the straight lines to pass through the origin, but in all cases the intercept with the abscissa is negative. On the null hypothesis these intercepts would be expected to be equally often positive or negative, so that the probability of a deviation as extreme as this occurring by chance is 1 in 2^7 , which is highly significant. This presents a problem whose explanation is not immediately evident.

If the differencing strategy were applied, then, in the absence of further considerations, we would expect the straight lines given by probit analysis to fit well. For these curves the summed value of chi square was 20.33 (df = 18), which is not significant. However, when nine pulsed FCP curves obtained in a further experiment by Leshowitz, Taub, and Raab (1968) are added in, the overall summed chi square rises to 123.69 (df = 44, $p < 0.001$).

To examine whether this high value of chi square resulted from variability or from a consistent departure from the expected shape, all the pulsed FCP curves were plotted in Fig. 9. In each case $z(C)$, the standardized normal deviate corresponding to $P(C)$, was plotted against ΔI_v , standardized by division by the value of the SD given by the probit analysis for that curve. When the data are plotted in this way it is clear that there is a nonlinear tendency. Because of the randomization involved in the experimental design, we would not expect $P(C)$ for $\Delta I_v = 0$ to depart significantly from 0.5 [giving $z(C) = 0$]. This value of ΔI_v was not used, but it is evident that for low values of ΔI_v the data fall along a line passing through or near the point (0,0). However, as ΔI_v increases, there is an increasing tendency for the values of $z(C)$ obtained to fall below the level that would be predicted by this initial straight line. This results in a bowing of the curve, roughly illustrated by the two line segments that have been fitted by eye. It is evident that the negative intercepts with the abscissa shown in Figs. 6 and 7 must result from fitting straight lines to this bowed curve.

Fig. 9. The pulsed forced-choice data shown in Figs. 6 and 7, and nine further curves obtained by Leshowitz, Taub, and Raab (1968) are shown on a single plot. $z(C)$ is the standardized normal deviate corresponding to $P(C)$. The values of ΔI_v have been standardized by division by the SD given by the probit analysis of the data for each curve. The parameters of the curves, $\log I_N$ in log mL, and t_i in msec, are also shown.

Why does the curve depart from the expected straight line? The most plausible explanation would be that the bowing is an effect of the inadequate light adaptation obtaining in the pulsed forced-choice procedure. Save at the moments when I_N or I_V are actually being presented, the eye is exposed to an unilluminated background that will be the main determinant of its state of adaptation. The greater I_V is, the poorer adaptation to it will be. If poor adaptation effectively increases $\sigma_{E(pfc)}$, this should produce an unduly poor performance at high values of I_V . There might also be an effect of the disparity between I_V and I_N . When the first stimulus of the trial is given, this might induce some degree of adaptation to its own level, which might facilitate response to the second stimulus when ΔI_V is small, but impair it when ΔI_V is large.

It is instructive to compare Figs. 8 and 9. As well as the difference in shape in the two cases, the data are more widely scattered in Fig. 8. This would be expected if double detection were applied in the continuous FCP since variation in the criterion (E_c) employed for different curves would result in their being laterally dispersed when plotted on the same abscissa. No similar factor would operate to disperse curves given by differencing.

(3) Can information about the strategy employed be derived from the slopes of the functions obtained? If we compare the lines fitted to pulsed and continuous data where they have the same parameters and scales, the latter appear to give steeper slopes. In Fig. 6, curve e is steeper than c, and f is steeper than d. In Fig. 7, f is steeper than a; an increase in t_i produces a steeper curve (compare curves g, h, and i, in Fig. 6), yet curve i, with $t_i = 32$ msec, is steeper than d with $t_i = 320$ msec.

If the differencing strategy were applied in the continuous FCP, the slope of the resulting psychometric function would depend on its SD, $\sigma_{I_{cfc}}$. If we refer to Fig. 2, it is evident that this must depend on the SD of the distribution of differences, $\sigma_D = 2^{1/2}(1 - r_E)^{1/2}\sigma_E$, and the transducer function $E = f(I)$. Making the usual linear assumption for the latter, and taking $\sigma_{I_{cfc}} = I_{0.84} - I_{0.50}$, by an argument identical to that embodied in Eqs. 15 to 19 we get

$$\sigma_{I_{cfc}} = \sigma_D / sf A_i t_i = 2^{1/2}(1 - r_E)^{1/2}\sigma_E / sf A_i t_i. \quad (23)$$

If the double detection strategy is applied in the continuous FCP, the resulting psychometric function is simply related to that obtained when such a detection procedure as the method of constant stimuli (MCS) is employed (see Fig. 1). The SD of the latter, σ_I , corresponds to $E_{0.84} - E_{0.50}$, and by an argument parallel to that leading to Eq. 3 we get

$$\sigma_I = I_{0.84} - I_{0.50} = \sigma_E / sf A_i t_i. \quad (24)$$

In Fig. 5 the abscissa is scaled in units of σ_I . If the MCS psychometric function were plotted on this scale, the $z(C)$ transformation would of course be a straight line with a slope of 1. For this figure we have taken $\sigma_{I_{cfc}} = 2^{1/2}\sigma_I$, and accordingly the $z(C)$ transform of the differencing psychometric function has a slope of $1/2^{1/2} = 0.71$. The double detection psychometric function is related to the MCS function by Eq. 22; the latter lies between the asymptotes 0 and 1, the former between $0.5(1 - \&)$ and $0.5(2 - \&)$ and it is correspondingly flatter. For example, the stimuli that give $P(Y) = 0.50$ and $P(Y) = 0.84$ when the MCS is applied, will give (ignoring $\&$) $P(C) = 0.75$ and $P(C) = 0.92$ for double detection. This would correspond to a slope, on Fig. 5, of 0.73. With $\& = 0.04$, the slope for this portion of the curve would be 0.67. The dashed approximating straight line shown for the $z(C)$ transform of the double detection curve has a slope of 0.61.

Evidently, the slope of the probit line is not likely to

distinguish between the two strategies. But if both strategies would give probit lines of similar slope, what can we say of the evidence noted above that the continuous data give steeper lines than the pulsed data? Two reasons for this might be suggested: (a) The differencing strategy is employed in both procedures, and the slope for the continuous FCP is determined by $\sigma_{I_{cfc}}$, as given by Eq. 23. For the pulsed FCP the slope is determined by the SD of the pulsed FCP psychometric function, $\sigma_{I_{pfc}}$. Referring to Fig. 2 and Eqs. 6 to 11 we see that this is given by

$$\sigma_{I_{pfc}} = 2^{1/2}(1 - r_E)^{1/2}\sigma_E(pfc) / sf A_i t_i. \quad (25)$$

Then the continuous FCP might give steeper slopes because r_E is greater in the continuous procedure than in the pulsed procedure. However, there is no obvious reason why this should be the case, and since the two stimuli on each trial are given at a spacing of 0.8 sec, we should not expect r_E to be large for either procedure. Thus this explanation does not appear plausible. (b) Whether differencing or double detection is employed in the continuous FCP, the slope will depend on σ_E (Eqs. 23 and 24), and the slope in the pulsed FCP will depend on $\sigma_E(pfc)$ (Eq. 25). Comparing Eq. 9 with Eq. 2 (for the double detection strategy) or Eq. 18 (for differencing) suggests that $\sigma_E(pfc)$ will be less than or equal to σ_E for the continuous cases. However, we have noted earlier that in the pulsed FCP adaptation to I_N is imperfect, and this would tend to give increased values of $\sigma_E(pfc)$ and so a flatter slope for the pulsed FCP. Thus the effect of inadequate light adaptation in the pulsed procedure could explain both the steeper Weber functions and the flatter psychometric functions.

Concluding this section, it appears that our model is capable of predicting the main findings for the continuous forced-choice procedure, as for the others previously considered. We have distinguished two strategies that S s might employ, and have obtained some evidence that the double detection, rather than the differencing strategy, is the preferred one in the continuous procedure.

VI. THE PEDESTAL FORCED-CHOICE PROCEDURE

In this procedure a background of intensity I_N is constantly present; an increment, ΔI_V , is given at one or the other of two indicated spatial or temporal locations; at each location an increment I_p , the "pedestal," is also given, whether or not ΔI_V is added at that location. This procedure is of interest since I_N is constantly present and so the state of light adaptation may be better than in the pulsed procedure, especially when I_N is large as compared with I_p . But the S is likely to employ the differencing strategy when I_p is at all large, since the double detection strategy would then give an unduly high proportion of detections at both locations on each trial. Thus the use of the differencing strategy may be dissociated from the effects of poor light adaptation.

Making the same assumptions about the sampling and decision mechanisms as before, we obtain

$$E_N = sf[A_i t_i(I_N + I_n) + A_i t_i I_p] \quad (26)$$

and

$$E_{0.75} = sf[A_i t_i(I_N + I_n) + A_i t_i(I_p + \Delta I_{ped})], \quad (27)$$

where ΔI_{ped} is the increment giving $P(C) = 0.75$. If the differencing strategy is employed, Fig. 2 will apply, and we will have

$$E_{0.75} - E_N = sf A_i t_i \Delta I_{ped} = 0.6745 \sigma_D. \quad (28)$$

Since

$$\sigma_{E(ped)}^2 = r_s \sigma_s^2 f^2 [A_r t_r (I_N + I_n) + A_i t_i I_p]^2 + (s^2 + \sigma_s^2) f [A_r t_r (I_N + I_n) + A_i t_i I_p] \quad (29)$$

we will have

$$\begin{aligned} \Delta I_{ped} &= 0.6745 \cdot 2^{1/2} (1 - r_E)^{1/2} \sigma_{E(ped)} / sf A_i t_i \\ &= \frac{0.9539(1 - r_E)^{1/2}}{f A_i t_i} \left\{ r_s v^2 f^2 [A_r t_r (I_N + I_n) + A_i t_i I_p]^2 \right. \\ &\quad \left. + (1 + v^2) f [A_r t_r (I_N + I_n) + A_i t_i I_p] \right\}^{1/2}. \end{aligned} \quad (30)$$

When $f[A_r t_r (I_N + I_n) + A_i t_i I_p]$ is small this will reduce to

$$\Delta I_{ped} = \frac{0.9539(1 - r_E)^{1/2}}{f^{1/2} A_i t_i} [A_r t_r (I_N + I_n) + A_i t_i I_p]^{1/2} \quad (31)$$

and when it is large to

$$\Delta I_{ped} = \frac{0.9539(1 - r_E)^{1/2}}{A_i t_i} r_s^{1/2} v [A_r t_r (I_N + I_n) + A_i t_i I_p]. \quad (32)$$

Also

$$\sigma_{I_{ped}} = I_{0.84} - I_{0.50} = \frac{2^{1/2} (1 - r_E)^{1/2} \sigma_{E(ped)}}{sf A_i t_i}. \quad (33)$$

We can now derive predictions for this procedure.

Variation in t_i . Consider I_N large, so that Eq. 32 applies and I_n is negligible. Then for $t_i \geq \tau$, ΔI_{ped} is constant. For $t_i < \tau$, the equation can be simplified to $\Delta I_{ped} = k/t_i + k'$, with $k \propto I_N$ and $k' \propto I_p$. The shape of the temporal summation curve will then depend on the relative magnitudes of k and k' . For $I_p = 0$ (the continuous procedure) we would have Bloch's law, with a slope of -1 when $\log \Delta I_{ped}$ is plotted against $\log t_i$. For $k' > 0$ the curve will be flat for values of t_i not much less than τ , the absolute value of the slope increasing as $\log t_i$ decreases, with an asymptotic value of -1 for t_i very small. The larger k' the more marked the departure from Bloch's law would be.

When Eq. 31 applies, if $t_i \geq \tau$ the equation can be written as $\Delta I_{ped} = k/t_i^{1/2}$, giving a slope of -0.5 on a log-log plot. For $t_i < \tau$, when $I_N = 0$ (the pulsed FCP case) we have a slope of -0.5 for all values of t_i ; when $I_p = 0$ (the continuous FCP case) we have a slope of -1 ; when both I_N and I_p are greater than zero, as t_i decreases we have a transition from a slope of -0.5 to an asymptotic value of -1 .

Variation in A_i . This gives analogous predictions.

Variation in I . If we vary I_N with I_p held constant, when Eq. 31 applies, if $A_i t_i I_p \gg A_r t_r I_N$ then ΔI_{ped} will at first increase very slowly as I_N increases, giving a slope, on log-log coordinates, not much greater than 0. As I_N increases the slope will increase towards an asymptote of 0.5. When Eq. 32 holds, if $A_i t_i I_p \gg A_r t_r I_N$ the slope of $\log \Delta I_{ped}$ against $\log I_N$ will again be low, increasing as I_N increases, to an asymptote of 1. An identical set of predictions can also be derived for the " I_p function" that would be given if $\log \Delta I_{ped}$ were plotted against $\log I_p$ with I_N constant. All these predictions must be qualified, however, by the consideration that when I_p is small as compared with I_N the eye will be well adapted and these equations will apply as they stand; but when I_p is relatively large the situation will be closer to that obtaining in the pulsed FCP: light adaptation will be relatively inadequate and ΔI_{ped} will increase more rapidly than would otherwise be predicted, for the reasons discussed in Section IV.

Fig. 10. Results obtained by Leshowitz, Taub, and Raab (1968) for Ss BL and HT. Data are shown for the continuous (filled symbols) and pedestal (open symbols) FCPs with the best-fitting lines given by probit analysis, and for the pulsed FCP the best-fitting line is shown alone as a dashed line. In each case $\log I_N = 2.3 \log \text{mL}$ and $t_i = 32 \text{ msec}$, and in the pedestal procedure $\log I_p = 2.3 \log \text{mL}$. $z(C)$ is the standardized normal deviate corresponding to $P(C)$. The coordinates shown in dotted lines correspond to $I_v = I_N (\Delta I_v = 0)$ and $z(C) = 0$.

Data against which these predictions could be tested are not yet available, but other predictions can be derived from the equations above for which relevant data exist:

(1) Comparison of ΔI_{ped} and ΔI_{cfc} for the same I_N . (a) We first derive predictions on the assumption that differencing is the strategy employed in the continuous FCP. Let us compare Eq. 30 with Eq. 19. If I_N has the same value in each case, and I_p is added in Eq. 30, we can evidently predict (i) ΔI_{ped} will be greater than ΔI_{cfc} . (ii) If we compare Eqs. 33 and 23 (taking note of Eqs. 29 and 18) it is also evident that the slope of the psychometric function will be steeper for the continuous FCP than when the pedestal is added. If light adaptation is imperfect in the pedestal procedure this will tend to reinforce both predictions. (b) Let us assume that the double detection strategy is employed in the continuous FCP. We then compare Eq. 30 with Eq. 3. If we take Eq. 3 as giving an estimate of ΔI_{cfc} , we see that the two equations differ in two ways. Firstly, z in Eq. 3, which may be of the order 2-6, corresponds to $0.9539(1 - r_E)^{1/2} \leq 1.35$ in Eq. 30; this difference will tend to make ΔI_{cfc} greater than ΔI_{ped} . Secondly, Eq. 30 includes terms containing I_p that will tend to raise ΔI_{ped} , as will any deterioration in light adaptation consequent on the use of a pedestal. Thus we can predict: (i) Under some conditions ΔI_{ped} may be less than ΔI_{cfc} . (ii) The slope of the pedestal psychometric function is determined by $\sigma_{I_{ped}}$, given by Eq. 33. If the S employs a double detection strategy in the continuous FCP, we have seen that the slope of the probit line that might be fitted to the resulting data would depend on σ_I and on the flattening effect of transposing the normal ogive to the asymptotes $0.5(1 - \epsilon)$ and $0.5(2 - \epsilon)$; this effect will depend on ϵ , which is unknown. From these

considerations we cannot derive any definite prediction about the relations of the two slopes.

Data relevant to these predictions, obtained by Leshowitz, Taub, and Raab (1968) for two Ss, are shown in Fig. 10. The figure shows data for the pedestal and continuous procedures, with $\log I_N = 2.3 \log \text{mL}$, and $t_i = 32 \text{ msec}$, in each case, and with a pedestal of intensity $2.3 \log \text{mL}$ added in the pedestal procedure. The best-fitting lines given by probit analysis are also shown. The most striking effect is that the threshold for the pedestal procedure is markedly lower than that for the continuous procedure in each case. (The 75% threshold is the value $(I_v - I_N)$ corresponding to $z(C) = 0.6745$.) This is contrary to what was predicted if the S employed differencing in the continuous procedure, but in agreement with the prediction for double detection. The slopes are similar in the two cases, being slightly flatter, but not significantly so, in the continuous procedure. This again is not in accord with the prediction for differencing.

Leshowitz, Taub, and Raab (1968) report a more shallow slope for the pedestal than for the continuous procedure. This is not in disagreement with the present description, as they plotted the data against $\log I_v$, not I_v as here.

These results strongly support the evidence obtained in the previous section that Ss may employ double detection rather than the differencing strategy in the continuous procedure.

The dashed line is the best-fitting line for the pulsed FCP with the same parameters, in each case (curve c in Fig. 6 for BL, and curve a in Fig. 7 for HT). The discussion up to this point suggests the following overall explanation for the relations between the three procedures shown in Fig. 10. In the continuous FCP the S prefers the double detection strategy. In the pedestal procedure he is forced to adopt the differencing strategy; with these parameters its greater sensitivity more than compensates for the increased central variance due to I_p , and for any impairment in light adaptation. Thus "negative masking" (Leshowitz, Taub, & Raab, 1968), a fall in threshold consequent on the addition of I_p , results from a forced change in decision strategy, rather than from any peripheral sensory effect of the pedestal. The pulsed procedure does not have the continuous $2.3 \log \text{mL}$ background illumination that is present in the pedestal procedure. The advantage in reduced overall intensity is far outweighed by the resulting disadvantage of inadequate light adaptation, here apparently a much greater disadvantage than in the pedestal procedure, with the result that the psychometric function is flatter, and the 75% threshold correspondingly higher than in the pedestal procedure. (It is possible that the absence of adequate light adaptation impairs performance more for the higher values of I_v , as was suggested above. If so, this would contribute to making the line fitted to the pulsed FCP appear flat.)

(2) The effect of variation in I_p . If we add a pedestal to the continuous FCP, what will the effect on threshold be as a function of the magnitude of I_p ? If the S employs the double detection strategy in the continuous FCP, then when the added pedestal is sufficiently large he will switch to differencing, the strategy appropriate to the pedestal procedure, and if parameters such as those in Fig. 10 apply, the threshold will fall (or if ΔI_v is held constant, $P(C)$ will rise).

What will happen if I_p is small? In the discussion of the double detection strategy, we assumed that if a detection occurred at locus a or at b the S would respond accordingly; but if detections occurred at both or at neither he would guess. When double detections occur frequently this becomes unsatisfactory and the S might then switch strategies as a response to this. However, a more attractive idea is that he normally applies a "combined" strategy: when a detection occurs at a or at b he responds in accordance with the double detection strategy, but when

Fig. 11. Values of $P(C)$ for $\log I_N$ constant at $2.3 \log \text{mL}$, $\log \Delta I_v$ constant at $2.0 \log \text{mL}$, and I_p varied; means of the data for two Ss who gave similar results (Leshowitz, Taub, & Raab, 1968).

detections occur at both or neither his response is determined by the difference between the two central effects. When double detections are infrequent the results of the combined strategy would be very similar to the double detection strategy as we have described it. But if I_p is increased gradually above 0, the proportion of trials giving two detections and so resulting in the application of differencing will increase smoothly, reaching a maximum when I_p is always detectable. Thus, as I_p is increased above 0, there should first be a continuous fall in threshold [or rise in $P(C)$ for a fixed value of I_v]. When differencing is fully established, further increase in I_p should result in a rise in threshold [or fall in $P(C)$], in accordance with Eq. 30.

Figure 11 shows data relevant to this prediction (Leshowitz, Taub, & Raab, 1968). The background was kept at a fixed value of $2.3 \log \text{mL}$, $\log \Delta I_v$ was kept fixed at $2.0 \log \text{mL}$, and $P(C)$ was determined for a series of values of I_p . As I_p increases there is a smooth increase in $P(C)$ to a peak at $\log I_p = 2.3 \log \text{mL}$. This is an appropriate value, as the mean of ΔI_{cfc} for $I_N = 2.3 \log \text{mL}$ for these two Ss was $2.15 \log \text{mL}$ (see Fig. 4), so that at the peak in Fig. 11, I_p must itself be detectable on most or all trials. The next rise in I_p produces the fall in $P(C)$ which was predicted for high values of I_p .

VII. CONCLUSION

We have outlined a model for visual discrimination that has previously been found to predict the laws of spatial and temporal summation and the form of the Weber function for traditional detection procedures, such as the method of constant stimuli. The model was then extended to the forced-choice procedures. When this is done it predicts departures from the classical laws of spatial and temporal summation and an increase in the slope of the Weber function for the pulsed forced-choice procedure. Data are presented for temporal summation and the Weber function that agree with these novel predictions. Data obtained with the continuous forced-choice procedure are examined. Predictions made by the model are also confirmed in this case. Two strategies that the S may follow are distinguished, and it is shown that the evidence suggests that in the continuous forced-choice procedure (at any rate with the long ISI that applied to the data examined here) the S employs the "double detection strategy," i.e., he either detects or fails to detect an increment at each of the two alternative locations, and rests his final response on this, rather

than taking the difference between his observations of the two locations (the "differencing strategy"): there is a bias to detect rather than compare. An analysis of the forced-choice procedure with a pedestal is also given, and a comparison of results obtained with this procedure and with the continuous procedure further supports the conclusion that the S employs the double detection strategy in the latter.

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NOTES

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