Temperature sensitivity: One subjective continuum or two?¹

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The full range of nonpainful temperatures (approximately 0 deg C to 52 deg C) was scaled by magnitude estimation with the standard at physiological zero (32 deg C). Apparent magnitude bore an exponential relationship to temperature over this range, although the same Os gave power functions when judging warmth or cold alone. A transformation is presented, which, when applied to the appropriate sections of the exponential relationship for the full range, produced functions indistinguishable from the power law for warm and cold ranges. The conclusion Stevens and Stevens (1960) drew from the results of scaling warmth and cold alone, namely that a ϕ_0 threshold correction should be applied to the physical scale of all continua to derive power law functions from magnitude estimations, is shown to be an invalid generalization from temperature scaling.

In a report of the scaling of sensations of temperature by the method of magnitude estimation, Stevens and Stevens (1960) presented separate psychophysical functions for the ranges above and below "physiological zero" (approximately 32 deg C). They were able to describe their results in terms of the power law, found to hold in many sensory continua (Stevens, 1961), but the power law fitted the data only after the temperature continuum was recalibrated so that judgments were plotted against degrees away from physiological zero. Equation 1 expresses the version of the power law they used:

$$\psi = k |\phi - \phi_0|^N \quad (1)$$

where ψ is apparent magnitude, ϕ is stimulus intensity in deg C, and ϕ_0 is the intensity corresponding to subjective zero. With assumed physiological zero as ϕ_0 , the exponent, n , was about 1.6 for the range above ϕ_0 (warm stimuli) and about 1.0 for the range below ϕ_0 (cold stimuli). The fit of Eq. 1 to the data Stevens and Stevens obtained in a number of different scaling situations was quite good and the exponents for the two ranges were reasonably constant over these conditions.

Although they considered no other functions or versions of the power law in their report, Stevens and Stevens concluded both that the power law's applicability was proven in yet two more instances and also that the rescaling procedure they chose is, in general, the proper one for correcting the deviations from the power law often seen near threshold in many continua. They drew this conclusion about the threshold correction because they assumed that the only special characteristic of the warm and cold senses is that the zero point of the psychological scale is not near the zero point of the physical scale. The need for a threshold correction on the physical scale is thus brought out in bold relief, and its generality for other continua was assumed because warm and cold were considered unique only in the placement of their thresholds. The present study attempts to show that the entire thermal continuum, while giving rise to qualitative differences in sensations at different temperatures (Woods, Griffith, Page, & Rodier, 1967), can be reliably scaled as a single subjective continuum. Furthermore, it is shown that if thermal sensitivity forms a single continuum, the ϕ_0 correction as applied by Stevens and Stevens produces the power law only because of the way they divided the temperature continuum into two ranges, and the validity of this correction is, indeed, unique to the temperature sense.

A special feature of the scaling procedure in this study is the use of a stimulus at physiological zero as the standard for magnitude estimations of the full thermal range. This temperature was used as the standard in order to provide a direct test of the assumption that warm and cold are distinct subjective continua. If this assumption is correct a stimulus at physiological zero should be subthreshold on both continua, have a subjective value of zero ($-\infty$ in the log transformation), and be, therefore, a stimulus against which reliable ratio estimates cannot be made. The fact that Os were able to use physiological zero as a standard adds to the evidence that it is not a true threshold.

METHOD

Apparatus and Technique

Aluminum cups with a circular base about 17 cm² in area, insulated on sides and top with polystyrene and containing about three-quarters of a cup of water, were the stimulators. Temperature of stimulus was controlled by mixing room-temperature water (approximately 23 deg C) with hot (approximately 80 deg C) or cold (0 deg C) water in the cups. There were 11 cups used: 5 for cold (below 32 deg C), 5 for warm (above 32 deg C) and 1 neutral (kept close to 32 deg C). Before each series of stimuli was presented, water was mixed in the cups in different proportions to provide an even spread of temperatures over the range being used at the time. Before and after each presentation the temperature of the water in the cup was measured and the average—there was rarely a difference—was recorded as the temperature presented. Os put their right forearms through a screen, and placed them palm side up on a table surface. The base of the cup was rested on the ventro-mesial surface of the forearm, midway between wrist and elbow. Stimulation lasted 3 sec, after which time the judgment was made, and at least 5 sec was allowed between presentations. Usual magnitude estimation instructions were administered and the standard always had a value of 10. It was presented at the start of every judgment series, after every 10 judgments, and whenever requested by O.

Procedure

All Os performed in three different judgmental tasks: (1) Warm judgments: magnitudes of stimuli between 33 deg C and 52 deg C were estimated in reference to a standard of 38 deg C, with warmer stimuli receiving the higher numerical estimates; (2) Cold judgments: stimuli between 30 deg C and 0 deg C were estimated against a standard at 25 deg C, with higher estimates going to the colder stimuli; and (3) Full judgments: stimuli between 0 deg C and 52 deg C were estimated against a standard at 32 deg C, with higher estimates going to the warmer stimuli. The Warm and Cold tasks replicate conditions reported in the Stevens and Stevens article and were performed to compare the present methodology to theirs. In both the Warm and Cold conditions each O made 10 judgments and in the Full condition, 40 judgments. Stimuli were presented in an irregular order and covered the ranges fairly evenly. The three tasks were presented in an order counter-balanced across Os. There were 12 male Johns Hopkins University undergraduate volunteers in the experiment, paid \$1.25 for their services.

RESULTS AND DISCUSSION

The temperature continuum for data from both the warm and

Fig. 1. Magnitude estimation of apparent warmth and cold in separate warm and cold series. Each point represents about 30 temperature-judgment combinations, provided by 12 Os who judged 10 temperatures in each series. The assumed neutral point was 31 deg C for both series. Coordinates are log-log.

cold series was divided into four consecutive intervals so that about the same number of judgments fell in each interval. Within each interval the arithmetic mean of the judgments and temperatures was taken. Thus, the 120 warm and 120 cold judgments were reduced to four points in each series, with approximately 30 temperature-judgment combinations represented by each point. These points are plotted in log-log coordinates in Fig. 1 with an assumed neutral temperature of 31 deg C for both warm and cold. The lines fitted by eye to the two sets of points have slopes of 0.6 and 1.0 for cold and warm, respectively. The Stevens and Stevens findings were essentially replicated: the power law (Eq. 1) describes the results well, the assumed neutral point is close to that used by Stevens and Stevens, and the exponent for cold is about two-thirds that for warm. The only difference is in the absolute magnitude of the warm and cold exponents.

For summary of the data from the Full condition, the temperature continuum was divided into 10 intervals containing four temperatures for which judgments had been obtained. An 11th interval at the warmest end of the scale contained five judged temperatures. Within each interval the geometric mean of the judgments and the arithmetic mean of the temperatures was taken, and the resulting 11 points, summarizing 480 judgments,

Fig. 2. Magnitude estimation of apparent temperature over the full range of thermal sensitivity. The point at the warmest end of the scale summarizes the data obtained with the five warmest stimuli judged while the remaining 10 points each represents the data from an interval containing four judged temperatures. Each point is plotted at the geometric mean of the judgments and the arithmetic mean of the temperatures in the interval it represents. The same 12 Os who judged the warm and cold series also judged the full series, each O making 40 estimates in the full range. Coordinates are semi-log.

are seen in Fig. 2, where apparent magnitude is plotted against stimulus temperature in deg C. Of many representations attempted, none gave so straight a line, and no manipulation of constants in Eq. 1 fit the data very well. The appropriateness of the linear fit is illustrated by the fact that, for log judgment vs linear temperature (before averaging), $\eta^2 = .74$ and $r^2 = 0.69$. Moreover, individual plots were made for each of the 12 Os in log-lin coordinates and all were quite linear, although there were differences in slope. The evidence seems clear that an exponential function is necessary to describe the relationship between subjective temperature and temperature measured according to the conventions of the physical sciences. The function,

$$\psi = ke^{\alpha\phi}, \quad (2)$$

expresses the response of the subjective thermometer over the range of tolerable temperatures. The line fitted to the points in Fig. 2 obeys this function where $k = 2.56$ and $\alpha = .0425$ (Lewis, 1960, pp. 61-64).³

It is of some interest to determine how the power law can hold for the warm and cold ranges when scaled separately, while an exponential function describes the growth of subjective magnitude over the full range of thermal sensitivity. The two results can be shown not to be in contradiction if O is assumed to make some transformation of his judgments to conform to the different requirements of the two judgmental tasks. For the sake of exposition, Eq. 2 will be taken as the expression of the basic relation between the subjective and physical thermal scales. Faced with the task of judging amount of "warmth" (hotness) against a standard somewhere in the middle of a warm series, O must recalibrate his judgments so that (a) a judgment of zero falls at or below the lower limit of warm temperature sensations and (b) his numerical estimate of the standard is 10. The following transformation of "basic" subjective sensations (J_f) of the warm half of the continuum into estimations of warmth (J_w) will satisfy the requirements:

$$J_w = n(J_f - J_o), J_f > J_o, \quad (3)$$

where J_o is the basic subjective value of the stimulus of zero warmth and n is a constant that puts the judgment of the standard at 10. An analogous transformation of basic judgments for the cold range into judgments of coldness (J_c) is

$$J_c = m(J_o - J_f), J_f < J_o. \quad (4)$$

In Fig. 3 the straight lines represent points derived from warm

Fig. 3. Transformations of warm and cold ranges of the fitted line of Fig. 2 according to Eqs. 3 and 4, respectively. The transformed empirical points are also shown. The assumed neutral point in these recalibrations was 32 deg C for the warm range and 30 deg C for the cold range. Coordinates are log-log.

and cold regions of the fitted line of Fig. 2 according to Eqs. 3 (with $n = 3$) and 4 (with $m = 2$), respectively. The temperatures against which the transformed judgments are plotted are degrees away from neutral, assumed to be 32 deg C for warm and 30 deg C for cold, and the coordinates are log-log. The empirical points from the Full condition were transformed appropriately and are also plotted in Fig. 3. The slopes for the transformed warm and cold ranges are slightly greater than those found in the Warm and Cold conditions. Stevens and Stevens (1960, p. 188) also found an increase in slope when warm and cold stimuli were judged in the same series.

The transformed points in log-log coordinates are very well approximated by straight-line functions and are therefore well described by power functions. It is not the case, however, that either transformation converts the basic exponential into a power function; the transformations simply provide good approximations. The approximation is excellent when the zero point is anywhere above 25 deg C for warm recalibrations or below 35 deg C for cold, but the fact that it is only an approximation becomes apparent when one goes beyond this range. It is also possible, by suitable converse transformations, to begin with "basic" power functions for the two ranges and recalibrate each one to assemble a function indistinguishable from an exponential for the entire range. The question, then, as to which is truly the basic function cannot be answered on the basis of these data alone. An O could start with an exponential perception of temperature and recalibrate to conform to instructions to judge only warmth or coldness, or conversely, he could start with separate power law perceptions of warm and cold and integrate them to produce an exponential full-range function. Both the power and exponential versions of thermal sensitivity may, in fact, be based on more fundamental perceptions of temperature, derived from a variety of sensory cues and inaccessible through magnitude estimation of temperatures alone.

The results of this study indicate that magnitude estimations of temperatures considered as deviations away from some point near the middle of the temperature scale will obey a power law, while full-range scaling will result in an exponential function. Stevens and Stevens cannot, therefore, validly conclude from their results

with scaling of warmth and cold that a ϕ_0 (threshold) correction should be used to "bring zeros of the physical and psychological scales into coincidence [Stevens & Stevens, 1960, p. 191]" in other continua. It appears that the striking utility of ϕ_0 in producing power functions of warmth and cold is unique to the temperature sense and is not a result of ϕ_0 's being a threshold. Rather, ϕ_0 seems to be the boundary of the range being scaled and merely serves to recalibrate a basically exponential relationship into one indistinguishable from a power function. Further evidence that physiological zero is not a threshold in the sense accorded this term in other continua is the fact that Os were able to use a stimulus at physiological zero quite reliably as a standard stimulus for magnitude estimation. Such a temperature must, therefore, occupy a point some distance from zero on the subjective scale of temperature, whatever qualitative sensation it arouses.

Finally, in light of the present study, we should consider the question asked in the title of this paper: Are there one or two subjective thermal continua? Central to the Stevens and Stevens study is the assumption that there are two subjective continua, warmth and cold. Although the physiological evidence points to separate sensory mechanisms (Zotterman, 1959), the identity of the transducers is not established, and a single sensory mechanism may be responsible for sensations of both warmth and cold (Geldard, 1953, pp. 224-232). Subjective continua do not, however, seem necessarily to be limited by underlying sensory mechanisms, at least insofar as magnitude estimation establishes such continua. Magnitude estimation is, for example, insensitive to the duplex nature of human vision and reveals a single subjective continuum, brightness, which spans energy ranges detected by two separate sensory mechanisms (Warren, 1969, p. 28). If we are willing to assume that whenever we find a power law relationship between ψ and ϕ values we are dealing with a single subjective continuum, then we should accept the Stevens and Stevens results as showing that there are two *subjective* thermal senses, whatever the physiology of the matter may be. This approach has, however, the unfortunate result that a stimulus near the middle of the temperature range should have a subjective value of zero, even though Os know it has a temperature of some finite value. The single-continuum view of thermal sensitivity, on the other hand, takes account of the fact that thermal stimuli vary along a single dimension and that the evidence is poor for a subjective "hole" in the middle of the scale. The sole defect of this view, it seems, is the fact that it does not result in a power law. While the power law has been verified in a large number of prothetic continua, there is no compelling theoretical reason to expect it to hold in every one. Besides, if Stevens and Stevens are allowed their transformation of the temperature scale that gives them a power law, we should be allowed ours (cf Note 3).

REFERENCES

- GELDARD, F. A. *The human senses*. New York: Wiley, 1953.
- LEWIS, D. *Quantitative methods in psychology*. New York: McGraw-Hill, 1960.
- STEVENS, J. C., & STEVENS, S. S. Warmth and cold: Dynamics of sensory intensity. *Journal of Experimental Psychology*, 1960, 60, 183-192.
- STEVENS, S. S. To honor Fechner and repeal his law. *Science*, 1961, 133, 80-86.
- WARREN, R. M. Visual intensity judgments: An empirical rule and a theory. *Psychological Review*, 1969, 76, 16-30.
- WOODS, P. J., GRIFFITH, B. A., PAGE, R. P., & RODIER, P. M. Human responses to various conditions of water temperature. *Perception & Psychophysics*, 1967, 2, 157-160.
- ZOTTERMAN, Y. Thermal sensations. In J. Field, H. W. Magoun, and V. E. Hall (Eds.), *Handbook of physiology*. Vol. 1. Washington, D.C.: American Physiological Society, 1959. Pp. 431-458.

NOTES

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3. The arbitrary "thermometer" measure of temperature has been accepted both by Stevens and Stevens and by the present author. If we assume that temperature should actually be measured differently and that "real" temperature (T_t) is proportional to the logarithm of temperature in conventional units (T_c , measured in degrees away from some zero point) we have the following relationship:

$$T_t = A \log T_c. \quad (5)$$

Now if the "true" measure is substituted for conventional temperature in Eq. 2, the operating characteristic for the complete range of thermal sensitivity, we get:

$$\psi = k e^{\alpha (A \log T_c)}. \quad (6)$$

The advantage of this logarithmic measure of temperature, if it be an advantage, is the fact that with it the results of the full-range scaling obey the power law. This is shown when logarithms are taken on both sides of Eq. 6 and the following linear relationship results:

$$\log \psi = B \log T_c + C, \quad (7)$$

where $B = \alpha (A \log e)$ and $C = \log k$.

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