Generating complex waveforms*

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A wide variety of complex waveforms can be generated by approximating the desired analog waveform from an array of digital values. Some basic properties of these digital approximations are discussed in terms of pulse amplitude modulation and sampling theory. The waveforms are generated by transferring the digital values to a digital-to-analog converter followed by a low-pass filter. This usually requires the dedicated use of a computer. We have built a device, incorporating solid state memory, that can store, time, and transfer previously computed digital values, so that a computer is no longer necessary to generate the waveforms. Specifications of the digital-to-analog converter and appropriate settings of the filter are discussed, along with a simplified procedure for calculating waveforms that have line spectra. An adaptation of this procedure enables the device to be used as a high-speed programmable pure-tone source.

INTRODUCTION

The majority of research in hearing over the last 100 years has involved single-component sine waves as signals. Although the convenience and economy of tuning forks prior to the advent of the electronic age and of pure-tone oscillators since then have contributed to this preoccupation with simple sinusoids, implicit faith in Ohm's acoustic law (Schouten, 1970) as interpreted by Helmholtz (Plomp, 1970) has impeded research on both the perception of complex tones and techniques for generating such complex signals (Plomp, 1964). Two factors have stimulated interest in the perception of complex tones. First, under the impetus of Schouten's work (1940), many workers in Holland have been pursuing a non-Helmholtz point of view (cf. Plomp & Smoorenburg, 1970). Second, the recent work in the electrophysiology of the cochlea has identified and described the essential nonlinear function of hair cells (Whitfield, 1967; Engebretson & Eldredge, 1968; Dallos, 1969). The fact that the transfer function (mechanical to electrical) of the hair cells is of multinomial form (Eldredge & Miller, 1971) should give strong impetus to the study of the perception of tonal complexes.

METHODS OF GENERATING A TONAL COMPLEX

Generation of multiple-tone complexes has traditionally involved adding or multiplying (i.e., modulating) the output of pure-tone

oscillators. Plomp (1964) has shown that by constructing the resultant waveform from its constituent parts (i.e., the reverse of Fourier analysis) multiple-tone complexes having well defined line spectra can be generated with a minimum of equipment. Several laboratories currently are synthesizing waveforms that have either continuous or line spectra with independent control of frequency amplitude and phase of components. With the aid of a computer, waveforms are calculated

and stored in digital form as a series of equally spaced data points. To generate the waveform, the data points are converted into a stepped-analog voltage by a digital-to-analog converter (DAC) and then smoothed to an approximation of the desired waveform by means of a low-pass filter.

SOME PROPERTIES OF STEPPED-ANALOG WAVEFORMS

The stepped waveform seen at the output of the DAC is composed of a series of rectangular pulses of duration T , the reciprocal of transfer rate. The magnitude and polarity of these pulses vary as a function of the desired waveform. This stepped waveform has an amplitude spectrum that conforms to a $(\sin X)/X$ function with intercepts on the frequency axis at intervals of $1/T$, where T , in this case, is the duration of the pulse. If the desired waveform contains only frequencies less than $1/2T$, then the combined spectrum will contain the desired components plus sidebands of these components at intervals of $1/T$, introduced by the modulation of the rectangular pulses (Black, 1953). The amplitude and phase of all components in the combined spectrum is determined by the spectrum envelope of the rectangular pulse carrier. This is shown in Fig. 1.

Fig. 1. The upper part of this figure illustrates the amplitude spectrum of the signal to be synthesized by means of a stepped waveform. The lower drawing shows real spectrum of that waveform when generated at a pulse frequency (i.e., transfer rate) of $1/T$, where T , in case of a stepped waveform, is the duration of the step. This spectrum has a $\sin X/X$ envelope with zeros at integer multiples of the pulse frequency.

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Fig. 2. Waveform and partial spectra for: (a and b) two single component signals, f_1 and f_2 , and (c) the composite of f_1 and f_2 , all with the same pulse frequency, $f_p = 1/T = 2f_2$. The amplitude of the f_2 component exceeds the boundaries of the $(\sin X)/X$ envelope by 6 dB because it coincides with the lower sideband of f_p (i.e., $f_p - f_2$).

Fig. 3. The same three signals of Fig. 2, at twice the pulse frequency ($f_p = 4f_2$).

Figure 2 shows the waveforms and spectra of two single component signals (2a and 2b) and their complex (2c). The pulse frequency is the same for all three waveforms, $f_p = 1/T = 2f_2$. The spectrum of these signals shows that there is no energy (i.e., a zero) at multiple integrals of the transfer rate, f_p , and that the sideband components are distributed symmetrically around these zeros. The square wave shown in Fig. 2a represents a special case of stepped waveform in that the lower sideband of f_p coincides with a desired frequency component.

The sideband component separation in Figs. 2a and 2b makes it relatively easy to eliminate the unwanted sidebands by passing the stepped waveforms through low-pass filters with cutoff frequencies of $2f_2$ and $2f_1$, respectively.

The spectrum of the two tone complex in Fig. 2c shows that, when we introduce additional components while maintaining a pulse frequency of $2f_2$, only one lower sideband frequency coincides with a desired component, and the others appear as distortion components between f_2 and f_p . As any desired component approaches the highest frequency (f_H) in the complex, the unwanted lower sideband of f_p gets closer to f_H and becomes increasingly difficult to eliminate by low-pass filtering.

One solution to this problem is shown in Fig. 3. Here the same three waveforms are approximated, this time with the pulse frequency and, therefore, the number of data points increased by a factor of 2 (i.e., $f_p = 4f_2$). The amplitude of f_2 in Fig. 3a is less than that in Fig. 2a, because the lower sideband of f_p no longer coincides with f_2 . This loss in amplitude is somewhat offset by the fact that, for a higher pulse frequency, the $(\sin X)/X$ amplitude function has been extended relative to the desired components, moving them closer to the maximum amplitude of the curve.

In Fig. 3c we can see that increasing the pulse frequency to $4f_2$ effectively limits the lower sideband of f_p to frequencies of $3f_2$ and higher. A low-pass filter cutoff frequency of $\sqrt{2}f_2$ should effectively eliminate all unwanted sidebands.

THE COMPLEX WAVEFORM GENERATOR

The typical laboratory computer can be used to compute and generate the stepped waveform but is not yet fast enough to perform both jobs concurrently. Waveforms must be calculated in advance and stored either in local memory or on a bulk storage device. When the waveforms are being generated, computer time is so

Fig. 4. Circuit diagram of one bank of solid-state memory. Memory inputs come from the data bus common to all circuits (BAC 17-3). One side of the complimentary data outputs of each chip go to the DAC. Memory address lines, MAO-8, come from the memory address counter. The memory control logic is used for bank selection and to disable memory either under program control or on overflow of the signal duration counter.

severely restricted by maintaining a precise transfer rate (pulse frequency) that it is impossible to service or monitor other devices or perform any other task. This is a problem in laboratories such as ours, where the computer is called upon to control and service several devices during the course of any experiment. The availability of a direct memory access channel (DMA) provides a partial solution to this problem. The single-cycle break facility (or equivalent) will permit high-rate data transfers between the computer's core memory and the DAC on a cycle stealing basis. The DMA device controller (break interface) must contain memory address, word count, and priority logic. Waveforms are generated with the aid of an external programmable clock to pace the transfers. With similar logic, and the addition of external memory, we have constructed an essentially independent device that can store and generate the waveforms without calling on the computer each time a data point is transferred and without interfering with the DMA channel priorities. The device is designed to be supported by a computer which supplies it with waveform values, transfer rate, address limits, and starting time information. Once the device is started, its operation is no longer dependent on

Fig. 5. Memory access logic. The initial and terminal memory addresses are held in a single 16-bit memory address limit register. The outputs of the memory address counter and the most significant half of this register (bits 0-7) are connected to comparator logic. When a match is detected, the counter is reset from the least significant half of the limit register (bits 8-15) and the signal duration counter is incremented by one.

card with wire wrappable integrated circuit sockets; a short multiconductor cable is used to connect this hardware to the computer I/O bus and the buffered DAC. A diagram of the interface to a DEC PDP-15 I/O bus is shown in Fig. 8.

THE DAC

Speed, stability, and resolution are the primary characteristics to be considered when selecting a DAC. In this application, where the DAC is always followed by a low-pass filter that retards and stabilizes the waveform, conversion rate and settling time become secondary to the ability of the DAC to resolve small amplitude increments in the waveform. Resolution is a function of two variables: the number of bits in the converter and the number of bits involved in the conversion. For example, an 8-bit converter has the same resolution as a 16-bit converter at the 8-bit level (over a range from 0 to 256). Thus, in waveform applications we can state two important considerations. First, buy as much resolution (i.e., bit capacity) as you can afford, and second, scale the data as close as possible to the maximum level of the converter. The first consideration poses some problems because, beyond 12 bits, the DAC cost increase is proportional to the square of the resolution increase. Each bit added to the converter doubles resolution but quadruples cost. This also implies that the memory that holds the waveform data must agree in word length with the converter, or that some method of multiple-word transfer must be employed. To comply with the second consideration, the waveform data points should be computed at the maximum resolution available, either on a large desk calculator or with computer arithmetic routines, using double or triple precision if necessary. In scaling the waveform values, keep in mind that the individual data points

Table 1
Minimum Number of Data Points, MED, and Transfer Rate for Complexes With f_H Equal to 500, 1000, 2000, and 4000 Hz, and a Transfer Rate Equal to $4f_H^*$

f_{GCD} (Hz)	$f_H = 500$	$f_H = 1000$	$f_H = 2000$	$f_H = 4000$	MED (Sec)
1	2000	4000	8000	16000	1.0
2	1000	2000	4000	8000	0.5
5	400	800	1600	3200	0.2
10	200	400	800	1600	0.1
16	125	250	500	1000	0.0625
25	80	160	320	640	0.04
50	40	80	160	320	0.02
100	20	40	80	160	0.01
200	10	20	40	80	0.005
250	8	16	32	64	0.004
500	4	8	16	32	0.002
$f_p = 4f_H =$	2 kHz	4 kHz	8 kHz	16 kHz	

* f_{GCD} is the frequency of the greatest common divisor in the complex.

LOW-PASS FILTERING
The minimum cutoff frequency of

are the sum of the component magnitudes and should not exceed the maximum signed value the converter can handle. The converter used in our device has a bipolar output, with 14 bits of data plus 1 bit for the sign. An internal 15-bit register, loaded by the memory control logic, holds the data for conversion.

the filter is defined by the highest frequency in the complex, the number of data points in the waveform, and the transfer rate. The transfer rates given in Table 1 are for a minimum number of data points and assume a filter cutoff at $\sqrt{2} f_H$. If the same transfer rate is used in computing several complex waveforms, only one filter cutoff frequency is necessary for all of the waveforms. For example, two complexes, one with f_H equal to 1000 Hz and the other with f_H equal to 500 Hz, can use the same 1400-Hz cutoff if the 500-Hz complex is computed with twice the minimum number of data points.

Analog filters in general cause varying degrees of phase shift to all signals within the pass band. If relative phase of components in a complex is an important parameter, some means of equalizing the filter-induced phase shift is necessary. Simple analog phase equalizing (i.e., all pass) filters can be used to change the phase of components empirically after low-pass filtering. If the filter characteristics are well defined, phase equalization can be accomplished by adjusting the phase term in the expression used to calculate the waveform.

Fig. 8. Basic programmed I/O interface to a DEC PDP-15 computer. The PDP-15 I/O bus at the input to the M510 is bilateral. The M510 buffers all output transfer from the AC (BAC0-17) and is common to several devices.

Fig. 9. Relative amplitude spectrum of 11 single component waveforms, from 125 to 8000 Hz, with a 24-kHz transfer rate.

MINIMUM DATA POINT CALCULATIONS FOR WAVEFORMS WITH LINE SPECTRA

When a waveform is constructed by the addition of several components having the same time reference, $1/T$, the spectrum of the composite waveform will be the sum of the spectra of the individual components (Lande, Davis, & Albrecht, 1957). We can approximate a waveform having more than one sinusoidal component by adding the data points of each component using the following expression:

$$V(k) = \sum_{i=1}^N A_i \sin [(f_i/f_H)k\pi/2 + \phi_i]$$

for $k = 1, 2, 3, \dots, m$

where $V(k)$ are the data point values; N is the number of components; A_i , f_i , and ϕ_i are the amplitude, frequency, and phase of individual components; f_H is the highest frequency in the complex; and m is the minimum number of data points. This equation assumes a sine phase reference for all components. Other phase references, such as cosine phase to duplicate amplitude modulated signals, can be

effected through the use of the phase term or by replacing the sine with a cosine function. The $k\pi/2$ term sets the transfer rate, $1/T$, at $4f_H$ to prevent unwanted sideband components from intruding into the filter pass band.

The number of data points necessary to approximate a given waveform is dependent upon the spacing of spectral components and the transfer rate. For any combination of sinusoids, the total number of data points necessary to represent their complex waveform is a function of the length of time required for all components to return to their starting values simultaneously. Thus it takes two periods of a 1000-Hz tone to complete one period of a 500-Hz tone. With a transfer rate of $4f_H$, eight data points would be necessary to describe a 500- and 1000-Hz complex. The minimum time necessary for multiple tones to "resynchronize" we have called the minimum epoch duration (MED). For any number of tones in a steady-state phase-locked complex, the MED is equal to the reciprocal of the greatest common divisor of the individual frequencies. The minimum number of data points necessary in a tonal complex, while maintaining sufficient spacing between desired and reflected components, is four times the highest frequency in Hz times the MED in seconds. Table 1 lists the minimum number of data points, the MED, and the transfer rate for several waveforms with a variety of component separations. It should be remembered that the number of data points beyond eight and the MED are not determined by the number of components but by the frequency relationship between components.

A variation of this simplified computation permits the waveform generator to serve as a fast-access programmable pure-tone source. The

waveform values of the desired frequencies are computed from the same expression but not summed. The result is a set of waveforms that have a common reference frequency (f_H) and transfer rate ($4f_H$). The reference frequency is not necessarily a member of the set. The MED for each pure tone is the reciprocal of the greatest common divisor of the pure tone and the reference. The transfer rate is maintained at $4f_H$ by computing waveform values at the same intervals of k (i.e., $k = 1, 2, 3, \dots, m$), so that only one filter cutoff frequency, $\sqrt{2} f_H$, will serve for the entire set. To take advantage of the lower transfer rate requirement of single component waveforms, a transfer rate of $3f_H$ could be used and the filter cutoff moved to slightly above the highest frequency. An example of this is illustrated in Fig. 9, where the waveform generator contains waveforms for the 11 audiometric frequencies, computed at a 24-kHz transfer rate. The spectrum shows a one-octave guard band between the highest desired component, 8000 Hz, and the lowest sideband component, 16 kHz.

The waveform generator was designed to store and generate any stepped waveform compatible with the limits of memory and the definition of the programmable clock. Waveforms with either line or continuous spectra, from single sinusoids to Huffman sequences (Patterson & Green, 1970) or pseudorandom noise, can be accommodated by the device. With waveforms having adequate component spacing requiring a minimum number of data points, say less than 40 (see Tables 1 and 2), it becomes reasonable to consider eliminating the computer entirely from the device. Data points could be calculated on a desk calculator and loaded into memory by means of a switch register. The generator diagrams show that a common data bus is used for all data coming from the computer. Load control pulses such as load memory address, write in memory, load clock, etc., route the data to the proper register. Each time new data is written into memory, that same data is strobed into the DAC, and then the memory address register is incremented by one.

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Table 2
Number of Data Points Required for Each of 11 Single-Frequency Waveforms With a Common Transfer Rate ($3f_H = 24$ kHz), and Filter Cutoff Frequency (> 8000 Hz)

Frequency	Number of Locations
8000	3
6000	4
4000	6
3000	8
2000	12
1500	16
1000	24
750	32
500	48
250	96
125	192

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