CIE x,y coordinates from an inexpensive projection colorimeter*

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The present paper reports an extension of the design for a projection colorimeter originally proposed by Riggs, 1964. A technique and the necessary calculatory formulas are presented through which the definitions of colors produced by the instrument may be defined in terms of CIE x,y coordinates of chromaticity space. Data on the accuracy of measurements obtained from a random sample of naive Os are also reported.

Riggs (1964) has suggested a simple and inexpensive projection colorimeter which was primarily designed for classroom and laboratory demonstrations of the various phenomena of color mixing and matching. The basic design can be extended to research applications or to more sophisticated demonstrations of color by a method to be suggested in the present paper through which CIE x,y coordinates of colorimetric space can be generated to define the colors produced by the instrument.

The basic colorimeter consists of a 35-mm slide projector from which the condensing lenses have been removed. Two achromatic lenses are mounted in the tube of the projector. As shown in Fig. 1, three filters representing the primary colors of red, green, and blue are mounted on a mechanical stage and stationed several inches in front of the slide projector, directly in the path of the projected light beam.

The stage must be placed such that the filaments of the projection bulb are clearly in focus upon the three filters. By placing a 35-mm negative in the slide holder from which a 1/4-in. circular opening has been cut, a beam of light can be produced which passes through the three filters to a screen located several feet away. With an adjustment of the projection lens of the projector, the beam of light may be focused on the screen as a homogeneous circle of color. The color of the circle is then defined by the relative proportions of the three primary filters contributing to the beam. Through adjustments of the vertical and horizontal positions of the mechanical stage, the proportional

contribution of each filter can be varied. Thus, a wide variety of colors can be produced. The basic design is shown schematically in Fig. 2. For details of the construction of the colorimeter consult the paper by Riggs (1964). The Argus 300 Model III is no longer available. We have found the Viewlex Model V27 to be an acceptable substitute.

By the attachment of speedometer cables between the vertical and horizontal adjustment screws of the mechanical stage containing the filters and two corresponding knobs on a control panel, Os are able to comfortably adjust the stage to various proportions of the three filters.

The measurement of the CIE x,y coordinates defining the color produced on the screen is accomplished by measuring the amount of light passing through each filter in the mounting stage after a specific color has been produced. These measurements can be taken by mounting the photosensor from a sensitive photometer on a movable flange such that it can be positioned to interrupt the light beam directly behind the mechanical stage. In turn, the contributions of the red, blue, and green filters are measured by placing the same red, blue, and green filters over the entrance of the photosensor and recording the amount of light passed by each filter. An excellent instrument for this purpose is the Model IL 200 UV Visible Photometer with a PT 100 vacuum diode with the diffusing glass removed from the photosensor. For more accurate measurements, as required in research applications, the Model IL 600 Research Photometer with the PT 200 B photosensor is recommended. Again, the diffusing glass should be removed from the entrance to the photosensor and replaced with an open A200 hood. This equipment is produced by the International Light Co.

The three photometer readings (R, G, and B) are entered into Transformation Eqs. 1a, 1b, and 1c.

$$X_m = R(X_r) + G(X_g) + B(X_b) \quad (1a)$$

$$Y_m = R(Y_r) + G(Y_g) + B(Y_b) \quad (1b)$$

$$Z_m = R(Z_r) + G(Z_g) + B(Z_b) \quad (1c)$$

These are then translated into CIE, x, y, and z coordinates by Eqs. 2a, 2b, and 2c.

$$x = X_m / X_m + Y_m + Z_m \quad (2a)$$

$$y = Y_m / X_m + Y_m + Z_m \quad (2b)$$

$$z = Z_m / X_m + Y_m + Z_m \quad (2c)$$

The weighting factors in Eqs. 1a, 1b, and 1c represent corrections for the spectral transmittances of the three filters, spectral sensitivity of the photosensor, spectral transmittance of the heat filter, and spectral transmittance energy distribution of the projection lamp.

For the design described here, in which a CWA (G.E.) projection lamp with a color temperature of 3200°K, Viewlex Heat Filter HFR-1, PT 200B photosensor, and Wratten Filters 26, 61, and 48 were employed, the weights are given in Eqs. 3a, 3b, and 3c.

$$X_m = R(295.2196) + G(236.5339) + B(156.9175) \quad (3a)$$

$$Y_m = R(145.9268) + G(831.2083) + B(74.7733) \quad (3b)$$

Fig. 1. Arrangement of the three filters on the mounting stage.

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Fig. 2. Diagram of the projection colorimeter (after Riggs, 1964).

$$Z_m = R(.1000) + G(87.4892) + B(927.8351) \quad (3c)$$

If the photosensor PT 100 is used, the weights given in Eqs. 4a, 4b, and 4c should be applied.

$$X_m = R(986.1444) + G(482.8463) + B(165.2207) \quad (4a)$$

$$Y_m = R(484.1873) + G(1427.2581) + B(88.0669) \quad (4b)$$

$$Z_m = R(.3248) + G(119.4267) + B(987.1321) \quad (4c)$$

The cost of the projection colorimeter is approximately \$180.00, and the photometer units suggested here currently list by International Light Co. at \$325 for Model IL 200 and approximately \$1,000 for the IL 600 with a PT 200B photosensor and hood assembly. Thus, the simpler unit would cost about \$500, with the more advanced design costing about \$1,200.

The design employed here with the Wratten 26, 61, and 48 filters can provide a wide range of color matches, as indicated by the positions of the three filters on the chart of chromaticity space shown in Fig. 3. Any color contained within the confines of the triangle interconnecting the three filters used in the colorimeter can be produced.

Naturally, any set of three or more filters, photometer, projection bulb, or heat filter can be used with the design. In some instances, very exact measurements and matches from a small portion of the spectrum may be required, in which case the three filters in the colorimeter could be chosen from the desired area (e.g., three shades of red). The procedure for the computation of the weighting factors to be applied in Eqs. 1a, 1b, and 1c is outlined below.¹

(1) Obtain the percentage transmittance as a function of wavelength in 10-nm intervals for each filter used. ($R_{\lambda\Delta\lambda}, G_{\lambda\Delta\lambda}, B_{\lambda\Delta\lambda}$). For Kodak Wratten filters, this data may be obtained from Kodak Publication B-3.

(2) Obtain the percentage spectral sensitivity curve for the photosensor and for the heat filter ($PS_{\lambda\Delta\lambda}, HF_{\lambda\Delta\lambda}$). Data on the spectral transmission characteristics of the Viewlex HFR-1 heat filter were not available. An estimate of these characteristics was made by measuring the spectral sensitivity of the projection lamp without the heat filter in place

Fig. 3. Diagram of chromaticity space showing the positions of Wratten Filters 26, 61, and 48 and for white. Through the appropriate setting of the three-filter mechanical stage, all hues located within the confines of the triangle can be produced by the colorimeter.

when the beam was passed through an Oriel Model F-11-10 monochromator and comparing the percentage reduction in light when the filter was in place. The results indicated that the filter was insensitive to the short end of the spectrum with a reduction of 18% at 400 nm, 16% at 410 nm, 9% at 420 nm, 6% at 430 nm, 4% at 440 nm, and 1% at 450 and 460 nm.

(3) Multiply the percentage spectral sensitivity for the photosensor by the corresponding transmittance values for all three filters in 10-nm intervals (see Eqs. 5a, 5b, and 5c).

$$T_{\lambda\Delta\lambda} = (R_{\lambda\Delta\lambda})(PS_{\lambda\Delta\lambda})(HF_{\lambda\Delta\lambda}) \quad (5a)$$

$$T_{\bar{g}\lambda\Delta\lambda} = (G_{\lambda\Delta\lambda})(PS_{\lambda\Delta\lambda})(HF_{\lambda\Delta\lambda}) \quad (5b)$$

$$T_{\bar{b}\lambda\Delta\lambda} = (B_{\lambda\Delta\lambda})(PS_{\lambda\Delta\lambda})(HF_{\lambda\Delta\lambda}) \quad (5c)$$

(4) Values from Eqs. 5a, 5b, and 5c must then be multiplied by the CIE color matching functions ($\bar{x}, \bar{y}, \bar{z}$) which have already been weighted by the spectral energy distribution of the projection lamp. The values may be found in Wyszecki and Stiles (1967), Tables 3.9-3.26 for numerous light sources and Table 1.19 for tungsten light.

$$X_r = \Sigma(T_{\lambda\Delta\lambda})(\bar{x}_{\lambda}H_{\lambda\Delta\lambda}) \quad (6a)$$

$$Y_r = \Sigma(T_{\lambda\Delta\lambda})(\bar{y}_{\lambda}H_{\lambda\Delta\lambda}) \quad (6b)$$

$$Z_r = \Sigma(T_{\lambda\Delta\lambda})(\bar{z}_{\lambda}H_{\lambda\Delta\lambda}) \quad (6c)$$

$$X_g = \Sigma(T_{\bar{g}\lambda\Delta\lambda})(\bar{x}_{\lambda}H_{\lambda\Delta\lambda}) \quad (7a)$$

$$Y_g = \Sigma(T_{\bar{g}\lambda\Delta\lambda})(\bar{y}_{\lambda}H_{\lambda\Delta\lambda}) \quad (7b)$$

$$Z_g = \Sigma(T_{\bar{g}\lambda\Delta\lambda})(\bar{z}_{\lambda}H_{\lambda\Delta\lambda}) \quad (7c)$$

$$X_b = \Sigma(T_{\bar{b}\lambda\Delta\lambda})(\bar{x}_{\lambda}H_{\lambda\Delta\lambda}) \quad (8a)$$

$$Y_b = \Sigma(T_{\bar{b}\lambda\Delta\lambda})(\bar{y}_{\lambda}H_{\lambda\Delta\lambda}) \quad (8b)$$

$$Z_b = \Sigma(T_{\bar{b}\lambda\Delta\lambda})(\bar{z}_{\lambda}H_{\lambda\Delta\lambda}) \quad (8c)$$

(6) The photosensor readings are then entered into Transformation Eqs. 1a, 1b, and 1c and multiplied by the weighting factors obtained through Eqs. 6a, 6b, 6c, 7a, 7b, 7c, and 8a, 8b, 8c.

COLORIMETER TEST

Method

In order to test the colorimeter, a

Table 1
Mean and Standard Deviation of x and y Coordinates for Seven Test Filters and the Actual x and y Coordinates for Each of the Filters

Wratten Number	Filter		Mean Matches				Mean Differences	
	x	y	x	SD _x	y	SD _y	X _m - X _f	Y _m - Y _f
4	.244	.583	.251	.014	.624	.028	.007	.041
11	.220	.630	.220	.008	.658	.018	.000	.028
22	.484	.469	.372	.030	.526	.028	.112	.057
30	.369	.302	.346	.061	.367	.067	-.023	.065
38A	.171	.386	.177	.009	.382	.079	.006	-.004
57A	.213	.668	.206	.002	.677	.016	-.007	.009
85	.258	.496	.273	.005	.558	.053	.015	.062

group of 49 students was asked to match Wratten Filters 4, 11, 22, 30, 38A, 57A, and 85 with the projection colorimeter. The Os adjusted the colorimeter to match the hue of each filter projected on a screen located 10 ft from the O. The test hue was projected as a circle subtending 1.33 deg of arc located immediately to the right of the colorimetric projection. The size of the colorimetric projection was also 1.33 deg of arc. Before each trial, the colorimeter was adjusted to white ($x = .2222$, $y = .4777$), and the O then matched the test projection. The order of presentation of the filters was randomized across Os.

Results

The x,y coordinates of each O for all seven filters were then calculated via Eqs. 3a, 3b, 3c and 2a, 2b, 2c. Mean x and y coordinates and standard deviations were then computed. These results appear in Table 1. Also contained in Table 1 are the x and y coordinates generated by Eqs. 3a, 3b, and 3c for each filter used in testing when the actual filter was substituted for the tristimulus filter of the colorimeter. A series of such measurements was taken over 7 days during different times of day. The standard deviation of these measurements never exceeded .0002 for either x or y.

Discussion

The matches, as can be seen in the table, are fairly accurate. Some of the variance would be due to differences in brightness between the test and colorimetric projections. Technically, one cannot speak of color matches when an intensity difference exists between the colorimeter and target to be matched. This problem can be resolved by mounting a variable neutral density wedge between the projector and the mounting stage containing the three filters. In order to avoid an intensity gradient in the

field, the wedge must be placed in the plane of the filament images in front of the projector. An additional weight for the spectral transmission characteristics of the wedge should then be entered into Eqs. 5a, 5b, and 5c. The only filter presenting great difficulty seemed to be Wratten 22 (orange), in which the difference between observed and expected mean was rather large. An exact match of this filter is not possible with the colorimeter. The x and y coordinates obtained when the filter was substituted for the tristimulus filter of the colorimeter fall outside the range of matches possible with the three filters employed in the colorimeter. As can be seen in Fig. 3, a plot of x and y for Filter 22 falls just above the line connecting the red and green filters of the colorimeter.

REFERENCES

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NOTE

1. The formulas use the following notations: $\lambda = 400-700$ nm, $\Delta\lambda = 10$ nm, R = percentage spectral transmittance of red filter, G = transmittance of green filter, B = transmittance of blue, PS = percentage spectral transmittance of photosensor, HF = percentage transmittance of heat filter, T_r = corrected spectral transmittance for red, $T_{\bar{g}}$ = corrected spectral transmittance for green, $T_{\bar{b}}$ = corrected spectral transmittance for blue, $\bar{x}$, $\bar{y}$, and $\bar{z}$ = 1931 CIE color matching functions. The transformation equations of Formulas 1a-2c are those suggested by Burnham, Hanes, and Bartleson (1963), whereas those presented in Formulas 5a-8c are derived from Wyszecki & Stiles (1967). The notations in the transformation equations are based on Wyszecki and Stiles (see their Table 3.27). Exceptions are the use of PS for the photosensor and HF for the heat filter.