Individual half-judgment brightness functions¹

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Nine Ss made half-judgments of each of nine brightness standards. Individual half-judgment brightness functions were constructed, one for each of the two threshold forms of the power law. The ψ -Law (translation on the psychological axis) provided a better fit to the half-judgment data than did the ϕ -Law (translation on the intensity axis). A test of scale consistency for the fractionation method was confounded with the power law hypothesis, but the data are interpreted as providing fair support for the method independent of the form of the power law. The effect of standard on exponent estimates was significant for both forms of the law, and the effect of standard on threshold estimates was significant for the ϕ -Law but not for the ψ -Law. Both forms of the law contain a so-called threshold parameter, but the interpretation of this parameter as a threshold was rejected for the ϕ -Law and accepted for the ψ -Law.

Stevens (1957) presented (his Table 1) a list of studies in evidence that the power function holds for a number of perceptual continua. An inspection of the methods used in these studies indicates that ratio production (fractionation and multiplication) was used up to that time almost as frequently as magnitude and ratio estimation. Since that time there has been vastly accelerated research on the latter methods but relatively little attention to ratio production. As a result of such intense study of the numerical response methods, detailed information is becoming available raising questions as to the implications of observed effects of various independent variables (Poulton, 1968). Problems may also arise due to the way in which Ss use numbers, leading Curtis, Attneave, and Harrington (1968) to postulate a two-stage model of magnitude judgment involving an "output" exponent (which they claim is related to S's use of numbers) as well as the usual sensory or "input" exponent.

Although ratio production has also been shown to be sensitive to changes in certain independent variables such as the effect of context (Garner, 1954) and standard (Stewart, Fagot, & Eskildsen, 1967), there is one important difference between the methods which may limit generalizations. This is the fact that with magnitude and ratio estimation, S produces a number and with ratio production (as well as bisection), S produces a stimulus (i.e., performs an operation which produces a stimulus standing in a specified relation to another stimulus). The point is that there are basic differences in axiomatizations of formal systems which involve operations as primitives as contrasted to systems involving only relations (see Suppes & Zinnes, 1963, especially Section 3.5). The two classes of methods should not, therefore, be casually linked.

A somewhat neglected problem in the study of the direct scaling methods is that of scale internal consistency, which in the case of a formal model would consist of tests of the axioms. This critical problem has received some attention recently in the case of the numerical methods (e.g., Mashhour, 1961; Sjöberg, 1965; Svenson and Åkesson, 1966, 1967) but not in the case of ratio production. Earlier studies of internal consistency (e.g., Geiger & Firestone, 1933; Hanes, 1949a, b) compared fractionation and multiplication, but none of these studies led to unequivocal results regarding the validity of the method.

One problem with tests of ratio production is that for an arbitrary set of stimuli and fixed fraction or multiple, there are

no built-in criteria for rejecting the scale. However, if a particular form of the psychophysical function is assumed, then a test can be made, although in this case scale validity will be confounded with the power function hypothesis.

The simple power function has been complicated with the introduction of a threshold parameter, first via translation on the intensity axis—the ϕ -Law—as apparently first suggested by Ekman (1958), and takes the form $\psi = a(\phi - t)^k$, where ψ is psychological magnitude, ϕ is intensity, and t is usually interpreted as a "threshold parameter." A second form of the law—the ψ -Law—takes the form $\psi = a(\phi^k - t^k)$ and involves translation on the psychological axis. The ψ -Law was first used by McGill (1960) and also suggested by Ekman (1961). The prevalent view has been that the ϕ -Law is the appropriate form of the law, but contradictory evidence has been presented by McGill (1960) for loudness; Jameson (1965), Stewart, Fagot, and Eskildsen (1967), and Fagot and Stewart (1968) for brightness; Ekman and Åkesson (1965) for gustatory intensity; and Irwin and Corballis (1968) for loudness and softness. Much research is still needed to give a definitive answer to the question of the appropriate form of the law, and to the role played by modality and scaling method. For brightness, the recent evidence cited definitely appears to favor the ψ -Law when the method of magnitude estimation is used, but the study by Stewart, Fagot, and Eskildsen (1967) does not give conclusive support for the ψ -Law when fractionation is used.

The purpose of this study is to construct and assess individual half-judgment brightness functions. For each S, two theoretical half-judgment functions will be constructed, one based on the ϕ -Law and one based on the ψ -Law, and the fit of the two laws to the half-judgment data will be statistically compared. We shall also investigate the effect of the standard fractionated on estimates of the thresholds and exponents of the two power laws.

Theoretical r-Judgment Functions

In the application of fractionation or multiplication procedures, S is presented with a standard ϕ_S and instructed to produce a stimulus ϕ_r which is a given fraction or multiple r of the standard stimulus, i.e.,

$$(1) \psi_r = r\psi_S$$

where ψ denotes psychological magnitude. The precise form of the r -judgment function depends on the form of the psychophysical function. If the ϕ -Law holds, then the r -judgment function is of the form:

$$(2) \phi_r = r^{1/k}\phi_S + (1 - r^{1/k})t \quad (r > 0),$$

where t and k are threshold and exponent parameters, respectively, of the ϕ -Law. The r -judgment function for the simple power law is, of course, obtained by setting $t = 0$.

If the ψ -Law holds, then the r -judgment function is of the form:

$$(3) \phi_r = \left[r\phi_S^k + (1 - r)t^k \right]^{1/k} \quad (r > 0),$$

where t and k are threshold and exponent parameters, respectively, of the ψ -Law. In this study we focused on half-judgment functions, i.e., $r = 1/2$.

If t is inferred from the psychophysical function, then its value will in general be smaller in Eq. 3 (Fagot & Stewart, 1968). If t is the same value in both equations (if, for example, t is interpreted as an independently measured threshold), then the exponent in Eq. 3 will be smaller than in Eq. 2.

Ekman (1958) has pointed out that if a power function holds then the r -judgment function is linear. His statement holds only for Eq. 2, not for the ψ -Law. Likewise, Treisman's (1960) argument that a Weber function applies to fractionation data is not valid if the ψ -Law rather than the ϕ -Law applies.

METHOD

Subjects

Subjects were one male graduate student in psychology, and eight undergraduates, two male and six female, for a total of nine Ss. One of the females and two of the males were psychology majors. Two of the female Ss (SP and BE) had previously participated in a brightness experiment using fractionation.

Apparatus

The apparatus, described in detail elsewhere (Eskildsen, 1963), provided two luminous white circular targets 13 mm in diam, placed horizontally and separated by 90 mm, center to center. The luminance (ϕ) was continuously variable in each of the targets from 0.1 ft-L to 600 ft-L. Ss were presented a pair of targets on each trial with the right-hand target always designated as the standard, while S could vary the luminance of the left-hand target continuously over the entire stimulus range by means of a control switch. On each trial the standard was set at one of the following nine luminances: .1, .3, .9, 2.5, 7.5, 23, 68, 200, and 600 ft-L, which is an approximate log spacing. For standards of 23 ft-L or less, the left target was covered by a Kodak Wratten neutral density filter (with a factor of .08454) which permitted S to set the left-hand target below the .1 ft-L lower limit of the unfiltered target. S's control switch activated a cam mechanism which varied the luminance of the light at a rate which depended on the shape of the particular cam being used. In the present experiment an "0.3 exponent" cam was used, which means the cam was cut to provide a uniform rate of change in brightness (not luminance) if the psychophysical function is a power function with exponent equal to 0.3 (see Eskildsen, 1963; and Fagot, Eskildsen, & Stewart, 1966).

Procedure

Each S participated in five sessions lasting about 1 h, all

sessions being held on separate days. Within each session S made five half-judgments for each standard, a total of 45 judgments per session and 25 judgments per standard over the five sessions. The nine standards were presented in random order within each block of nine trials with the restriction that the last standard of a sequence could not be the first standard of the next sequence. Order of presentation varied over days but was constant over Ss. For each standard the starting point for each day's trials was a different random permutation of the set of luminances given approximately by $x/6$, $2x/6$, $3x/6$, $4x/6$, and $5x/6$, where x is the luminance of the standard. For standards of 23 ft-L and less these luminances were, of course, corrected for the filter factor. Prior to the first session, the following set of instructions was read to each S:

"You will be presented with two lights on each trial. The light on the right will be called the standard. Your task will be to adjust the brightness of the light on the left so that it appears half as bright as the standard. The standard will always change from one trial to the next.

"There are certain things which you should be cautioned against doing. Do not make a judgment on the basis of the size of the lights—they tend to appear larger or smaller to some people as the intensity changes—nor on the distance of the lights—they tend to appear further away or closer to some people as the intensity changes—nor on the basis of time. What I mean by that is, it will take the light a certain amount of time to change from its initial brightness to the brightness which you perceive as half that of the standard. Since we will start the light at a different place for each judgment, you will just increase your error if you take into consideration how long it is taking. If by chance the light is started on a point which you think is just right, you *must* move the light away from that point, even if you move it right back to the same place. You will not be limited in the amount of time you take to make an adjustment or the number of times you increase or decrease the brightness of the light. Are there any questions? Now let's run through the procedure a few times."

Each session of fractionation was preceded by a brief dark adaptation period of 15 min. Ss were seated at a distance of 12 ft from the targets in an ordinary classroom desk seat. Adjustments in the intensity of the variable light were made by means of a control switch, and at the conclusion of each observation, S pressed a button which closed a shutter over the stimulus lights and signaled to E that the observation was completed. After recording, E reset the luminance at a starting point previously determined by the randomizing procedure, and opened the shutter which signaled S to make the next observation.

Table 1
Tests of Individual Half-Judgment Functions

ϕ -Law					ψ -Law				
S	t	k	MSD	MSW	F ^a	t	k	MSD	F ^a
SP	.0286	.376	6.645	.1746	38.06***	.0108	.320	5.279	30.23***
BE	.0101	.305	0.739	.1919	3.85**	.0004	.287	0.876	4.57***
LH	.0270	.254	2.491	.2905	8.57***	.0035	.197	0.369	1.27
DT	.0150	.371	0.482	.0817	5.90***	.0016	.352	0.543	6.65***
DW	.0069	.293	0.296	.3765	0.79	.0001	.281	0.539	1.43
KS	.0097	.359	0.585	.2866	2.04	.0003	.353	0.981	3.42**
SC	.0224	.167	7.241	.3532	20.50***	.0014	.084	0.854	2.42*
RW	.0201	.369	0.157	.1486	1.06	.0022	.352	0.658	4.43***
AC	.0413	.454	0.346	.0602	5.74***	.0166	.421	0.305	5.06***
Mean	.0201	.328				.0041	.294		

^a Based on 6 and 216 df

* $p < .05$

** $p < .01$

*** $p < .0005$

Fig. 1. Mean log of half-judgments plotted as a function of log values predicted from the theoretical half-judgment functions based on the ϕ -Law.

RESULTS

The data for each S consisted of 25 half-judgments of each of nine standards. Equations 2 and 3 (with $r = 1/2$) were fitted to the data. $\log \phi_{1/2}$ was treated as the dependent variable, and a least squares iterative solution provided joint estimates of k and t , separately for each law and each S. An F test was then applied to test for the significance of the deviations from the theoretically best-fitting half-judgment functions. The results are presented in Table 1. The F tests do not appear to give strong support to the half-judgment functions regardless of which form of the law one assumes. For each law, only three of the nine Fs fail to reach significance at the .01 level, and for the ψ -Law, one of these three is significant at the .05 level. However, in the final discussion below, we question the appropriateness of evaluating models in isolation by such goodness-of-fit tests; in any event, the plots of half-judgment functions to follow give substantial support to the fractionation method.

Plots of the half-judgment functions were constructed for individual Ss and the nine individual plots were combined into a single plot for each law. Figure 1 shows a plot of the mean log of the half judgments ($\phi_{1/2}$) plotted as a function of log values predicted from the theoretical half-judgment functions (one for each S) constructed assuming a ϕ -Law (Eq. 2). This over-all plot contains 81 points—nine Ss times nine standards. Figure 2 shows the comparable plot based on the ψ -Law (Eq. 3).

Inspection of Figs 1 and 2 suggests that the deviations from the theoretical ϕ -Law half-judgment functions are somewhat larger than the deviations from the ψ -Law half-judgment functions. The total mean square deviations for Figs. 1 and 2 can be obtained by summing the sums of squares for the individual Ss and dividing by the total degrees of freedom—54. These calculations show that the total deviations mean square for the ϕ -Law is 1.82 times as large as the deviations mean square for the ψ -Law. Unfortunately, these mean squares are not independent and hence their ratio is not distributed as F. We did, however, obtain a statistical comparison of the fit of the two laws by the following procedure.

Let $d_i(\phi)$ denote the absolute value of the difference between the $\phi_{1/2}$ predicted from the ϕ -Law half-judgment function for data point i ($i = 1, \dots, 81$) and the corresponding observed (geometric mean) value; and $d_i(\psi)$ the corresponding difference based on the ψ -Law. According to the central limit theorem, the differences $D_i = d_i(\psi) - d_i(\phi)$ should be approximately normal. Then the mean difference is $\bar{D} = .034$, the standard error is equal to .017, and this mean

Table 2
 k for Simple Power Law Based on Fractionation of Single Standard

S	Standard (Ft-L)									
	0.1	0.3	0.9	2.5	7.5	23.0	68.0	200	600	Mean
SP	.782	.429	.422	.493	.542	.446	.427	.309	.252	.456
BE	.425	.349	.297	.314	.330	.351	.296	.287	.288	.326
LH	.512	.373	.341	.291	.287	.277	.238	.238	.218	.308
DT	.550	.421	.390	.371	.406	.415	.361	.331	.367	.401
DW	.383	.304	.311	.287	.299	.317	.279	.291	.296	.307
KS	.508	.354	.343	.340	.392	.386	.369	.369	.353	.379
SC	.374	.286	.235	.209	.200	.193	.167	.156	.138	.217
RW	.626	.449	.371	.359	.377	.374	.382	.390	.363	.410
AC	1.080	.625	.507	.532	.460	.450	.489	.438	.402	.554
Mean	.582	.399	.357	.355	.366	.357	.334	.312	.297	.373

difference is significant ($p < .05$, two-tail test). Hence, on the basis of this test, we are able to choose the ψ -Law as providing a better fit than the ϕ -Law to the half-judgment data.

Close inspection of Fig. 2 seems to indicate possible systematic deviations from the ψ -Law half-judgment functions, namely a tendency toward runs of points alternating above and below the theoretical line. This indicates primarily a tendency for predicted values to be too high at the low end of the range (but not at the extreme low end) and too low at the high end of the range, although the magnitude of the deviations is not large. The apparently systematic deviations from the ϕ -Law half-judgment functions are perhaps less marked and consist primarily of a tendency for predicted values to be somewhat too low over a wide middle range. These trends in the latter case are due primarily to the three worst Ss (SP, LH, and SC) for the ϕ -Law, whereas the trends seem to be present to some degree in most of the individual plots based on the ψ -Law.

Effect of Standard on Exponent Estimates

On the basis of the fractionation of a single standard, one parameter of the psychophysical function can be estimated (Fagot, 1966). One condition which should be imposed on the half-judgment function is that estimates of the parameters be independent of the standard. This invariance test was applied to the half-judgment functions for each of the assumed power laws. Estimates of the parameters were based on Eqs. 1, 2, and 5 in Stewart, Fagot, and Eskildsen (1967).

For the simple power law, estimates of the exponents based on fractionations of each standard are presented in Table 2 for each S.

Fig. 2. Mean log of half-judgments plotted as a function of log values predicted from the theoretical half-judgment functions based on the ψ -Law.

Table 3
 $\hat{k}$ for ϕ -Law Based on Fractionation of Single Standard given $\hat{t}$ Based on Total Range

S	$\hat{t}$	Standard (Ft-L)										Mean
		0.1	0.3	0.9	2.5	7.5	23.0	68.0	200	600		
SP	.028556	.401	.320	.388	.481	.538	.444	.426	.309	.252		.395
BE	.010115	.308	.310	.284	.309	.329	.350	.296	.287	.288		.307
LH	.027023	.117	.264	.306	.278	.283	.275	.237	.238	.218		.246
DT	.014997	.375	.365	.371	.365	.404	.415	.361	.331	.367		.373
DW	.006891	.304	.276	.302	.283	.298	.316	.279	.291	.296		.294
KS	.009685	.399	.318	.331	.336	.390	.386	.369	.369	.353		.361
SC	.022407	.047	.165	.192	.190	.192	.188	.166	.159	.146		.160
RW	.020116	.380	.374	.346	.350	.374	.373	.381	.390	.363		.370
AC	.041268	.422	.462	.457	.514	.454	.448	.489	.438	.402		.454
Mean		.306	.317	.331	.345	.362	.355	.334	.312	.298		.329

The ϕ -Law and the ψ -Law contain two parameters and not both can be estimated from a single standard. The procedure followed separately for each S and law was to use estimates of t based on data from the total range, given in Table 1, and estimate k for each standard using this single estimate of t . The results are given in Table 3 for the ϕ -Law and in Table 4 for the ψ -Law.

The data were analyzed by means of a two-way (Standards by Ss) factorial design (one observation per cell) with standards treated as a fixed effect and Ss a random effect, and exponent estimates as the dependent variable. A separate analysis was performed for each of the assumed power laws. The analysis of variance summary is presented in Table 5. All effects were significant ($p < .01$) for both laws. Figure 3 shows mean exponents plotted as a function of log standard for each law, and these plots show a less marked effect of standard for the ϕ -Law. The greater variability of the estimates for the simple power law and the ψ -Law appears to be due primarily to an appreciably higher estimate for the lowest stimulus (0.1 ft-L). For each law, the plots for individual Ss appear to be very similar, and the maximum at the low end of the range for both the simple and ψ -Law (see Tables 4 and 6) is observed for all nine Ss. This similarity in plots results, of course, in small interaction mean squares and is largely responsible for the failure of the invariance tests.

Effect of Standard on Threshold Estimates

In this case estimates of the exponents based on data from the total range (Table 1) were used to estimate the threshold t relative to each standard, separately for each S and each law. The results are given in Table 6 for the ϕ -Law and Table 7 for the ψ -Law.

Note in Table 6 that a large number of the threshold estimates based on the ϕ -Law are *negative*, and in fact for six of the nine Ss the mean t for the nine standards is negative, and the mean of the 81 estimates in the table is appreciably negative. In the case of the ψ -Law (Table 7) there are only a scattering of estimates below zero and only one S (SP) had a mean below zero. The mean of all Ss is also negative, but this is due entirely to this one S, and the mean t for the remaining eight Ss is .008 ft-L.

Table 5
 Analysis of Variance Test of Exponent Invariance

Source	df	- Simple Law		ϕ -Law		ψ -Law	
		MS	F	MS	F	MS	F
Std	8	.06325	15.37***	.00445	2.92**	.01973	6.86***
S	8	.08619	20.94***	.06936	45.45***	.09739	33.87***
Std x S	64	.00411		.00153		.00287	

** $p < .01$

*** $p < .0005$

Table 4
 $\hat{k}$ for ψ -Law Based on Fractionation of Single Standard given $\hat{t}$ Based on Total Range

S	$\hat{t}$	0.1	0.3	0.9	2.5	7.5	23.0	68.0	200	600	Mean
SP	.010775	.378	.035	.245	.427	.516	.421	.410	.282	.220	.326
BE	.000384	.338	.273	.231	.276	.307	.339	.282	.276	.281	.290
LH	.003543	.232	.149	.202	.177	.213	.225	.187	.202	.186	.197
DT	.001631	.421	.311	.319	.324	.384	.403	.349	.320	.362	.355
DW	.000097	.330	.248	.276	.258	.281	.306	.269	.284	.292	.277
KS	.000274	.462	.292	.302	.312	.380	.379	.363	.365	.350	.356
SC	.001419	.122	.070	.054	.060	.093	.111	.088	.089	.072	.084
RW	.002201	.501	.334	.279	.299	.345	.353	.370	.383	.358	.358
AC	.016590	.673	.335	.340	.459	.407	.419	.476	.426	.393	.436
Mean		.384	.227	.250	.288	.325	.328	.310	.292	.279	.298

The analysis of variance test of threshold invariance is summarized in Table 8, using the same model as in the case of the test of exponent invariance. The effect of standard was significant for the ϕ -Law ($p < .01$) but not for the ψ -Law, whereas the effect of S was not significant for either law.

DISCUSSION

We have previously (Fagot & Stewart, 1968) expressed concern with the use of tests of goodness-of-fit for evaluating models (such as the power law hypothesis) in isolation, because of general power considerations. In particular, when the S is viewed as an instrument for testing the validity of the power law, large variability of response (a highly undesirable feature of a measurement procedure) favors acceptance of the law. This being the case, we might ask whether acceptance of the law (nonsignificant F) is being purchased at the expense of high error variance. In an earlier study (Fagot & Stewart, 1968) this was not the case: The "good" Ss (low Fs) tended also to have smaller error variance. In the present study measures of S consistency— $\rho = 1 - (MS_{\text{within}})/(MS_{\text{total}})$ —ranged from .91 to .99, with only one S below .95, indicating rather high consistency of response, and showing no tendency for low ρ s (high error variance) to be associated with low Fs.

In this study it was not possible to test the internal consistency of the fractionation judgments independent of a test of a particular form of the power law, and hence a rejection of the half-judgment functions would have left open the question of which assumption should be abandoned or modified. However, Figs. 1 and 2 appear to give fair support for the fractionation method, regardless of which is the appropriate form of the power law.

An important consideration in evaluating psychological models is whether the parameters can be given a psychological interpretation. In the case of the power law, the parameter t has usually been referred to as a "threshold parameter" and sometimes more specifically as the "absolute" or "effective" threshold. A critical question to ask is how the parameter t inferred from the psychophysical function compares with an independently *measured* threshold. Although we do not have measured thresholds on these Ss, data is available on 11 other Ss for thresholds measured by the method of minimal change on the same apparatus. The mean absolute threshold ($\bar{t}_0$) for 11 Ss (Stewart, Fagot, & Eskildsen, 1967; Fagot & Stewart, 1968) was $\bar{t}_0 = .0005$ ft-L; the mean "effective" threshold in the presence of a masking stimulus of 10 ft-L was $\bar{t}_{10} = .0008$ ft-L (for six Ss from Fagot & Stewart, 1968); and the mean effective threshold in the presence of a masking stimulus of 500 ft-L was $\bar{t}_{500} = .0041$ ft-L (for five Ss from Stewart, Fagot, & Eskildsen, 1967). Comparison of these values with the threshold values in Table 1 inferred from the psychophysical functions indicates much greater support for the ψ -Law: If we consider the ratio of inferred threshold to

Table 6
 $\hat{t}$ for ϕ -Law Based on Fractionation of Single Standard given $\hat{k}$ Based on Total Range

S	$\hat{k}$	Standard (Ft-L)									
		0.1	0.3	0.9	2.5	7.5	23.0	68.0	200	600	Mean
SP	.376	.030	.015	.038	.258	1.068	1.442	3.113	-12.451	-67.597	-8.232
BE	.305	.010	.011	-.006	.019	.156	.902	-.539	-3.086	-8.617	-1.239
LH	.254	.021	.029	.063	.073	.193	.403	-.828	-2.378	-15.435	-1.984
DT	.371	.015	.013	.015	.000	.239	.919	-.627	-7.361	-2.188	-.997
DW	.293	.008	.003	.013	-.013	.035	.454	-.777	-.445	1.388	.074
KS	.359	.013	-.001	-.013	-.043	.223	.574	.607	1.858	-2.996	.025
SC	.167	.014	.022	.034	.051	.119	.267	-.013	-.812	-5.659	-.664
RW	.369	.021	.021	.001	-.024	.052	.091	.774	3.852	-3.182	.178
AC	.454	.039	.043	.044	.175	.044	-.087	2.199	-3.028	-29.688	-3.362
Mean		.019	.017	.021	.055	.236	.551	.434	-2.650	-14.886	-1.800

Table 7
 $\hat{t}$ for ψ -Law Based on Fractionation of Single Standard given $\hat{k}$ Based on Total Range

S	$\hat{k}$	Standard (Ft-L)									
		0.1	0.3	0.9	2.5	7.5	23.0	68.0	200	600	Mean
SP	.320	.01191	.00176	.00446	.04440	.23043	.19096	.37290	-.00193	-2.45174	-.1774
BE	.287	.00083	.00025	.00000	.00015	.00193	.02076	.00010	.00000	.00000	.0027
LH	.197	.00405	.00241	.00380	.00231	.00578	.01104	.00188	.00577	.00077	.0042
DT	.352	.00279	.00072	.00045	.00021	.00990	.04487	.00073	-.02510	.02622	.0068
DW	.281	.00034	.00000	.00006	.00000	.00009	.00290	.00000	.00026	.00390	.0008
KS	.353	.00167	.00000	-.00001	-.00008	.00417	.00867	.00335	.01054	.00000	.0032
SC	.084	.00174	.00125	.00094	.00088	.00178	.00353	.00173	.00192	.00058	.0016
RW	.352	.00523	.00167	.00007	.00001	.00120	.00251	.01778	.10428	.01101	.0160
AC	.421	.02177	.01153	.00644	.03017	.00953	.01456	.29424	.03586	-.17682	.0275
Mean		.00559	.00218	.00180	.00867	.02942	.03331	.07697	.01462	-.28734	-.0127

measured threshold, we find that this ratio is closer to unity in the case of the ψ -Law for all nine Ss relative to $\bar{t}_0$ and $\bar{t}_{10}$, and for six of the nine Ss relative to $\bar{t}_{500}$. The ratio of mean inferred threshold to mean measured threshold is also closer to unity for the ψ -Law for all three measured thresholds. Note also that the mean inferred threshold for the ψ -Law (.0041) is equal to $\bar{t}_{500}$. In general, the ϕ -Law estimates tend to be much too high for measured thresholds; in fact, for two Ss (LH and SC), the mean half-judgment for the lowest standard was actually lower than the inferred threshold estimate.

Table 7 makes it even more difficult to interpret t as a threshold parameter in the ϕ -Law: The mean estimates of t were negative for six of the nine Ss, and the remaining three Ss produced estimates which were much too high for measured thresholds. The fact that the ϕ -Law estimates of t based on the total range (Table 1) were much too high, together with the large number of negative estimates relative to individual standards, indicates that estimates of this parameter are very unreliable. However, there is no evidence to suggest that these are simply large random fluctuations about a reasonable threshold value. Finally, the effect of standard on threshold estimates was significant in the case of the ϕ -Law.

Table 6, on the other hand, presents a favorable case for the interpretation of t as a threshold parameter for the ψ -Law: Six of the nine Ss produce estimates which are of the order of magnitude of measured thresholds, and the mean for these six

Ss is .003 ft-L compared to a mean of .002 ft-L for the measured threshold of the 11 Ss (in previous studies) averaged over the three masking stimuli. In addition, the nonsignificance of the standard and S effect for the ψ -Law (Table 8) gives further strong support. Finally, Fagot and Stewart (1968) report that ψ -Law thresholds were closer to measured thresholds than the comparable measures for the ϕ -Law [for six Ss making magnitude estimations (with standard) of brightness]; and Fagot and Stewart have reported (unpublished) that estimates of t for the ϕ -Law were negative for all six Ss making magnitude estimations of differences in brightness.

Although the over-all "mean deviation difference" test picked the ψ -Law as providing a better fit to the half-judgment data than the ϕ -Law, and the interpretation of t as a threshold parameter favors the ψ -Law, there are two aspects of the data that tend to favor the ϕ -Law and raise questions about the

Fig. 3. Mean exponent plotted as a function of log standard for each law.

Table 8
Analysis of Variance Test of Threshold Invariance

Source	df	ϕ -Law		ψ -Law	
		MS	F	MS	F
Std	8	225.000	4.09**	.10049	1.22
S	8	63.949	1.16	.03500	.42
Std x S	64	55.046		.08252	

** $p < .01$

ψ -Law. First, in spite of the fact that the over-all deviations mean square is much smaller for the ψ -Law, inspection of Figs. 1 and 2 suggests more apparent systematic trends in the half-judgment functions based on the ψ -Law, and for this we have no explanation. Second, although the effect of standard on exponent estimates was significant for both laws, Fig. 3 suggests a more serious problem for the ψ -Law. However, to further complicate the picture, in a test of exponent invariance using a measured threshold, Stewart, Fagot, and Eskildsen (1967) found that the ψ -Law fared somewhat better. Nevertheless, Fig. 3 is disturbing for the ψ -Law. If we can ignore the slight hump around the middle of the standards, the very sharp drop in the ψ -Law exponent curve from 0.1 to 0.3 ft-L (which is hard to dismiss since it was observed for all nine Ss) suggests the possibility that two different psychophysical functions may be required. In view of the fact that the transition region from scotopic to photopic vision is about 0.01 to 0.1 ft-L, it is tempting to conjecture that different psychophysical functions hold for scotopic and photopic vision. Against this view is Ekman's (1963) report that work in the Stockholm laboratory concerning the brightness of monochromatic light produced estimates of exponents close to 0.35 in both scotopic and photopic vision. On the other hand, a study by Ekman and Gustafsson (1967) is consistent with the conjecture: Scaling brightness entirely within the scotopic region, they reported that the psychophysical function was approximately linear. Further investigation of this hypothesis would appear to be warranted.

In summary, we interpret these data as providing fair support for the fractionation method for brightness, and somewhat stronger support for the ψ -Law compared to the ϕ -Law. The interpretation of the parameter t in the ϕ -Law as a brightness threshold was rejected, and possible systematic trends in the half-judgment functions, especially for the ψ -Law, were noted. Finally, the exponents of both laws exhibited dependence on the standard fractionated, but this appears to be a more serious problem for the ψ -Law.

REFERENCES

- CURTIS, D. W., ATTNEAVE, F., & HARRINGTON, T. L. A test of a two-stage model of magnitude judgment. *Perception & Psychophysics*, 1968, 3, 25-31.
- EKMAN, G. Two generalized ratio scaling methods. *Journal of Psychology*, 1958, 45, 287-295.
- EKMAN, G. A simple method for fitting psychophysical power functions. *Journal of Psychology*, 1961, 51, 343-350.
- EKMAN, G. Contributions to the psychophysics of color vision. *Studium Generale*, 1963, 16, 54-64.
- EKMAN, G., & ÅKESSON, C. Saltiness, sweetness and preference: A study of quantitative relations in individual subjects. *Scandinavian Journal of Psychology*, 1965, 6, 241-253.
- EKMAN, G., & GUSTAFSSON, U. The absolute threshold and the psychophysical function in brightness vision. Report of the Psychology Laboratory, University of Stockholm, 1967, No. 233.
- ESKILDSSEN, P. An apparatus for studies of brightness which controls variation of intensity with time. *American Journal of Psychology*, 1963, 76, 321-323.
- FAGOT, R. F. Alternative power laws for ratio scaling. *Psychometrika*, 1966, 31, 201-214.
- FAGOT, R. F., ESKILDSSEN, P. R., & STEWART, M. R. Effect of rate of change in physical intensity on bisection and fractional judgments of brightness. *Journal of Experimental Psychology*, 1966, 72, 880-886.
- FAGOT, R. F., & STEWART, M. R. An experimental comparison of stimulus and response translated power functions for brightness. *Perception & Psychophysics*, 1968, 3, 297-305.
- GARNER, W. R. Context effects and the validity of loudness scales. *Journal of Experimental Psychology*, 1954, 48, 218-224.
- GEIGER, P. H., & FIRESTONE, F. A. The estimation of fractional loudness. *Journal of the Acoustical Society of America*, 1933, 5, 25-30.
- HANES, R. M. A scale of subjective brightness. *Journal of Experimental Psychology*, 1949a, 39, 438-452.
- HANES, R. M. The construction of subjective brightness scales from fractionation data: A validation. *Journal of Experimental Psychology*, 1949b, 39, 719-728.
- IRWIN, R. J., & CORBALLIS, M. C. On the general form of Steven's Law for loudness and softness. *Perception & Psychophysics*, 1968, 3, 137-144.
- JAMESON, D. Threshold and supra-threshold relations in vision. *Proceedings of the International Colour Meeting Lucerne*, 1965, 1, 128-136.
- MASHHOUR, M. On the validity of scales derived by ratio and magnitude estimation methods. Report of the Psychology Laboratory, University of Stockholm, 1961, No. 105.
- McGILL, W. A. The slope of the loudness function: A puzzle. In H. Gulliksen and S. Messick (Eds.), *Psychological scaling: Theory and applications*. New York: Wiley, 1960.
- POULTON, E. C. The new psychophysics: Six models for magnitude estimation. *Psychological Bulletin*, 1968, 69, 1-19.
- SJÖBERG, L. On ratio estimation. Report of the Psychology Laboratory, University of Stockholm, 1965, No. 191.
- STEVENS, S. S. On the psychophysical law. *Psychological Review*, 1957, 64, 153-181.
- STEWART, M. R., FAGOT, R. F., & ESKILDSSEN, P. R. Invariance tests for bisection and fractionation scaling. *Perception & Psychophysics*, 1967, 2, 323-327.
- SUPPES, P., & ZINNES, J. L. Basic measurement theory. In R. D. Luce, R. R. Bush, and E. Galanter (Eds.), *Handbook of mathematical psychology*. Vol. 1. New York: Wiley, 1963, Chap. 1.
- SVENSON, O., & ÅKESSON, C. A. Fractional and multiple estimates in ratio scaling. Report of the Psychology Laboratory, University of Stockholm, 1966, No. 202.
- SVENSON, O., & ÅKESSON, C. A. A further note on fractional and multiple estimates in ratio scaling. Report of the Psychology Laboratory, University of Stockholm, 1967, No. 224.
- TREISMAN, M. The nature of the psychophysical law and its significance for the scaling of sensory magnitudes. Presented at 16th International Congress of Psychology, Bonn, 1960.

NOTES

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