Modeling a tactile sensory register¹

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A detailed analysis was made of the data of a previous paper by Bliss et al (1966). That paper describes a series of experiments, in which 2 to 12 simultaneous air-jet stimuli were presented on the 24 phalanges of both hands. In addition, new data from an early blind S, whose performance was significantly better, is compared to the data of the original four Ss. Memory properties of all five Ss estimated by whole- and partial-reporting techniques gave evidence for a tactile short-term memory that (1) has a duration of a few seconds, (2) varies considerably in size (but not in temporal characteristics) for different Ss, and (3) has a capacity limited by spatial resolution. Tactile masking did not account for the limited spatial resolution, but a stimulus-spreading (or lateral-excitation) model did.

Numerous results from mostly visual experiments have led several investigators to suggest models for human memory consisting of a system of several interacting parts. For example, according to the model of Atkinson and Shiffrin (1967), the memory system is divided into three components: a sensory register, a short-term store, and a long-term store. Sperling's (1963) and Massa's (1964) models are similar, but they call the sensory register "visual information storage" or a "short-term memory." In terms of these models, the best known evidence for a sensory register is the short-term visual afterimage investigated by Sperling (1960), Averbach and Coriell (1961), Estes and Taylor (1964, 1966), and others. When it comes to other sense modalities, Atkinson and Shiffrin (1967) point out, "There is not much one can say about registers in sensory modalities other than visual. A fair amount of work has been carried out on the auditory system without isolating a registration mechanism comparable to the visual one. On the other hand, the widely differing structures of the different sensory systems makes it questionable whether we should expect similar systems for registration."

Figure 1 shows the specific model used as a framework for the analyses described in this paper. According to this model, when a tactile pattern is presented to the system, a filtered image of the pattern is transferred to the sensory register, where it begins to decay. As the image in the sensory register is decaying, a limited amount of this information is processed and transferred to the short-term store or immediate memory. Either the transferal process or the size of the short-term store limits the amount of information retained in the short-term store. This information can be stored without rehearsal for tens of seconds, and with rehearsal for indefinite periods. All responding by the S is based on the information in his short-term store and thus does not directly reflect sensory register limitations. It is the purpose of this paper to present evidence for a tactile sensory register and to estimate quantitative parameters for its capacity, and its temporal and spatial properties. This evidence evolves from an extensive analysis of data reported in a previous paper (Bliss et al, 1966).

Early in our experiments it became clear that widely different levels of performance could be obtained from different Ss, perhaps because of differences in training or development of the tactile modality. In order to attempt to obtain some idea of the range of performance possible, and yet run only a small number of Ss, we selected our Ss in a special way. Preliminary results from a standard echo-detection test conducted by Dr. Charles E. Rice indicated that the performance of a group of five early blind Ss (blindness occurring before the age of two) was superior to the performance of a group of six late blind Ss. We selected a S (M_4) from the late blind and a S (JK) from the early blind groups used by Rice for our tactile experiment. Obviously, with only two Ss, conclusions regarding the effects of blindness could not be obtained. However, since the memory of blind persons is almost legendary, we felt that an experiment that measured memory properties would be particularly sensitive to any differences among these Ss.

Fig. 1. Framework for the memory-system model.

SUMMARY OF PREVIOUS EXPERIMENTS

In Bliss et al (1966), we described two experiments involving brief (100-msec) tactile point stimuli applied to a number n of the 24 interjoint regions of the fingers of both hands (thumbs excluded). The n interjoint positions were simultaneously stimulated and n was (1) varied between 2 and 12, (2) always constant during a session, and (3) known by the S. After each stimulus presentation the S was required to make either a whole report or a partial report. When a whole report was required the S's task was to name all n stimulus locations. In the previously reported Experiment 2 (with which we will be mainly concerned), the interjoint regions of the fingers were labeled as shown in Fig. 2 and the Ss, after training, were required to give their responses in alphabetical order. The maximum number of positions that can be correctly reported, after allowance for guessing behavior, is a measure of the short-term store capacity or the span of immediate memory.

When a partial report was required, a marker stimulus occurred at -0.85, 0, 0.1, 0.3, 0.8, or 2.0 sec following the termination of the tactile stimulus. The marker stimulus, usually one of three lights (or three tones for the blind Ss), indicated which portion of the stimulus positions to name. The upper light indicated that only the fingertips stimulated were to be named, the middle light indicated that only the middle phalanges stimulated were to be named, and the bottom light indicated that only the proximal phalanges stimulated were to be named. The partial report version of the experiment permitted investigation of the properties of any hypothetical tactile afterimage or sensory register (that is, an eidetic or pictorial memory lasting a few seconds).

Included in that paper are data from three normally sighted Ss (A, K, and S) on Experiment 1, and data from three normally sighted Ss (M_1 , M_2 , and M_3) and one late blind S (M_4) on Experiment 2 (which was similar to Experiment 1 but with improved procedures). All of these Ss gave remarkably similar results except for Subject S. This S had considerably more experience in tactile experiments than the others.

Fig. 2. Finger labeling for two hands (dorsal view).

Below, we report on an analysis of the results of the original four Ss plus an additional S who participated in Experiment 2. The fifth S (JK) has been blind since birth and gave results (not previously reported) sufficiently different from the other Ss so as to suggest some modifications and extension of our previous conclusions.

CAPACITY OF THE SHORT-TERM STORE

Since our objective in this paper is to determine the characteristics of sensory mechanisms (in contrast to guessing strategies employed by the Ss) and since we would like to compare these characteristics with analogous characteristics determined from visual experiments, a correction of the data for stimulus-position guessing by the S is necessary. The probability of responding correctly by guessing alone varies from about 0.08 to 0.5 as n is varied from 2 to 12. To determine the percentage of correct responses that resulted from perception of the stimulus alone requires knowledge of the S's strategy for responding when he perceives imperfect information about the stimuli. Fortunately most, if not all, reasonable models for guessing behavior yield practically the same results. Guessing Model II, described in Appendix A, was used to correct all the data in this paper.

An analysis of variance was made on the data to test hypotheses regarding the effect of n and the similarity of the Ss. The hypotheses about the subjects were

H_1 : The mean number of positions perceived by the late blind S (M_4) was the same as the mean number of positions perceived by the sighted Ss (M_1, M_2, M_3).

H_2 : Same as H_1 except that the early blind S (JK) is compared to the sighted Ss.

H_3 : The number of positions perceived by the late blind S (M_4) satisfied the same functional relation as the number of positions perceived by the sighted Ss within an additive constant.

H_4 : Same as H_3 except that the early blind S (JK) is compared to the sighted Ss.

A 3 by 6 (Sighted Subjects by Values of n) analysis of variance of the number of positions perceived was performed, the results of which are shown in Table 1. This analysis revealed that the late blind S did not differ significantly from the sighted Ss in number of positions perceived. However, the data indicated that the early blind S (JK) perceived a larger number of positions than the sighted Ss ($p < 0.025$) and also differed in the functional relation he produced between number of positions perceived and n ($p < 0.001$).

Figure 3 gives the number of positions perceived (estimated by Model II described in Appendix A) as a function of n for these Ss. Since the results of the late blind S were not significantly different from those of the sighted Ss, the results of these four Ss are averaged together. The data for Subject JK is shown separately. Figure 3 also suggests the reason H_4 was rejected; the slope of a line drawn through Subject JK's data is much steeper than the line drawn through the other S's data.

The number of positions perceived in this whole-report experiment depended mainly on two factors: (1) the value of n

Fig. 3. Whole-report data and models.

and (2) the length of time that the S had been on the task. The dependence on time is suggested by the "zig-zag" lines connecting the points in Fig. 3. If these points had been connected in the order in which the corresponding sessions occurred (i.e., $n = 2, 6, 10, 12, 8$, and 4), the lines would have described an open "hysteresis" loop instead of a "zig-zag" line. That is, the number of positions perceived later in the experiment is greater than interpolation between the earlier values would predict.

Two models were hypothesized regarding the functional relationship among P (the number of positions perceived), n (the number of positions stimulated), and d (the number of test days the S had been on the task). A value of d for each value of n was obtained by averaging the number of test days the subject had been on the experiment when the n stimuli were presented. The values of d , in numerical order from $n = 2$ through $n = 12$, were 1, 6, 1, 5, 1.67, and 3.5.

The equations corresponding to these two models and resulting from the linear regression on the data from the three sighted Ss (M_1, M_2, M_3) and one late blind S (M_4) are:

$$P_1 = 1.306 + 0.190n + 0.066d$$

$$P_2 = 1.476 + 0.155n(1 + 0.080d)$$

The corresponding equations for the early blind S (JK) are:

$$P_1 = 1.042 + 0.534n + 0.121d$$

$$P_2 = 1.355 + 0.487n(1 + 0.037d)$$

The linear increase in P as a function of n is a major difference between these tactile results and Sperling's (1960) results for vision. Sperling found that for n greater than 4, Ss correctly reported about 4.5 letters independent of n . To examine this difference in more detail, the data in the whole report curves of Fig. 3 were sorted by phalanx—that is, into fingertip, middle, and proximal interjoint regions with the use of Model III described in Appendix A. Figure 4 shows the separate plots for each of the three phalanx groups for the average of the four Ss (M_1, M_2, M_3, M_4) and for Subject JK. The most striking feature of Fig. 4 is that the number of positions perceived on the fingertips increases linearly with n , while the positions perceived on the other phalanges approach a constant value with increasing n . This difference is most marked in the results of Subject JK, for whom the number of positions perceived on the fingertips corresponds to essentially perfect responses and the number of positions perceived on each of the other two groups of phalanges levels off at 1.6 positions. The slope of the positions perceived on the fingertips of the other four Ss was significantly less, and the

Table 1
Summary of Analysis of Variance of Positions
Perceived in the Whole Report

SOURCE OF VARIATION	df	MEAN SQUARE	F RATIO	SIG.
H_1	1	0.1689	< 1	
H_2	1	25.2038	67.66	$p < 0.025$
Between M_1, M_2, M_3	2	0.3735		
Between n	5			
Linear Component	1	7.8807	193.82	$p < 0.001$
Remainder	4	0.551	1.35	
H_3	5	0.0310	< 1	
H_4	5	1.3427	33.02	$p < 0.001$
$M_1, M_2, M_3, \times n$	10	0.0407

Fig. 4. The number of positions perceived by (a) the four M Ss, and (b) Subject JK on each group of phalanges. The dashed line represents perfect perception by each group.

saturation level of their middle and proximal phalanges was only 0.7 positions. In other words, this sorting separates the interjoint regions into two different groups: one that gives an increasing number of points perceived with increasing n , and another that gives a constant number of points perceived with increasing n . Thus, the upward slope of the curves of Fig. 3 is due to the increasing number of positions perceived on the fingertips alone. Learning effects, as indicated by the zig-zags in the curves of Fig. 4, seem to be mainly on the middle and proximal phalanges.

In our earlier paper it was suggested that the superior performance of Subject S on Experiment 1 was due to an ability to recode the stimulus patterns into larger units, or "chunks" of information, much as in visual experiments in which enhanced performance is obtained by recoding binary numbers into octal numbers. The effect of pattern organization on the perception of visual patterns has been studied by Attneave (1955, 1959), Massa (1961), and others. Attneave found that Ss perceived random and organized patterns equally well when responses were given immediately after the pattern presentation, while reports made $\frac{1}{2}$ to 1 sec after pattern presentation show significantly higher reporting accuracy from organized than from random patterns. Massa found that the organized patterns were always reported more accurately than random patterns.

We conducted three different kinds of tests to determine if pattern organization was a significant factor in the performance of the five Ss of Experiment 2. All three tests considered the set of 188 12-position patterns presented to each S in the whole-report experiment. While the tests are described in detail by Hill (1967), the major conclusions are: (1) averaging across Ss, there was a slight increase ($p < 0.01$) of about 0.7 more positions perceived on patterns judged to be the most organized, than on those judged to be the least organized, (2) no S, including JK, utilized pattern

organization any more than the others, (3) Subject JK's concept of an organized random pattern was similar to that of sighted Ss, and (4) there were no groups of patterns more or less accurately reported by any pair of Ss. Thus we have been unable to detect any strong influence of pattern organization in these experiments.

INFORMATION TRANSMITTED BY THE SHORT-TERM STORE

While it is difficult to estimate directly the information that the whole-report responses give about the stimuli, five lower bounds on this information are described by Hill (1967). These bounds are shown in Figs. 5(a) and 5(b). An important feature of Figs. 5(a) and 5(b) is that with one or two positions per pattern, almost all the information of the stimuli patterns, $H(S)$, is transmitted, while for n greater than four, less and less information is transmitted by the responses.³

The basic difference between the information transmitted by Subject JK and the other four Ss can be described by noting that for all values of n where the information transmitted is not limited by the source, $H(S)$, Subject JK transmits 6-7 bits more information than the other Ss.

None of the bounds computed, except the finger-pattern bound, consider the additional information the errors give about the stimulus. In order to more completely specify the information transmitted, an upper bound as well as a lower bound should be obtained. The information content of the source, $H(S)$, serves as one upper bound, but not a very restrictive one. If Ss were made to guess additional locations of the patterns until they had correctly reported the patterns, an upper bound similar to that described by Shannon (1951) could be obtained.

CAPACITY AND TEMPORAL PROPERTIES OF THE SENSORY REGISTER

An analysis of variance was made on the partial-report results, to test hypotheses similar to those tested in the analysis of whole-report results. The analysis indicated that the late blind S (M_4) again did not differ significantly from the sighted Ss (M_1 , M_2 , and M_3) in the number of positions available. On the other hand, the early blind S (JK) had a significantly higher ($p < 0.025$) number of positions available than the sighted Ss. The difference between Subject JK and the three sighted Ss can be adequately described by an additive constant. The data showed that Subject JK had 2.29 and 5.88 more positions available with $n = 6$ and $n = 12$, respectively, than did Subjects M_1 , M_2 , and M_3 .

Another hypothesis tested was that the number of positions perceived on each row of phalanges was the same. The analysis indicated that there were significant differences between the three rows, the distal phalanges being the most accurate and the middle phalanges the least.⁴

Still another hypothesis tested was that the number of positions available to M_1 , M_2 , and M_3 was the same for each value of

Fig. 5. Information bounds from the whole-report data of (a) Subjects M_1 , M_2 , M_3 , M_4 , and (b) Subject JK.

Fig. 6. The number of positions available to (a) the four M subjects, and (b) Subject JK, as a function of marker delay. The crosshatched bar represents the timing of the stimulus and the solid bars represent the whole report from the partial-report patterns.

marker delay. The analysis indicated that there were indeed different numbers of positions available with different marker delays for both $n = 6$ and $n = 12$. The data from M_1 , M_2 , M_3 , and M_4 were averaged together and are shown in Fig. 6(a). The corresponding data for Subject JK is shown in Fig. 6(b). The decreasing number of positions available with marker delay shown in Fig. 6 is evidence for a tactile sensory register, and the time course of these curves describes the temporal characteristics of this sensory register.

Since the only variables affecting the number of positions available in this partial-report experiment are marker delay and subject, a simple linear-effects model will adequately describe the data. Such a model is

$$P = P(S) + P(m) + \epsilon \quad (1)$$

where P is the sum of (1) a constant, $P(S)$, dependent on the S , (2) an effect dependent on the marker, $P(m)$, and (3) a random variable, ϵ . The random variable is normally distributed with zero mean and standard deviation of 0.3 positions for both $n = 6$ and $n = 12$.

The second term of Equation (1) can be modeled with the following equation:

$$P(m) = A + B \exp(-m/T), \quad m \geq 0 \quad (2)$$

where m is the time in seconds from the termination of the stimulus to the beginning of the marker, A is the number of positions stored in the immediate memory, B is the difference between the capacity of the sensory register and the capacity of the short-term store, and T is the time constant of the sensory register in seconds, arbitrarily assuming that the time course of information loss in the sensory register can be described by an exponential. Table 2 shows model coefficients obtained by fitting the data using the least squares criteria.

The coefficient values shown in Table 2 indicate that the span of immediate memory of Subject JK is greater than seven positions, so that with $n = 6$ this S perceives all locations of the patterns correctly and the sensory register does not enter into the results. With $n = 6$ the value of B for Subjects M_1 , M_2 , M_3 , and M_4 was small (0.5 position), but still significantly larger than zero. With this small value for B it was not possible to determine a value for T .

Table 2
Partial Report Model Coefficients for Equation (2)

SUBJECTS	n	A	B	T
JK	6	5.6		
Average of M_1, M_2, M_3, M_4	6	2.7	0.5	
JK	12	7.5	3.9	1.5
Average of M_1, M_2, M_3, M_4	12	3.3	1.6	1.3

For $n = 12$, the sensory register of all five S s showed two features in common. One was that the capacity of the sensory register was about 50% larger than that of the immediate memory (52% for Subject JK and 48% for the other four S s). The other common feature was that the time constant of the sensory register T was about 1.3 sec. This compares to a value of about 3 sec, roughly, estimated from Sperling's data with dark pre- and postfields (1960). The feature not shared by the S s was memory capacity; Subject JK showed a memory capacity more than twice as great as that of the other four S s.

LOCALIZATION ACCURACY IN THE SENSORY REGISTER

The detailed stimulus-response data from this experiment also contains information about the S 's ability to detect and locate particular stimulus positions within the patterns of stimuli. This information further characterizes the tactile sensory register and sheds light on properties of the underlying neurological mechanisms. This subsection describes a spread correlation analysis for determining the set of locations where a typical stimulus position will be reported.

When stimulus patterns consisting of only one position are presented to a S and only one response is given, the S 's ability to localize each position can be estimated in a straightforward manner. For example, the percentage of positions correctly identified, or identified to the right, to the left, upward, downward, etc., from the stimuli can be simply tabulated. Elithorn, Piercy, and Crosskey (1953) made this type of analysis of single tactile stimuli applied to the fingers.

When several positions are stimulated simultaneously and several responses are given, the problem is more difficult. One approach to the problem is to devise a 1-to-1 mapping of the stimulus pattern onto the response pattern. Saslow (1962) used this technique in an analysis of patterns with one, two, or three positions applied to a single fingertip. He matched the stimulus and response patterns by eye to determine the S 's errors in locating the stimulus positions. The difficulty in extending this approach to patterns with a large number of positions is that there are large numbers of mappings. For example, in patterns with n stimuli, there are $n!$ mappings between each particular pair of stimulus and response patterns, and the methods for selecting the most likely mapping from this large set of mappings are somewhat arbitrary. The results of a pattern-matching analysis could depend as much on the method chosen as on the S 's responses.

To overcome these difficulties, a statistical approach to the problem will be presented here. This method assumes that there is a certain probability that a stimulus presented at one location of a set of locations will be reported at each location in the set. For example, consider the middle and index fingertips of the left hand, Positions C and D (cf. Fig. 2). We assume that there is a certain probability, $P(C \rightarrow D)$, possibly zero, that a stimulus presented at Position C will be reported at Position D. The data for the analysis, to continue with the specific example, are a list of occurrences or nonoccurrences of stimuli at Position C with a particular set of patterns, and a similar list for responses at Position D. If there is a relationship between stimuli occurring at Position C and responses occurring at Position D, then we would expect that the probability of a response at Position D, given that a stimulus occurred at Position C, $\Pr(D | C)$, would be different from the probability of a response at Position D, $\Pr(D)$. The relationships among these three probabilities are developed in Appendix B.

Also in Appendix B, a function is developed that permits estimation from the data of the probability, $P(S_i \rightarrow R_j)$, that a stimulus at one particular location causes a position to be reported in another specific location. It is shown that this function bears a strong resemblance to a correlation function, and for this reason the resultant values are termed spread correlations (abbreviated SC in the remainder of this paper). This SC function is shown to have the following properties: (1) If a stimulus (e.g., C) always results in a response (e.g., D), then the SC for this stimulus-response pair has a value of one. (2) If the occurrence of a particular response is

Fig. 7. Confusions made by (a) the four M Ss, and (b) the early blind S in the whole report. The width of the arrows is proportional to the average SC between fingers and phalanges. Dark arrows represent SC significantly greater than zero (5% t test), while open arrows represent nonsignificant SC. Dorsal view.

independent of the occurrence of a particular stimulus, then the SC for this stimulus-response pair has a value of zero. (3) The sum of the SC for a particular stimulus and all 24 response locations is a number between zero and one. This sum represents the fraction of time the stimulus was detected.

A preliminary analysis of the average SC between all possible pairs of positions on the two hands was made using the whole report data of Experiment 2. The two basic results of this preliminary analysis were: (1) locations separated by three or more finger-phalanx displacements were not confused significantly often, and (2) when Ss made location mistakes, they most frequently reported stimuli in the immediate neighborhood of a stimulus.

Because of these results, only the 13 SC values with the shortest displacements between stimulus location and response location were considered in detail. These displacements are as follows, in order of decreasing magnitude of the SC: (1) Zero displacement. SC values computed when stimuli and responses are the same location. (2) One-distal and one-proximal displacement. (3) One-left and one-right displacement. (4) The group of eight SC values that have a total displacement of two. The four orthogonal displacements in this group are two-left, two-right, two-distal, and two-proximal. The four displacements at 45 deg are one-right and one-distal, one-right and one-proximal, one-left and one-distal, and one-left and one-proximal.

The values of SC for each particular displacement can depend on several variables. Besides the subject and the number of stimulus positions n in the whole report, or marker delay, m , in the partial report, there are in general two spatial variables: finger and phalanx. In the SC analysis concerned with the effects of the spatial variables, only the four closest displacements (the most significant displacements) were considered. Since the interactions between the spatial variables were not significant, the SC values across the hands may be adequately represented by the effects of each spatial variable (finger and phalanx) alone. A schematic representation of this analysis of the spatial data of the whole report is given in Fig. 7.

Several features of the confusions that Ss made in the whole-report experiments are made clear by comparing Figs. 7(a) and 7(b). One is the greater number of left-right confusions made by the four M Ss compared with the number made by Subject JK. Another feature of the data is that, except for Subject JK in the whole report, there are more distal than proximal confusions made by the Ss. The proximal confusions of Subject JK can be explained by the reporting procedure used in the whole report, if this S tended to repeat distal locations while reporting the middle and proximal stimulus locations. This conjecture is also consistent with the result that Subject JK made more distal confusions than proximal on the partial-report experiment, where responses are from only one phalanx row at a time. Krohn (1893) and others also have noted that, in general, tactile location errors on the extremities tend to be in the distal direction.

Several specific null hypotheses were tested regarding the directions of the Ss' spatial confusions, and the confusions mentioned in the preceding paragraph were all significant under certain of these hypotheses. In addition, there were significant confusions in the direction from the periphery of the area stimulated on each hand toward the center of this area. These inwardly directed confusions were significant in the left-right directions for all Ss in the whole report; and they were significant in the proximal-distal directions for all Ss in the partial report. Neither of these confusions changed direction in going from whole to partial report. Elithorn, Piercy, and Crosskey (1953) also found significant left-right confusions toward the center of each hand. In addition, the data of Saslow (1962) may be used to show significant inwardly directed proximal-distal confusions, although Saslow did not test his data for this effect. Another result of our analysis showed that more proximal-distal confusions than left-right confusions were made. Saslow (1962) also noted that proximal-distal confusions were significantly larger than left-right confusions for stimuli contained on a single fingertip. In addition, Krohn (1893) noted that radial-ulnar localization was superior to proximal-distal localization.

In the analysis discussed thus far, the SC values of the experimental variables (positions per pattern, n , or marker delay, m) were averaged together in order to obtain the spatial information from the data. When, instead, the SC values of the spatial variables (finger and phalanx) are averaged together, the effects of the experimental variables on the data can be investigated. Typical data obtained from this spatial averaging are given in Fig. 8, which show the average values of SC plotted as a function of displacement for the four M Ss and separately for Subject JK. These data are from the whole report, with $n = 12$. The presentations of Fig. 8 show the average probability of reporting a position correctly, of reporting a position to the left,

Fig. 8. Spread correlations of (a) the four M Ss, and (b) Subject JK, computed from the whole-report data with $n = 12$.

Fig. 9. Spread correlations for the four M Ss and Subject JK computed from the data of (a) the whole report, and (b) the partial report, with $n = 12$.

or of reporting a position with any given displacement. If a S reports all of the positions of the patterns perfectly, then a SC surface as in Fig. 8 would have a central peak of one, and all other displacements zero. However, if a S guesses at the positions, then all of his SCs would be zero. The shape of the SC surface shown in Fig. 8 indicates that the Ss most frequently reported the positions correctly and often reported positions next to the actual position. Seldom did they report positions two spaces away. Also, the central peak of Fig. 8(a) is higher than that of Fig. 8(b), indicating that Subject JK correctly identified more positions of the patterns than did the other Ss. The left- and right-side skirts of Fig. 8(b) are smaller than those of Fig. 8(a). This indicates that Subject JK made fewer left-right confusions than did the other Ss. The SC of all the Ss in both parts of the experiment are given in Fig. 9. The presentation form of Fig. 8 for any part of the experiments can be constructed with these data. Figure 9 contains the 5% two-sided t test for testing the difference between two points and the difference between each point and zero.

Two different effects of the experimental variables can be noted in Fig. 9: (1) the general decrease in the sum of all the SCs with n in the whole report, and (2) the large relative decay of the central SC with respect to the relatively constant surrounding SCs. The first effect (detection) occurs because Ss perceive fewer positions of the patterns with increasing n and m . The second effect (localization) occurs because Ss do not localize the positions of the patterns as accurately with increasing values of n and m .

MASKING VS FACILITATION

Two different ways in which stimuli at neighboring locations might affect the perception of a given stimulus position can be described as masking and facilitation. Masking, or inhibition, is a common concept in psychology, usually meaning that the application of one stimulus reduces the strength of the sensation derived from another. Numerous investigators have reported masking with tactile stimuli. For example, Békésy (1957, 1958, 1960) has shown that a model for tactile perception containing inhibition (the "neural unit") can explain a wide variety of tactile phenomena. He calculated the size and shape of the neural unit and

showed that inhibition extended over distances as great as 3 cm on the palm and lower arm. Uttal (1960), using electrically stimulated tactile sensations separated 1 msec in time has shown the presence of masking between each pair of fingertips on the hand. Ethel Schmid (1961), also using electrical stimuli, found that the maximum masking effect occurred between neighboring fingers with simultaneous or nearly simultaneous stimuli (interstimulus interval < 10 msec). Sherrick (1964), using simultaneous mechanical pulsed stimuli, found that masking was more than 20 dB between the index and little fingers of one hand. Thus, one could reasonably expect masking phenomena to influence the data of our Experiment 2.

Facilitation is also a common concept, but, not so widely reported in conjunction with tactile stimuli. We use facilitation here to mean that the application of one stimulus enhances the sensation derived from another.

In order to test whether masking could explain the low middle-phalanx accuracy of Experiment 2, two methods were used to compute the accuracy at an average middle phalanx position with each of the possible patterns of neighboring stimuli. The first method involved computing the spread correlation, SC, function and the second method involved computing the sensitivity parameter, d' , of the signal-detection model (Luce, 1963).

The data for these analyses were obtained by cataloging the stimuli and responses according to the following conditions: (1) Which one of the 16 neighboring stimulus patterns shown in Fig. 10 was present, (2) whether a stimulus was present or not present on the middle phalanx, and (3) whether a response was reported or not from the middle phalanx. On the extreme left and right fingers, the pattern was determined by assuming that the outside location was the nonstimulus condition. Using this method, eight data points were obtained from each stimulus pattern. These data points were assembled into sixteen 2-by-2 tables of the form shown in Table 3, each table containing the totaled detection data for only one of the 16 "+" patterns. Using the entries in each table, the values of d' and SC were computed from both the whole and partial report data.

To determine whether masking accounts for the relationship between the surrounding "+" patterns and the low middle-phalanx accuracy observed in the partial-report analysis, we tested the null

Fig. 10. The set of 16 "+" patterns for describing the local context of the tactile patterns.

Table 3
Detection Data Table for each Pattern

	STIM.	NO STIM.
Response	SR	$\bar{S}R$
No Response	$\bar{S}\bar{R}$	$\bar{S}R$

Fig. 11. Values of SC and d' plotted vs the number of surrounding positions stimulated in a given "+" pattern. Only the upper left regression line is significant ($p < 0.02$). Flags indicate the "+" pattern for each point. The data is from the partial report with $n = 12$.

hypothesis that accuracy does not depend on the number of neighboring stimuli, using both the SC and signal detectability methods of measuring the accuracy of the middle phalanges. The data are shown in Fig. 11. If inhibition influences reporting accuracy then we would expect the regression lines to have negative slopes; if facilitation influences accuracy then we would expect positive slopes. In three cases there was not enough evidence to show significant non-zero slopes, and in the remaining case there was sufficient evidence to show that a positive slope (or facilitation) explained the results.

A LINEAR SUMMATION MODEL

With the failure of masking to explain the responses in terms of local context in the stimulus patterns, the facilitation concept was investigated further. A linear model for these data was tested in which the presence of a stimulus at each of the five stimulus locations of the "+" patterns is considered to linearly increase the probability of giving a response at an average middle phalanx. In such a model, the reporting probability R is represented as the sum of five terms, plus other effects. That is,

$$R = R_C + R_P + R_D + R_L + R_R + R_{sub j} + \epsilon \quad (3)$$

where the subscripts C, P, D, L, and R refer, respectively, to the effects of the central, proximal, distal, left, and right cells of the "+" pattern. The last two terms are effects due to different Ss and to errors of measurement.

The hypothesis that Equation (3) is a proper model for the data (called the spread or facilitation model) was tested for all Ss with an analysis of variance, using the data of both the whole and the partial report. For the four M Ss, the deviation mean square, as well as the error mean square, could be computed.

For all of the Ss, the validity of the linear spread model of Equation (3) was very significant. The spread model describes the majority of the variance due to the different stimuli present in the "+" patterns. Each of the effects of Equation (3) as well as the standard deviation of the error, σ_ϵ , is given in Table 4. The results of this linear model are nearly identical with the spread-correlation analysis.

Several features of Table 4 are as follows: (1) The effect of the

Table 4
Magnitudes of the Effects of Equation (1)

	REPORT	R_C	R_D	R_P	R_L	R_R	σ_ϵ
FOUR M SUBJECTS	Whole	0.115	0.065	0.075	0.085	0.095	0.085
	Partial	0.179	0.119	0.141	0.077	0.117	0.096
JK	Whole	0.329	0.116	0.129	0.078	0.058	0.061
	Partial	0.677	0.012	0.155	0.031	0.048	0.085

central locations in the "+" patterns is the largest. (2) For the four M Ss in the whole report, all four surrounding effects are about the same. (3) For all Ss in the partial report the proximal stimuli have the second largest effect. (4) The effect of the proximal and distal locations is generally larger than the effect of the left and right locations.

Although the linear model describes the great majority of the variation in the data, it is not a complete model for the data. For the partial-report data of the four M Ss, the deviation mean square was significant, indicating that the model was not complete in this case. A closer investigation of the analysis showed that the main source of the deviation variance was the interaction between two and three stimuli at different locations of the "+" pattern. The sum predicted by the model when two or three stimuli are presented simultaneously in the "+" pattern is less than the sum actually obtained from the data. This interaction represents a nonlinearity of the model, not the effect of masking. The nonlinearity of the model could be investigated by applying different nonlinear transformations to the data and repeating the analysis summarized in Table 4. Stevens (1961) shows that sensation generally increases as a power function of the stimulus. A logarithmic or power-function transformation could linearize the data, allowing a more accurate model to be fit. This refinement was not carried out.

DISCUSSION

A wide range of performance has now been observed on the whole-report task of naming all the interjoint regions of the fingers that were simultaneously stimulated. Figure 12 gives these data for all eight Ss that participated in various versions of the experiment. With the exception of JK, the Ss with more experience performed better. Subject S had the most experience: he was involved continuously on a multiple tactile stimulation task with $n = 2$ for three months previous to the experiment. Subjects M_1 , M_2 , M_3 , M_4 , and JK had only one week of training. Subjects A and K had roughly one month of training. If Subject JK had obtained relevant tactile experience in his everyday life (e.g., by reading Braille and operating his Braille-writer), a learning hypothesis would explain the wide range of performance observed.

Previous multiple-tactile-stimulation data have not generally been presented in a form showing the span of immediate memory or sensory register capacities. A survey of multiple-tactile-perception data is shown in Fig. 13. Krohn's (1893) data were obtained directly from his paper. Alluisi et al (1965) and Saslow (1962) give only the fraction of erroneous patterns (abbreviated here by FEP). An erroneous pattern is a pattern containing one or

Fig. 12. Whole-report data of all eight Ss corrected for guessing with Model II.

Fig. 13. Comparisons of the number of positions perceived, estimated from the data of other experimenters.

more errors. If it is assumed that the frequency of errors at each stimulation site is the same, then it can be shown that the number of positions perceived is

$$\frac{n}{\sqrt{1 - \text{FEP}}}$$

(Alluisi stated that the frequency of errors was constant at different locations.) This equation was used to obtain the corresponding plots in Fig. 13. The data from Geldard and Sherrick (1965) deserve further explanation. Since Ss in this experiment were given two patterns with which to respond "same" or "different," some model of how the Ss distinguished between the patterns had to be assumed to obtain the plot of Fig. 13. Results from two different models are plotted: The first (results plotted with dots in Fig. 13) is that some stimulus location read into the sensory register on the first pattern was not read in the second pattern. With this assumption, the number of positions perceived is

$$n(1 - \% \text{ error with } n - 1 \text{ degree of communality}).$$

The second model (results plotted with triangles in Fig. 13) is that there is a constant probability of noticing a change at any stimulus location. With this assumption, the number of positions perceived is

$$n\sqrt{1 - \% \text{ error with } n - 1 \text{ degrees of communality}}.$$

Fig. 14. (a) Left-right localization function computed from data given by Bliss (1961) compared with the SC of the four M Ss whole-report data with $n = 2$, and (b) proximal-distal localization function computed from data given by Saslow (1962) for $n = 1, 2$, and 3.

Since the original data were not available, the adequacy of these models could not be checked and these assumptions must be considered speculations showing what may be obtained from this "same" and "different" data. It should be noted that we have assumed that the pattern comparisons are made in the sensory register and not in the short-term store. Both models give similar sensory-register-capacity estimates that agree well with our partial-report experiment. This agreement, between an experiment involving finger stimulation and an experiment involving stimulation sites on the body separated as widely as possible, suggests a central limitation rather than a peripheral one. This suggestion is also consistent with the observed improvement with training.

All of the data of Fig. 13, except for Krohn's, were obtained from Ss having only one to two weeks of practice. Since all of these data lay in a relatively narrow band across the figure, and since the data represent widely different spatial separations of the stimuli, the existence of a central spatial-decision mechanism that can be "focused" on different areas of skin surface, and that improves with practice, is further suggested.

The partial-report data show the existence of a tactile sensory register (i.e., more information was available than could be reported in a whole report). For all the Ss the sensory register had a capacity (1) that was about 50% greater than the span of immediate memory, and (2) that decayed with a time constant of 1.3 sec. The mechanism for this sensory register may be the same as that for tactile persistence (aftersensations) reported in the literature. The tactile sensory register has considerably less capacity than the visual sensory register determined using letters instead of dots. However, there is little comparable visual dot pattern data.

Spatial properties of the sensory register were determined by two separate analyses. The SC analysis considered the set of locations where a typical stimulus would be reported, and the linear analysis considered the set of stimulus positions that influence a typical report. In comparing the results of these two analyses, a basic similarity is seen. Both results show that a given stimulated position may be reported at a neighboring location as well as at the proper location. We have called this concept the spread model.⁵ This spread model, rather than masking, explains the poorer accuracy on the middle phalanges; there are more locations to which stimuli can spread on the middle phalanges than on the distal and proximal phalanges.

The results of applying the spread model to data collected by Bliss (1961, p. 51) using double simultaneous tactile-kinesthetic stimulation of six fingers is shown in Fig. 14(a), together with the comparable data of Experiment 2. Summarizing his results, Bliss stated that most errors resulted from confusions between adjacent fingers of the same hand. The data of Saslow (1962), collected with one, two, and three simultaneous airjet stimuli on the index fingertip, are shown in Fig. 14(b). The spread correlation was estimated from Saslow's data. The spread model seems to describe a range of multiple tactile stimuli experiments, and perhaps it is a property of the common "central decision maker."

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NOTES

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 2. Now at Stanford Research Institute, Menlo Park, California.
 3. In Fig. 5(a) it appears that the maximum mutual information $I(S;R)$ occurs at $n = 2$. However, more tactile perception data can be called upon at this point. In Bliss (1967) an experiment is described with $n = 2$ and $n = 3$, using the apparatus identical to that in the present experiment. The group of five Ss in this new experiment transmitted an average of 7.30 bits with $n = 2$, and 8.14 with $n = 3$. Assuming a similar increase, the information transmitted by the four Ss shown in Fig. 5(a) would have a maximum value of about 7.4 bits at $n = 3$.
- For Subject JK, whose lower bounds are shown in Fig. 5(b), it appears that the maximum information is transmitted with $n = 4$ or $n = 5$. No data on this S were taken with $n = 5$, and it must be assumed that the 12.1 bits transmitted with $n = 4$ is typical of his maximum. In either case, the

maximum transmission by Subject JK occurs with a higher value of n than that of the other Ss.

4. A possible explanation for the decreased middle-phalanx accuracy is that there is interference from stimuli on neighboring locations. On the middle phalanges, this interference or masking could come from adjacent locations on all four sides of a stimulus. On the proximal and distal phalanges, this masking could come from only three sides. As shown in the analysis of spatial properties of the sensory register, neither interference or masking appropriately describes the decreased accuracy of the middle phalanges, but a "spread" model does, in which the ability of Ss to localize the positions of the patterns decreases as n increases.

5. The spread model should not be confused with the spread function of the Fourier transformation. Other possible names for the spread model are (1) lateral excitation model (in contrast to the lateral inhibition or masking model), (2) crosstalk model, and (3) linear-distribution model in which a stimulated point can be reported over a set of locations described by a probability distribution.

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APPENDIX A METHODS FOR ESTIMATING NUMBER OF POSITIONS PERCEIVED

Because of the nature of the stimuli used in these experiments, Ss can guess a significant portion of the stimuli locations correctly even if they were not perceived. For $n = 12$, for example, a S could just guess 12 locations and, on the average, get six correct. In order to estimate memory capacities independently of the guessing behavior of the Ss, one needs to be able to estimate the number of positions actually perceived from the experimentally derived number of positions correctly reported. We have examined several models of the S's guessing behavior and derived guessing corrections for each. Below is a brief description of these models; more complete descriptions are given by Hill (1967).

Model I - Constant Number of Positions Perceived

Out of a set of S possible stimulus positions, n positions are presented on each trial. We assume that the subject perceives a constant, average number of them, P , and makes $n - P$ guesses randomly from the remaining $S - P$ unperceived locations with probability of success $g = (n - P)/(S - P)$ on each guess. The average number of positions correct per trial, C , is then,

$$C = P + (n - P) \frac{n - P}{S - P}$$

Solving this equation for P yields

$$P = \frac{SC - n^2}{C + S - 2n} \quad (A-1)$$

The values of P can easily be determined from the measured values of C , using this equation.

Model II - Constant Report Probabilities

On a given stimulus pattern, n positions are presented and the S is required to report n different positions. We assume that the probability of the i th report being a perceived position is a constant probability p_i ($i = 1, 2, \dots, n$), and that the probability of a successful guess when the i th report is an unperceived position is a constant probability g_i ($i = 1, 2, \dots, n$). The fraction of time that the i th report is correct, c_i , is then given by n equations of the form

$$c_i = p_i + (1 - p_i)g_i \quad i = 1, 2, \dots, n. \quad (A-2)$$

We need to solve these equations for the p_i , and then sum the p_i to get the total number of positions perceived. In general we wish to allow for unequal values of the g_i . However, in equation set (A-2) there are only n equations and we may only solve for n unknowns. Therefore, we cannot solve the equation set (A-2) for both the p_i and g_i simultaneously. We want solutions for the response probabilities, p_i , and hence we must evaluate the g_i in terms of the p_i in order to keep the number of unknowns equal to n . In order to express the values of g_i in terms of the p_i we must postulate a model for the guessing scheme.

One method for evaluating the g_i assumes that the S guesses uniformly from among the unperceived positions. If j positions are perceived on a trial,

then the guessing probability is $(n - j)/(S - j)$. If the probability of perceiving j positions when response i is guessed, is $P(j | \text{response } i \text{ guessed})$, then

$$g_i = \frac{\sum_{j=0}^n \frac{n-j}{S-j} P(j | \text{response } i \text{ guessed})}{S-j} \quad (\text{A-3})$$

As an approximation we can replace the average of the function $(n - j)/(S - j)$ by the function of the average value of j . Then Equation (A-3) reduces to

$$g_i \approx \frac{n - \bar{j}}{S - \bar{j}}$$

where $\bar{j}$ is the expected value of the number of positions perceived when the j th position is not perceived. If we further assume that the p_i are independent then

$$\bar{j} = \sum_{k=1}^n p_k$$

Since the function $(n - j)/(S - j)$ is not linear, the average of this function of j is in general unequal to the function of the average value of j (or $\bar{j}$). However, we will show this function is linear enough for small variations of j about $\bar{j}$, so that the resultant approximation is close enough for practical use. If we assume independent values for the p_i , the resultant distribution is wider than those obtained in the experiment and we can put bounds on the accuracy of this approximation.

When all of the p_i are either zero or one, the approximation is exact. The approximation is worst under variations in the p_i when $p_i = 1/2$ for all p_i , because this gives the broadest distribution of conditional probabilities. The approximation is also exact for $n = 2$, and gets progressively worse as n increases. The highest value of n used in the experiment is $n = 12$. In the worst case, $n = 12$, $p_i = 1/2$. In this case, g_i is 1.5% higher than the actual value $\bar{g}_i$, an amount of error that is tolerable in terms of the accuracy of the measurements made.

Equation set (A-2) with $\bar{g}_i$ instead of g_i can be reduced to a set of n linear equations in n unknowns and solved directly in theory. In practice, difficulties in the solution could easily arise since the equations are nearly singular for higher values of n . A simple and economical iterative solution of these equations is given by Hill (1967).

Model III - Constant Position Probabilities

We assume that each position of the array of S positions has associated with it a constant probability p_i ($i = 1, 2, \dots, S$) of perceiving the presence of a stimulus at the position and a constant probability g_i , ($i = 1, 2, \dots, n$) of guessing a stimulus. The fraction of correct responses, c_i , at the i th position is then

$$c_i = p_i + (1 - p_i)g_i$$

which may be solved for the S values of p_i ,

$$p_i = (c_i - g_i)/(1 - g_i)$$

where

$$c_i = \frac{\text{Number of stimuli with responses}}{\text{Total number of stimuli presented}}$$

$$g_i = \frac{\text{Number of responses when no stimuli present}}{\text{Total number of times that no stimuli were present}}$$

This model is not a complete model for this stimulus-response situation because there is no restraint on the total number of positions reported per pattern. This restraint would require that of the set of S positions that could produce responses to a stimulus pattern, only n produce responses. Even though this model does not fulfill this requirement, the average number of positions perceived per pattern or some fraction of a pattern (hands, fingers, rows, or individual positions) may be computed from this model. The average number of positions perceived per pattern, P , is n times the average value of p_i :

$$P = n\bar{p}_i = \frac{n}{S} \sum_{i=1}^S p_i$$

Comparison of Different Models

In order to compare the three different models, all three were used to

compute the average number of positions perceived per stimulus pattern on the whole-report data. In general the number of positions perceived computed from each of the models agreed to within a few per cent. Specifically, Model II and Model III agreed to within 1.5% of each other on all values of n . Model I gave consistently higher estimates of the number of locations perceived than the other two models. The difference varied directly with n , Model I giving 0.4% higher results with $n = 2$, and 4.5% higher results with $n = 12$ than Model II. Considering that the standard deviation of the number of positions perceived per pattern is about 7%, the differences between guessing corrections are smaller than the standard deviation. Thus, any conclusions drawn from the number of positions perceived, computed by one of the models, should apply to the number computed by the other models too. The three models can be considered equivalent from the point of view that they estimate the same number of positions perceived per stimulus pattern.

Models I and II can also be compared on their ability to explain the variance in the total number of positions correctly reported per pattern, C . The variance of C predicted by Model I is $g^2(S - n)^2/(S - P - 1)$. The variance of C predicted by Model II is the variance of the sum of the n probabilities of correct response, c_i ($i = 1, 2, \dots, n$). This variance can be expressed as

$$\sigma_c^2 = \sum_{i=1}^n \sigma_{c_i}^2 + 2 \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} \quad (\text{A-4})$$

where σ_{ij} is the covariance between c_i and c_j , and where $\sigma_{ii} = \sigma_{c_i}^2$ is the variance of each c_i .

If the c_i are independent, then $\sigma_{ij} = 0$ if $i \neq j$ and

$$\sigma_c^2 = \sum_{i=1}^n \sigma_{c_i}^2 = \sum_{i=1}^n c_i(1 - c_i) \quad (\text{A-5})$$

In general, the response probabilities, c_i , of Model II are not independent, and the possibility of non-zero values of covariance must be allowed for. The difference between the variance predicted by Equation (A-5) and the actual measured variance is the sum of the off-diagonal covariance terms of Equation (A-4). If this variance difference is significantly different from zero, then the difference is an estimate of the sum of these terms.

The variance predicted by Model I and Model II along with the actual variance of C for the three sighted Ss and for the early blind S (JK) on the different values of n are given in Figs. A-1(a) and A-1(b).

It can be seen in the figures that Model II with independent response probabilities explains the variance data with $n = 2$ and $n = 4$ for the sighted Ss, and with $n = 2$ through $n = 8$ for the early blind subject (JK). Model II, with nonindependent response probabilities, c_i , can be made to explain the data for the higher values of n too. In this case there must be negative covariances between the c_i . It was found that the average between-response correlation coefficient, r , computed from the data, was independent of all the experimental parameters except n . With a constant r model for determining the covariance of the c_i a better estimate of the variances can be obtained. The results of Model II with a constant-response correlation coefficient have been plotted in Fig. A-1 as a dotted line. Model I falls short of explaining the observed variance, and is not flexible enough in nature to permit adjustments that would make it fit. Model II is therefore judged to be the best model of the Ss' performance, as it gives both the number of points perceived and can give the variance of the number of correct responses per pattern.

Fig. A-1. Measured and model variance for (a) the three sighted Ss, and (b) Subject JK.

APPENDIX B

A FUNCTION FOR MEASURING RESPONSE LOCALIZATION

(The Spread Correlation)

$P(S_1 \rightarrow R_2)$ will be defined as the probability that a stimulus at one particular location (Location 1) in the pattern causes a position to be reported in another specific location (Location 2, possibly the same as Location 1). The conditional response probability, $Pr(R_2 | S_1)$, can be expressed in terms of $P(S_1 \rightarrow R_2)$ by the following equation:

$$P(R_2 | S_1) = P(S_1 \rightarrow R_2) + b[1 - P(S_1 \rightarrow R_2)] \quad (B-1)$$

where b is a bias parameter that depends on the experimental procedure.

The bias parameter, b , is restricted by the paradigm of Experiment 2. In both the whole and partial reporting schemes used in this experiment, the S s were informed of the number of stimulus positions occurring in each pattern and gave the same number of responses. Below, we first explain how the parameter b was chosen, and then describe some of the resulting properties of $P(S_1 \rightarrow R_2)$. The variables n and S are defined in Appendix A.

When stimuli and responses at the same location in the patterns are considered, then $b = n/S$. When stimuli and responses at different locations in the patterns are considered then the interdependence of the stimulus locations must be taken into account and we chose

$$b = P(R_2) - A_{12}[P(R_2 | S_2) - P(R_2)]$$

where

$$A_{12} = \frac{P(S_1 | S_2) - P(S_1)}{1 - P(S_1)}$$

In Experiment 2 the expected value of A_{12} is $1/(1 - S)$. In all of the analyses described in this paper the values of $P(R_j | S_i)$, $P(R_j)$, and A_{ij} were computed from the data for every pair of stimulus and response locations. The expected values were not used.

The resulting function $P(S_i \rightarrow R_j)$: (1) maintains some of the properties of a probability function, and (2) has many of the properties of a correlation function. The first point aids in the interpretation of $P(S_i \rightarrow R_j)$ and the second point aids in the analysis and significance testing of $P(S_i \rightarrow R_j)$.

There are two features of $P(S_i \rightarrow R_j)$ that allow it to be interpreted as a probability. One is that when stimuli and response are independently or directly related, $P(S_i \rightarrow R_j)$ has the intuitively correct values (zero or one). Equation (B-1) may be solved for $P(S_1 \rightarrow R_2)$ to give:

$$P(S_1 \rightarrow R_2) = \frac{P(R_2 | S_1) - b}{1 - b} \quad (B-2)$$

The independent and dependent limits of $P(S \rightarrow R)$ may be examined using Equation (B-2). The definition of pair-wise independence takes the form of two conditions

$$\begin{aligned} P(R_2 | S_1 S_2) &= P(R_2 | S_2) \\ P(R_2 | S_1 S_2) &= P(R_2 | S_1) \end{aligned}$$

When these conditions hold, it can be shown that with the constraints of Experiment 2, $P(S_1 \rightarrow R_2)$ is zero, the desired result. On the other hand, when every stimulus at Location 1 is followed by a response at Location 2, $P(R_2 | S_1)$ is one. From Equation (B-2) it follows that $P(S_1 \rightarrow R_2)$ is also one, the desired result.

Another feature is that the sum of the $P(S_i \rightarrow R_j)$ over j of Equation (B-2) is a number between zero and one. Specifically with an average S_i

$$\sum_{j=1}^S P(S_i \rightarrow R_j) = \frac{E[P(R_i | S_i)] - n/S}{1 - n/S}$$

When stimulus and response patterns are independent (pure guessing) then $\sum_j P(S_i \rightarrow R_j)$ is zero. When the stimulus and response patterns are identical (perfect responses), then the sum is one.

There are several features of $P(S_i \rightarrow R_j)$ that allow it to be interpreted as a correlation coefficient. $P(S_i \rightarrow R_j)$ defined by Equation (B-2) is a regression coefficient where the regression of stimuli at one location and responses at another location are considered (see Wine, 1964, p. 497). $P(S_i \rightarrow R_j)$ is also closely related to the usual correlation coefficient, r . Indeed, when $P(R) = P(S)$, the spread probability and the correlation coefficient are identical. In most of the cases considered in this paper $P(R_i)$ and $P(S_i)$ are approximately equal. When values of $P(S_i \rightarrow R_j)$ are close to zero, it may be shown that $P(S_i \rightarrow R_j)$ is approximately normally distributed, and hence, that $P(S_i \rightarrow R_j)$ may be analyzed using an analyses of variance. As $P(S_i \rightarrow R_j)$ is basically a difference between two random variables estimated from the data, negative as well as positive values can occur. Since negative correlations are more acceptable than negative probabilities, and since $P(S_i \rightarrow R_j)$ has such a close kinship with the correlation coefficient, $P(S_i \rightarrow R_j)$ defined by Equation (B-2) is referred to as the "spread correlation" (abbreviated SC) in this paper.