The basic concept of having S terminate the response inside the transport box is believed an essential requirement in studies employing aversive stimulation. It has been our experience that handling devices that are lowered over S, like the one described by Tighe (1965), only seem to work well when applied to nonshock situations.

Whatever solution is devised, the procedure of handling Ss between trials is not only unnecessary but experimentally undesirable.

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NOTE

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Probability gate statistics

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A probability gate presents a fixed but controllable transmission probability p to pulses arriving at its input. This paper describes some statistical properties of the transformation imposed by a gate on the input distribution of interpulse intervals. The output distribution of interpulse intervals is derived in terms of the input density and p . A simple relation is shown to hold between the moment generating functions, and the first four output moments are presented in terms of the input moments and p . Certain restrictions on the kinds of obtainable output distributions are discussed. In particular, a condition is established that specifies when the output density is a "replica" of the input distribution.

Aside from their roles as conceptual units in models of processes in such diverse areas as traffic flow (Haight, 1963) and signal detection (Luce, 1966), probability gates are used most frequently to increase the means of given, but usually unspecified, distributions of interevent intervals. For example, if a pulse train representing the responses of a rat in a

Skinner box is applied to the input of a probability gate and its output pulse train actuates a source of reinforcements, then, when the transmission probability is set to p , the probability that the rat will be reinforced for any response is just p . The reciprocal of the means of the input and output distributions of interpulse intervals, i.e., the response and reinforcement rates, are typically monitored during experiments using variable-ratio reinforcement schedules, and it is readily seen that reinforcement rate is given by p times the response rate.

The purpose of this note is to show how the temporal input and output probability distributions, their moment-generating functions (MGFs), and, hence, their moments are related in general. The results presented impose certain limitations on the kinds of output distributions that can be instrumented.

In the following derivations, the pulses referred to will be idealized as points, and a probability gate will be regarded as a device which at any moment presents a fixed transmission probability p to pulses arriving at its input. In addition, it will be assumed that the MGFs referred to exist.

First, the output probability density of interpulse intervals, say $y(t)$, will be obtained as a function of the input distribution, say $u(t)$, $0 < t$, and p . If a pulse has just appeared at the output, the probability that the next $n-1$ input pulses will be rejected while the n th pulse

is transmitted is given by the geometric distribution:

$$w(n) = pq^{n-1}, \quad q = 1 - p, \quad n = 1, 2, 3, \dots$$

The time between the initially transmitted pulse and the n th input pulse following it is the sum of n independent interpulse intervals, each of which is distributed as $u(t)$. The probability, conditional on n , that the total interval is t sec in duration is thus given by the n -fold convolution of $u(t)$ with itself:

$$v(t | n) = u(t)^{*n}.$$

The joint density of t and n is the product of $v(t | n)$ and $w(n)$, and summing this over all values of n gives the required distribution of output interpulse intervals as a function of the input distribution and p . Thus, all output distributions have the form:

$$y(t) = \sum_{n=1}^{\infty} u(t)^{*n} p q^{n-1}, \quad 0 < t. \quad (1)$$

A special case of this derivation has been used to show that if $u(t)$ is exponential with its origin at 0 and time constant β , then $y(t)$ is also exponential, but with time constant $p\beta$ (Cronholm, 1969).

A very simple relation exists between the MGFs of the input and output distributions. Using this, the moments of

$y(t)$ can be expressed in terms of the moments of $u(t)$ and p . The Laplace MGF of $y(t)$ is defined as:

$$M_y(\theta) = \int_0^\infty e^{\theta t} y(t) dt.$$

Substituting $y(t)$ from Eq. 1 and integrating term by term gives:

$$\begin{aligned} M_y(\theta) &= p \sum_{n=1}^{\infty} q^{n-1} \int_0^\infty e^{\theta t} u(t)^{*n} dt \\ &= p \sum_{n=1}^{\infty} q^{n-1} M_v(\theta). \end{aligned}$$

But $M_v(\theta) = M_u(\theta)^n$, since $v(t|n)$ is an n -fold convolution. Thus,

$$\begin{aligned} M_y(\theta) &= \frac{p}{q} \sum_{n=1}^{\infty} [q M_u(\theta)]^n \\ &= \frac{p M_u(\theta)}{1 - q M_u(\theta)}, \end{aligned} \quad (2)$$

which is the required relation between input and output MGFs. Conversely, it can be shown that $y(t)$ has the form given in Eq. 1 if its MGF can be put in the form $pM/(1 - qM)$; the proof is omitted. Thus, a density is an output density if and only if its MGF has this form.

The r th moment of the output distribution $u_{y,r}$ can be expressed in terms of the r th and lower moments of the input distribution and p by differentiating the left and right sides of Eq. 2 r times with respect to θ and then setting $\theta = 0$. The first four raw moments are:

$$\begin{aligned} \mu_{y1} &= \frac{1}{p} \mu_{u1} \\ \mu'_{y2} &= \frac{1}{p^2} (p \mu'_{u2} + 2q \mu_{u1}^2) \\ \mu'_{y3} &= \frac{1}{p^3} (p^2 \mu'_{u3} + 6pq \mu'_{u2} \mu_{u1} + 6q^2 \mu_{u1}^3) \\ \mu'_{y4} &= \frac{1}{p^4} [p^3 \mu'_{u4} + 2p^2 q (4 \mu'_{u3} \mu_{u1} + 3 \mu_{u1}^2) \\ &\quad + 36pq^2 \mu'_{u2} \mu_{u1}^2 + 24q^3 \mu_{u1}^4]. \end{aligned}$$

Note that the anticipated relation between means has emerged (also see Parzen, 1962, p. 47). The output and input variances also are related simply:

$$\sigma_y^2 = \frac{1}{p^2} (p \sigma_u^2 + q \mu_{u1}^2).$$

The ratio of the output to input variances,

$$\frac{\sigma_y^2}{\sigma_u^2} = \frac{1}{p} + \frac{q}{p^2} \frac{\mu_{u1}^2}{\sigma_u^2},$$

shows that the output variance is always greater than the input variance. It varies inversely with p and the square of the input coefficient of variation σ_u/μ_{u1} .

Obviously, the output distribution of interpulse intervals may differ considerably from the input distribution. It also may be very similar to the input distribution, for as the transmission probability p approaches 1, the density $y(t)$ and its MGF $M_y(\theta)$ approach $u(t)$ and $M_u(\theta)$, respectively. Thus, a graded sequence of output functions which differ from a parent density in a specifiable way may be generated.

There are, however, certain general restrictions on the kinds of output densities that can be generated. For example, the domain of t in the output distribution is $0 \leq t < \infty$, $p < 1$, where τ is the origin parameter of the input distribution. Thus, no output distributions that are truncated on the right exist, and those that do exist have their origins at τ .

A question of some practical importance concerns the class of "replicable" input densities. An input density $u(t)$ will be termed replicable, and the corresponding output density $y(t)$ its replica, when $u(t)$ and $y(t)$ have the same form, differing only in the values assigned their parameters. Of course, a replicable distribution is also a replica of its replicas, i.e., the relation is reflexive. The exponential distribution with its origin at zero is replicable, for example, but when its origin is at $\tau > 0$ it is not.

The problem of identifying the class of replicable densities might be approached using the relationship between $y(t)$ and $u(t)$ derived previously (Eq. 1), but the n -fold convolution is troublesome. Another point of contact involves finding the class of MGFs which satisfy functional Eq. 2 such that $M_y(\theta)$ is a replica of $M_u(\theta)$. It will be shown here that a density is replicable if and only if it is itself a replica of the output distribution of a probability gate.

The condition that a distribution $y(t)$ be an output distribution or the replica of one is equivalent to the condition that its MGF have the form of Eq. 2, since all output MGFs and no others have this form. But if $y(t)$ is in turn the input density of a second probability gate with transmission probability p' , the MGF of its output distribution $z(t)$ must be:

$$\begin{aligned} M_z(\theta) &= \frac{p' M_y(\theta)}{1 - q' M_y(\theta)}, \quad q' = 1 - p' \\ &= \frac{pp' M_u(\theta)}{1 - (1 - pp') M_u(\theta)}, \end{aligned}$$

which is a replica of $M_y(\theta)$ in Eq. 2. Thus, $z(t)$ is a replica of $y(t)$ and sufficiency is established. Necessity follows directly from the reflexive property, i.e., any replicable input density is a replica of its output replica.

If the range of values p can take for a particular probability gate does not include small enough values for some purpose, the replicability theorem shows that probability gates may be cascaded until a suitable replica of the first gate's output density is obtained. The effective transmission probability of a string of k gates will be the product of the k individual transmission probabilities.

Unfortunately, most, if not all, commercially available probability gates work on a principle that does not permit them to be cascaded (see Cronholm, 1969). Essentially, the input pulse train is and-gated with a *periodic* pulse train of variable duty cycle p . Thus, the effective transmission probability of a string of gates depends primarily on the first gate's p and the phases of the other periodic pulse trains. For the same reason, these gates cannot be used to produce "periodic" distributions, such as the geometric, with arbitrary epoch durations. A universally applicable probability gate much more like the idealized gates discussed in this paper should be developed. This could be achieved by and-gating the input density with an *aperiodic* pulse train of variable duty cycle, e.g., the output of a Schmitt trigger, with adjustable threshold p , driven by a noise voltage. Geometric distributions with parameter p and epoch duration δ could then be produced by driving the probability gate with clock pulses of period δ .

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