A multipurpose four-dimensional analysis of variance program*

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A computer program is described for a four-dimensional analysis of variance using the IBM 360/40 OS, or larger, machine.

Behavioral scientists often find themselves designing experiments which necessitate an extension of available statistical procedures and result in computations which are beyond the scope of the hand calculator. In addition, these extended designs are often not found in published programs such as those of Veldman (1967).

The present program was written, tested, and utilized for the statistical analysis necessary for a comparative study (Lippman, Galosy, & Thompson, 1970) which investigated the effects of three variables (handling, training, and species) over a series of repeated measures (days).

The statistical design is an extension of Lindquist's (1953) Type III ANOVA analysis of variance design, and of the computational procedures of Bruning and Kintz (1968). The extensions involve the addition of one between, or independent, comparison to the standard three-factor analysis, resulting in a four-dimensional ANOVA with three between comparisons and one within, or correlated, comparison. Because of the computational similarities of the four-dimensional, Type III, and Type I ANOVA (Bruning & Kintz, 1968; Lindquist, 1953), it was subsequently amended to perform any of the three ANOVAs.

PROGRAM CHARACTERISTICS

Programming Language

PL/I is the programming language.

Type of Hardware

IBM 360/40 OS, or larger, should be used.

Processing Performed

The program will perform, at the user's option, any one of the following ANOVAs: two-factor, repeated

measures on one; three-factor, repeated measures on one; and four-factor, repeated measures on one.

The program computes the mean squares, degrees of freedom, F ratios, and the probability of each F ratio. Individual cell means and standard deviations are also computed.

Storage Requirements

In most cases, 96K has been sufficient. However, if more than approximately 100 Ss are used, or more than approximately seven repeated measures are taken, auxiliary storage may be necessary.

Time

With about 30 Ss in a 2 by 2 by 2 by 4 four-dimensional analysis, the total job time is about 5 min. The smaller designs take less time.

Fig. 1. A three-dimensional representation of the card deck before submitting the program for a run.

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Table 1
Summary Table With Cell Means and Standard Deviations for the
Type 34 ANOVA Using Test Data

This is a Test Run Source	SS	DF	MS	F	P
Total	659.83	95.00			
Between Ss	274.33	23.00			
A (Handling)	0.67	1.00	0.67	0.07	0.85739
B (Species)	5.04	1.00	5.04	0.49	0.50182
C (Training)	13.50	1.00	13.50	1.33	0.26595
A by B	15.04	1.00	15.04	1.48	0.24063
A by C	16.67	1.00	16.67	1.64	0.21722
B by C	51.04	1.00	51.04	5.07	0.03780
A by B by C	9.38	1.00	9.38	0.92	0.64813
Error Bet	163.00	16.00	10.19		
Within Ss	385.50	72.00			
D (Days)	3.42	3.00	1.14	0.46	0.71637
A by D	1.08	3.00	0.36	0.15	0.95227
B by D	49.04	3.00	16.35	6.59	0.00109
C by D	75.42	3.00	25.14	10.14	0.00010
A by B by D	40.21	3.00	13.40	5.41	0.00309
A by C by D	47.08	3.00	15.69	6.33	0.00136
B by C by D	19.04	3.00	6.35	2.56	0.06481
A by B by C by D	31.21	3.00	10.40	4.20	0.01030
Error Win	119.00	48.00	2.48		

Cell Means and Standard Deviations

Cell				Mean	SD
A	B	C	D		
1	1	1	1	4.00	1.000
1	1	1	2	3.67	2.082
1	1	1	3	5.67	1.528
1	1	1	4	6.00	1.000
1	1	2	1	9.00	1.000
1	1	2	2	9.00	1.000
1	1	2	3	4.00	2.000
1	1	2	4	5.33	2.082
1	2	1	1	4.00	3.000
1	2	1	2	5.33	3.055
1	2	1	3	7.33	1.528
1	2	1	4	6.00	3.606
1	2	2	1	3.67	1.528
1	2	2	2	4.00	2.646
1	2	2	3	4.67	2.082
1	2	2	4	1.67	0.577
2	1	1	1	6.67	3.512
2	1	1	2	2.00	1.000
2	1	1	3	4.00	2.000
2	1	1	4	3.33	1.528
2	1	2	1	6.67	3.512
2	1	2	2	8.67	0.577
2	1	2	3	7.00	1.000
2	1	2	4	3.33	1.528
2	2	1	1	4.67	2.082
2	2	1	2	3.67	2.887
2	2	1	3	7.00	2.000
2	2	1	4	5.33	3.215
2	2	2	1	3.33	1.528
2	2	2	2	7.33	1.528
2	2	2	3	4.00	2.646
2	2	2	4	9.00	0.000

Input

The title, parameter, and data cards precede the data deck and are described in the comments at the beginning of the program listing. There is no program limitation for the number of variables (levels) which may be analyzed on each dimension. Also, there is no limit to the number of Ss used. However, there must be an equal number of Ss represented in each cell of the data matrix. As the program has no options for missing data, the user should either drop data from the analysis at random or should provide mean scores for the missing numbers before input of the data.

The data deck should be organized in blocks corresponding to rows, columns, and slices of a three-dimensional representation of the design. Figure 1 shows a representative deck structure. In all cases, including the four-dimensional design, the repeated measures factor is the basic input block index.

The raw input data must be numeric and in base 10, but can be integer or explicit floating point.

Output

The following information is output from the program: (1) standard summary table, including variable names, mean squares, degrees of freedom, F ratios, and probability of each F ratio; and (2) listing of mean and standard deviation for each cell (see Table 1).

Error Detection

The accuracy of output can best be determined by running a small deck of test data for which hand computations are available. Common sources of error involve keypunch errors in the source deck of the data, omissions of field-separating commas in the data, extra commas in the data, faulty parameter card, missing data, or incorrect input deck structure.

PROGRAM LISTING

```
ANOVA: PROCEDURE OPTIONS(MAIN);

/* THIS PROGRAM WILL PERFORM THE FOLLOWING ANALYSES:
A. A 4 DIMENSIONAL TYPE III ANALYSIS OF VARIANCE COMPOSED OF
  3 BETWEEN AND 1 WITHIN COMPARISON (TYPE 34).
B. A 3 DIMENSIONAL TYPE III ANALYSIS OF VARIANCE COMPOSED OF
  2 BETWEEN AND 1 WITHIN COMPARISON (TYPE 33).
C. A 2 DIMENSIONAL TYPE I ANALYSIS OF VARIANCE COMPOSED OF
  1 BETWEEN AND 1 WITHIN COMPARISON (TYPE 01).
IN ORDER TO USE THIS PROGRAM THE DATA CARDS MUST BE IN THE FOLLOWING
ORDER:
FIRST CARD: HOLDS THE NAME OF THE EXPERIMENT. THIS NAME MAY
BE UP TO 40 SPACES LONG AND MUST BE ENCLOSED BY SINGLE QUOTES
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```
AND FOLLOWED BY A COMMA.
SECOND CARD: HOLDS THE TYPE AND DIMENSION OF THE ANALYSIS
WHICH YOU WISH TO BE PERFORMED. THE TYPE COMES FIRST AND IS FOLLOWED
BY A COMMA, THEN THE DIMENSION WHICH IS ALSO FOLLOWED BY A COMMA.
THIRD CARD: HOLDS THE NAMES OF THE VARIABLES. EACH NAME MAY
BE UP TO 10 SPACES LONG. THEY MUST BE ENCLOSED IN SINGLE QUOTES
AND FOLLOWED BY A COMMA.
FOURTH CARD: HOLDS THE NUMBER OF VARIABLES (LEVELS) THERE ARE
FOR EACH DIMENSION. IN ADDITION THE NUMBER OF SUBJECTS PER
CELL IS REQUIRED, EACH CELL HAVING EQUAL NUMBERS OF SUBJECTS.
THE NUMBER OF VARIABLES FOR THE 'A', 'B', 'C', AND 'D' DIMENSIONS
COME FIRST, IN THAT ORDER, EACH FOLLOWED BY A COMMA. THEN THE NUMBER
OF SUBJECTS PER CELL FOLLOWED BY A COMMA.
```

FIFTH - N CARDS: THESE ARE THE DATA CARDS UPON WHICH THE RAW SCORES ARE ENTERED. EACH SCORE MUST BE FOLLOWED BY A COMMA. THE SCORES SHOULD BE LISTED AS FOLLOWS: SCORES FOR CELL A1,B1,C1,D1, THEN FOR A1,B1,C1,D2, ETC. MORE THAN ONE CARD MAY BE USED FOR EACH CELLS SCORES IF NECESSARY.

EXAMPLE: THIS IS FOR A TYPE I ANALYSIS HAVING 2 VARIABLES ON THE BETWEEN DIMENSION AND 2 VARIABLES ON THE WITHIN DIMENSION WITH 5 SUBJECTS PER CELL.

CARD 1: 'EXAMPLE FOR TYPE I',

CARD 2: 1,0,

CARD 3: 'BETWEEN','WITHIN',

CARD 4: 2,2,5,

CARD 5: N: 1,2,3,4,5,
6,7,8,9,10,
11,12,13,14,15,
16,17,18,19,20,

THE PROGRAM WILL NOW BEGIN */

```
DCL (TYPE,DIMEN,A,B,C,D,E,I,J,K,L,M,TN,N,DFT,DFBET,DFA,DFB,DFC,
DFAB,DFAC,DFBC,DFABC,DFACD,DFBCD,DFABCD,DFABD,DFEBET,DFWIN,
DFD,DFAD,DFBD,DFCD,FA,FB,FC,FD,FAB,FAC,FBC,FABD,FAD,FBD,
FCD,FABD,FACD,FBCD,FABCD,X,Y,Z,XX,YY,ZZ,XXX,YYY,ZZZ,DFEWIN,
TSXS,TSX,CT,SST,STS,CR,SSBET,TVA,SSA,TVB,SSB,TVC,SSC,TAB,
SSAB,TAC,SSAC,TBC,SSBC,SSBET,SSWIN,TDT,SSD,TAD,SSAD,TBD,SSBD,
TCD,SSCD,TABC,SSABC,TABD,SSABD,TACD,SSACD,TBCD,SSBCD,TABCD,
SSABCD,SSSEWIN,Q,R,MSA,MSB,MSC,MSD,MSAB,MSAC,MSAD,MSBC,MSBD,
MSCD,MSABC,MSABD,MSBCD,MSACD,MSABCD,MSEBET,MSEWIN,PROBA,
PROBB,PROBC,PROBAB,PROBAC,PROBBC,PROBABC,PROBD,PROBAD,
PROBBD,PROBCD,PROBARD,PROBACD,PROBSCD,PROBABCSD) DEC FLOAT
INITIAL(0);
DCL (VARA,VARB,VARC,VARD) CHARACTER(10);
DCL PRBF ENTRY(DEC FLOAT,DEC FLOAT,DEC FLOAT);
DCL (NAME) CHAR(40) VARYING;
GET LIST(NAME);
GET LIST(TYPE,DIMEN);
IF TYPE=3 THEN DO;
  IF DIMEN=4 THEN GO TO T34;
  IF DIMEN=3 THEN GO TO T33;
END;
IF TYPE=1 THEN GO TO T1;
T33: GET LIST(VARA,VARB,VARC);
  GET LIST(B,C,D,E);
  A=1;
GO TO START;
T34: GET LIST(VARA,VARB,VARC,VARD);
  GET LIST(A,B,C,D,E);
GO TO START;
T1: GET LIST(VARA,VARB);
  GET LIST(C,D,E);
  A=1;
  B=1;
START:
BEGIN;
  TN=(A*B*C*D*E);
  X=(A*B);
  Y=(A*C);
  Z=(B*C);
  XX=(A*D);
  YY=(B*D);
  ZZ=(C*D);
  XXX=(A*B*D);
  YYY=(A*C*D);
  ZZZ=(B*C*D);
DCL(DATA(A,B,C,D,E),DAYT(A,B,C,D),GPT(A,B,C),ST(A,B,C,E),AA(A),
BB(B),CC(C),AB(A,B),AC(A,C),BC(B,C),AD(A,D),BD(B,D),CD(C,D),
ABD(A,B,D),ACD(A,C,D),BCD(B,C,D),DT(D),MEAN(A,B,C,D),SD(A,B,C,
D),DAYXS(A,B,C,D),DAYTS(A,B,C,D)) DEC FLOAT;
MEAN=0;SD=0;DAYXS=0;DAYTS=0;DATA=0;DAYT=0;GPT=0;ST=0;AA=0;
BB=0;CC=0;AB=0;AC=0;BC=0;DT=0;AD=0;BD=0;ABD=0;BCD=0;CD=0;ACD=0;
GET LIST(DATA);
```

/* COMPUTE DAY TOTALS, TOTAL SUM X SQ, DAY SUM X SQ, TOTAL SUM X,
GROUP TOTALS, DAY TOTALS SQ */

```
L1: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO L=1 TO D;
  DO M=1 TO E;
  DAYT(I,J,K,L)=DAYT(I,J,K,L)+DATA(I,J,K,L,M);
  TSXS=TSXS+(DATA(I,J,K,L,M)**2);
  DAYSXS(I,J,K,L)=DAYSXS(I,J,K,L)+(DATA(I,J,K,L,M)**2);
END;
TSX=TSX+DAYT(I,J,K,L);
GPT(I,J,K)=GPT(I,J,K)+DAYT(I,J,K,L);
DAYTS(I,J,K,L)=(DAYT(I,J,K,L)**2)/E;
END L1;
```

/* COMPUTE CELL MEANS AND STD. DEV. */

```
L1A: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO L=1 TO D;
  SD(I,J,K,L)=SQRT((DAYSXS(I,J,K,L)-DAYTS(I,J,K,L))/(E-1));
  MEAN(I,J,K,L)=DAYT(I,J,K,L)/E;
END L1A;
```

/* COMPUTE SUBJECT TOTALS, CORR. TERM, SS TOTAL */

```
L2: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO M=1 TO E;
  DO L=1 TO D;
  ST(I,J,K,M)=ST(I,J,K,M)+(DATA(I,J,K,L,M));
END L2;
CT=(TSX**2)/TN; SST=TSXS-CT;
```

/* COMPUTE SS BETWEEN */

```
L3: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO M=1 TO E;
  STS=STS+(ST(I,J,K,M)**2);
END L3;
CB=STS/D;
SSBET=CB-CT;
IF TYPE=1 THEN DO;
  SSA=1;
  SSB=1;
GO TO L8;
END;
```

/* COMPUTE SSA */

```
L4: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  AA(I)=AA(I)+GPT(I,J,K);
END L4;
L5: DO I=1 TO A;
  TVA=TVA+((AA(I)**2)/(B*C*D*E));
END L5;
IF TYPE=3 & DIMEN=3 THEN SSA=1;
IF TYPE=3 & DIMEN=4 THEN DO;
  SSA=TVA-CT;
END;
```

/* COMPUTE SSB */

```
L6: DO J=1 TO B;
  DO I=1 TO A;
  DO K=1 TO C;
  BB(J)=BB(J)+GPT(I,J,K);
END L6;
L7: DO J=1 TO B;
  TVB=TVB+((BB(J)**2)/(A*C*D*E));
END L7;
SSB=TVB-CT;
```

/* COMPUTE SSC */

```
L8: DO K=1 TO C;
  DO I=1 TO A;
  DO J=1 TO B;
  CC(K)=CC(K)+GPT(I,J,K);
END L8;
L9: DO K=1 TO C;
  TVC=TVC+((CC(K)**2)/(A*B*D*E));
END L9;
SSC=TVC-CT;
```

/* COMPUTE SSAB */

```
L10: DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  AB(I,J)=AB(I,J)+GPT(I,J,K);
END;
TAB=TAB+((AB(I,J)**2)/(C*D*E));
END L10;
SSAB=TAB-CT-SSA-SSB;
```

/* COMPUTE SSAC */

```
L12: DO I=1 TO A;
  DO K=1 TO C;
  DO J=1 TO B;
  AC(I,K)=AC(I,K)+GPT(I,J,K);
END;
TAC=TAC+((AC(I,K)**2)/(B*D*E));
END L12;
SSAC=TAC-CT-SSA-SSC;
```

/* COMPUTE SSBG */

```
L14: DO K=1 TO C;
  DO J=1 TO B;
  DO I=1 TO A;
  BC(K,J)=BC(K,J)+GPT(I,J,K);
END;
TEC=TEC+((BC(K,J)**2)/(A*D*E));
END L14;
SSBC=TEC-CT-SSB-SSC;
```

```

/* COMPUTE SSABC */
L16: DO I=1 TO A;
      DO J=1 TO B;
      DO K=1 TO C;
      TABC=TABC + ((GPT(I,J,K)**2)/(D*E));
END L16;
SSABC=TABC-CT-SSA-SSB-SSC-SSAB-SSAC-SSBC;

/* COMPUTE SS ERROR BETWEEN */
IF TYPE=1 THEN DO;
  SSEBET=SSEBET-SSC;
END;
IF TYPE=3 & DIMEN=3 THEN DO;
  SSEBET=SSEBET-SSB-SSC-SSBC;
END;
IF TYPE=3 & DIMEN=4 THEN DO;
  SSEBET=SSEBET-SSA-SSB-SSC-SSAB-SSAC-SSBC-SSABC;
END;

/* COMPUTE SS WITHIN */
L17: DO L=1 TO D;
      DO I=1 TO A;
      DO J=1 TO B;
      DO K=1 TO C;
      DT(L)=DT(L)+DAYT(I,J,K,L);
END L17;
L18: DO L=1 TO D;
      TDT=TDT + ((DT(L)**2)/(A*B*C*E));
END L18;
SSD=TDT-CT;

/* COMPUTE SSAD */
L19: DO I=1 TO A;
      DO L=1 TO D;
      DO J=1 TO B;
      DO K=1 TO C;
      AD(I,L)=AD(I,L) + DAYT(I,J,K,L);
      END;
      END;
      TAD=TAD + ((AD(I,L)**2)/(B*C*E));
END L19;
SSAD=TAD-CT-SSA-SSD;

/* COMPUTE SSBD */
L21: DO J=1 TO B;
      DO L=1 TO D;
      DO I=1 TO A;
      DO K=1 TO C;
      BD(J,L)=BD(J,L) + DAYT(I,J,K,L);
      END;
      END;
      TBD=TBD + ((BD(J,L)**2)/(A*C*E));
END L21;
SSBD=TBD-CT-SSB-SSD;

/* COMPUTE SSCD */
L23: DO K=1 TO C;
      DO L=1 TO D;
      DO J=1 TO B;
      DO I=1 TO A;
      CD(K,L)=CD(K,L) + DAYT(I,J,K,L);
      END;
      END;
      TCD=TCD + ((CD(K,L)**2)/(A*B*E));
END L23;
SSCD=TCD-CT-SSC-SSD;

/* COMPUTE SSABD */
L25: DO I=1 TO A;
      DO J=1 TO B;
      DO L=1 TO D;
      DO K=1 TO C;
      ABD(I,J,L)=ABD(I,J,L) + DAYT(I,J,K,L);
      END;
      END;
      TABD=TABD + ((ABD(I,J,L)**2)/(C*E));
END L25;
SSABD=TABD-CT-SSAB-SSAD-SSBD-SSA-SSB-SSD;

/* COMPUTE SSACD */
L27: DO I=1 TO A;
      DO K=1 TO C;
      DO L=1 TO D;
      DO J=1 TO B;
      ACD(I,K,L)=ACD(I,K,L) + DAYT(I,J,K,L);
      END;
      END;
      TACD=TACD + ((ACD(I,K,L)**2)/(B*E));
END L27;
SSACD=TACD-CT-SSA-SSC-SSD-SSAC-SSAD-SSCD;

```

```

/* COMPUTE SSBCD */
L29: DO J=1 TO B;
      DO K=1 TO C;
      DO L=1 TO D;
      DO I=1 TO A;
      BCD(J,K,L)=BCD(J,K,L) + DAYT(I,J,K,L);
      END;
      TB CD=TB CD + ((BCD(J,K,L)**2)/(A*E));
END L29;
SSBCD=TB CD-CT-SSB-SSC-SSD-SSBC-SSBD-SSCD;

/* COMPUTE SSABCD */
L31: DO I=1 TO A;
      DO J=1 TO B;
      DO K=1 TO C;
      DO L=1 TO D;
      TABCD=TABCD + ((DAYT(I,J,K,L)**2)/(E));
END L31;
SSABCD=TABCD-CT-SSB-SSC-SSD-SSBC-SSAB-SSAC-SSAD-SSBD-SSCD-SSABC-SSABD-SSACD-SSBCD-SSA;

/* COMPUTE SS ERROR WITHIN */
IF TYPE=1 THEN DO;
  SSEWIN=SSWIN-SSD-SSCD;
END;
IF TYPE=3 & DIMEN=3 THEN DO;
  SSEWIN=SSWIN-SSD-SSBD-SSCD-SSBCD;
END;
IF TYPE=3 & DIMEN=4 THEN DO;
  SSEWIN=SSWIN-SSD-SSAD-SSBD-SSCD-SSABD-SSACD-SSBCD-SSABCD;
END;

/* COMPUTE DF */
DFT=TN-1;
DFBET=((A*B*C*E)-1);
IF TYPE=1 THEN DO;
  DFA=1;
  DFB=1;
END;
IF TYPE=3 & DIMEN=3 THEN DO;
  DFA=1;
  DFB=B-1;
END;
IF TYPE=3 & DIMEN=4 THEN DO;
  DFA=A-1;
  DFB=B-1;
END;
DFC=C-1;
DFAB=(DFA*DFB);
DFAC=(DFC*DFA);
DFBC=(DFB*DFC);
DFABC=(DFA*DFB*DFC);
IF TYPE=1 THEN DO;
  DFESET=DFBET-DFC;
END;
IF TYPE=3 & DIMEN=3 THEN DO;
  DFESET=DFBET-DFB-DFC-DFBC;
END;
IF TYPE=3 & DIMEN=4 THEN DO;
  DFESET=(DFBET-DFA-DFB-DFC-DFAB-DFAC-DFBC-DFABC);
END;
DFWIN=DFT-DFBET;
DFD=D-1;
DFAD=(DFA*DFD);
DFBD=(DFB*DFD);
DFCD=(DFC*DFD);
DFABD=(DFA*DFB*DFD);
DFACD=(DFA*DFC*DFD);
DFBCD=(DFB*DFC*DFD);
DFABCD=(DFA*DFB*DFC*DFD);
IF TYPE=1 THEN DO;
  DFEWIN=DFWIN-DFD-DFCD;
END;
IF TYPE=3 & DIMEN=3 THEN DO;
  DFEWIN=DFWIN-DFD-DFBD-DFCD-DFBCD;
END;
IF TYPE=3 & DIMEN=4 THEN DO;
  DFEWIN=DFWIN-DFD-DFAD-DFBD-DFCD-DFABD-DFACD-DFBCD-DFABCD;
END;

/* COMPUTE MEAN SQ. */
MSA=SSA/DFA;
MSB=SSB/DFB;
MSC=SSC/DFC;
MSD=SSD/DFD;
MSAB=SSAB/DFAB;
MSAC=SSAC/DFAC;
MSAD=SSAD/DFAD;
MSBC=SSBC/DFBC;
MSBD=SSBD/DFBD;
MSCD=SSCD/DFCD;
MSABC=SSABC/DFABC;

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MSABD=SSABD/DFABD;
MSBCD=SSBCD/DFBCD;
MSACD=SSACD/DFACD;
MSABCD=SSABCD/DFABCD;
MSEBET=SSEBET/DFEBET;
MSEWIN=SSEWIN/DFEWIN;
Q=MSEBET;

/* COMPUTE F RATIO */

FA=MSA/Q;
FB=MSB/Q;
FC=MSC/Q;
FAB=MSAB/Q;
FAC=MSAC/Q;
FBC=MSBC/Q;
FABC=MSABC/Q;
R=MSEWIN;
FD=MSD/R;
FAD=MSAD/R;
FBD=MSBD/R;
FCD=MSCD/R;
FABD=MSABD/R;
FACD=MSACD/R;
FBCD=MSBCD/R;
FABCD=MSABCD/R;

/* COMPUTE PROBABILITIES */

IF TYPE=1 THEN DO;
  PROBC=PRBF(DFC,DFEBET,FC);
  PROBD=PRBF(DFD,DFEWIN,FD);
  PROBCD=PRBF(DFCD,DFEWIN,PCD);
  GO TO P2;
END;
IF TYPE=3 & DIMEN=3 THEN GO TO P1;
  PROBA=PRBF(DFA,DFEBET,FA);
  PROBAB=PRBF(DFAB,DFEBET,FAB);
  PROBAC=PRBF(DFAC,DFEBET,FAC);
  PROBABCD=PRBF(DFABCD,DFEWIN,FABCD);
P1:  PROBB=PRBF(DFB,DFEBET,FB);
  PROBC=PRBF(DFC,DFEBET,FC);
  PROBD=PRBF(DFD,DFEWIN,FD);
  PROBCB=PRBF(DFCB,DFEBET,FCB);
  PROBCD=PRBF(DPCD,DFEWIN,PCD);
  PROBCD=PRBF(DPCD,DFEWIN,PCD);
P2:  UNO=MUO;
  F3:  FORMAT(X(6),A(5),X(9),4 F(11,2),X(5),F(7,5));
  F4:  FORMAT(X(20),A(30));
  F5:  FORMAT(X(10),A(6),X(10),A(2),X(10),A(2),X(8),A(2),X(10),
    A(1),X(10),A(1));
  F6:  FORMAT(X(2),A(5),X(13),2 F(11,2));
  F7:  FORMAT(X(4),A(16),2 F(11,2));
  F8:  FORMAT(X(6),A(1),X(1),A(1),A(10),A(1),4 F(11,2),X(5),
    F(7,5));
  F9:  FORMAT(X(6),A(3),X(11),4 F(11,2),X(5),F(7,5));
  F10: FORMAT(X(6),A(9),X(5),3 F(11,2));
  F11: FORMAT(X(4),A(15),X(1),2 F(11,2));
  PUT PAGE;
IF TYPE=1 THEN GO TO TABLE1;
IF TYPE=3 THEN DO;
  IF DIMEN=4 THEN GO TO TABLE34;
  IF DIMEN=3 THEN GO TO TABLE33;
END;

/* PRINT SUMMARY TABLES */

TABLE34:
  PUT EDIT(NAME) (X(10),A);
  PUT SKIP(2) EDIT('SUMMARY TABLE FOR TYPE 34 ANOVA') (R(F4));
  PUT SKIP (3);
  PUT EDIT('SOURCE','SS','DF','MS','F','P') (R(F5));
  PUT SKIP (2);
  PUT EDIT('TOTAL','SST','DFT') (R(F6));
  PUT SKIP (1);
  PUT EDIT('BETWEEN SUBJECTS','SSEBET','DFEBET') (R(F7));
  PUT SKIP EDIT('A','(','VARA','),'SSB,DFB,MSB,FB,PROBB) (R(F8));
  PUT SKIP EDIT('B','(','VARB','),'SSC,DFC,MSC,FC,PROBC) (R(F8));
  PUT SKIP EDIT('C','(','VARC','),'SSD,DFD,MSD,FD,PROBD) (R(F8));
  PUT SKIP EDIT('AXB','SSAB,DFAB,MSAB,FAB,PROBAB) (R(F9));
  PUT SKIP EDIT('AXC','SSAC,DFAC,MSAC,FAC,PROBAC) (R(F9));
  PUT SKIP EDIT('BXC','SSBC,DFBC,MSBC,FCB,PROBCB) (R(F9));
  PUT SKIP EDIT('AXBXC','SSABC,DFABC,MSABC,FABC,PROBABC) (R(F3));
  PUT SKIP EDIT('ERROR BET','SSEBET,DFEBET,MSEBET) (R(F10));
  PUT SKIP(2) EDIT('WITHIN SUBJECTS','SSWIN,DFWIN) (R(F11));
  PUT SKIP EDIT('D','(','VARD','),'SSD,DFD,MSD,FD,PROBD) (R(F8));
  PUT SKIP EDIT('AXD','SSAD,DFAD,MSAD,FAD,PROBAD) (R(F9));
  PUT SKIP EDIT('BXD','SSBD,DFBD,MSBD,FBD,PROBBD) (R(F9));
  PUT SKIP EDIT('CXD','SSCD,DFCD,MSCD,PCD,PROBCD) (R(F9));
  PUT SKIP EDIT('AXBXD','SSABD,DFABD,MSABD,FABD,PROBAD) (R(F3));
  PUT SKIP EDIT('AXCXD','SSACD,DFACD,MSACD,FACD,PROBACD) (R(F3));

PUT SKIP EDIT('BXCXD','SSBCD,DFBCD,MSBCD,FCBD,PROBCBD) (R(F3));
PUT SKIP EDIT('AXBXCXD','SSABCD,DFABCD,MSABCD,FABCD,PROBABC) (X(6),A(7),X(7),4 F(11,2),X(5),F(7,5));
PUT SKIP EDIT('ERROR WIN','SSEWIN,DFEWIN,MSEWIN) (R(F10));
PUT SKIP(5) EDIT('CELL MEANS AND STD DEV') (X(10),A(22));
PUT SKIP(2) EDIT('CELL','A','B','C','D','MEAN','STD DEV') (X(5),A(4),X(3),A(1),X(3),A(1),X(3),A(1),X(3),A(1),X(5),A(4),X(5),A(7));
DO I=1 TO A;
DO J=1 TO B;
DO K=1 TO C;
DO L=1 TO D;
PUT SKIP EDIT(I,J,K,L,MEAN(I,J,K,L),SD(I,J,K,L)) (X(11),F(3),X(1),F(3),X(1),F(3),X(1),F(3),X(3),F(6,2),X(3),F(7,3));
END;
END;
END;
GO TO STOP;
TABLE33:
  PUT EDIT(NAME) (X(10),A);
  PUT SKIP(2) EDIT('SUMMARY TABLE FOR TYPE 33 ANOVA') (R(F4));
  PUT SKIP(3) EDIT('SOURCE','SS','DF','MS','F','P') (R(F5));
  PUT SKIP(2) EDIT('TOTAL','SST','DFT') (R(F6));
  PUT SKIP EDIT('BETWEEN SUBJECTS','SSEBET,DFEBET) (R(F7));
  PUT SKIP EDIT('A','(','VARA','),'SSB,DFB,MSB,FB,PROBB) (R(F8));
  PUT SKIP EDIT('B','(','VARB','),'SSC,DFC,MSC,FC,PROBC) (R(F8));
  PUT SKIP EDIT('AXB','SSBC,DFBC,MSBC,FCB,PROBCB) (R(F9));
  PUT SKIP EDIT('ERROR BET','SSEBET,DFEBET,MSEBET) (R(F10));
  PUT SKIP(2) EDIT('WITHIN SUBJECTS','SSWIN,DFWIN) (R(F11));
  PUT SKIP EDIT('C','(','VARC','),'SSD,DFD,MSD,FD,PROBD) (R(F8));
  PUT SKIP EDIT('AXC','SSBD,DFBD,MSBD,FBD,PROBBD) (R(F9));
  PUT SKIP EDIT('BXC','SSCD,DFCD,MSCD,PCD,PROBCD) (R(F9));
  PUT SKIP EDIT('AXBXC','SSBCD,DFBCD,MSBCD,FCBD,PROBCBD) (X(6),A(5),X(9),4 F(11,2),X(5),F(7,5));
  PUT SKIP EDIT('ERROR WIN','SSEWIN,DFEWIN,MSEWIN) (R(F10));
  PUT SKIP(5) EDIT('CELL MEANS AND STD DEV') (X(10),A(22));
  PUT SKIP(2) EDIT('CELL','A','B','C','MEAN','STD DEV') (X(5),A(4),X(3),A(1),X(3),A(1),X(3),A(1),X(5),A(4),X(5),A(7));
  DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO L=1 TO D;
  PUT SKIP EDIT(J,K,L,MEAN(I,J,K,L),SD(I,J,K,L)) (X(11),F(3),X(1),F(3),X(1),F(3),X(3),F(6,2),X(3),F(7,3));
  END;
  END;
  END;
  END;
  GO TO STOP;
TABLE1:
  PUT EDIT(NAME) (X(10),A);
  PUT SKIP(2) EDIT('SUMMARY TABLE FOR TYPE 01 ANOVA') (R(F4));
  PUT SKIP(3) EDIT('SOURCE','SS','DF','MS','F','P') (R(F5));
  PUT SKIP(2) EDIT('TOTAL','SST','DFT') (R(F6));
  PUT SKIP EDIT('BETWEEN SUBJECTS','SSEBET,DFEBET) (R(F7));
  PUT SKIP EDIT('A','(','VARA','),'SSC,DFC,MSC,FC,PROBC) (R(F8));
  PUT SKIP EDIT('ERROR BET','SSEBET,DFEBET,MSEBET) (R(F10));
  PUT SKIP(2) EDIT('WITHIN SUBJECTS','SSWIN,DFWIN) (R(F11));
  PUT SKIP EDIT('B','(','VARB','),'SSD,DFD,MSD,FD,PROBD) (R(F8));
  PUT SKIP EDIT('AXB','SSCD,DFCD,MSCD,PCD,PROBCD) (R(F9));
  PUT SKIP EDIT('ERROR WIN','SSEWIN,DFEWIN,MSEWIN) (R(F10));
  PUT SKIP(5) EDIT('CELL MEANS AND STD DEV') (X(10),A(22));
  PUT SKIP(2) EDIT('CELL','A','B','MEAN','STD DEV') (X(5),A(4),X(3),A(1),X(3),A(1),X(5),A(4),X(5),A(7));
  DO I=1 TO A;
  DO J=1 TO B;
  DO K=1 TO C;
  DO L=1 TO D;
  PUT SKIP EDIT(K,L,MEAN(I,J,K,L),SD(I,J,K,L)) (X(11),F(3),X(1),F(3),X(3),F(6,2),X(3),F(7,3));
  END;
  END;
  END;
  END;
  GO TO STOP;
STOP: PUT PAGE;
END;

/* PROBABILITY FUNCTION */

PRBF: PROCEDURE(DA,DB,FR) FLOAT;
/* DA=NUM DF, DB=DNOM DF, FR=F RATIO */
DCL(DA,DB,FR,PROB) DEC FLOAT;
DCL(AP,BP,FP,AAP,BBP,ZP) DEC FLOAT INITIAL(0);
PROB=1.0;
IF ((DA*DB*FR)=0) THEN RETURN (PROB);
IF FR<1.0 THEN GO TO FIVE;
AP=DA;
BP=DB;
FP=FR;
GO TO TEN;
FIVE:
AP=DB;
BP=DA;
FP=1.0/FR;

```

```

TEN:
  AAP=2.0/(9.0*AP);
  BBP=2.0/(9.0*BP);
  IF(BBP=0) | (FP=0) | (AAP<0) THEN DO;
    PUT DATA(BBP,FP,AAP);
    PUT LIST('SQRT OF 0');
  GO TO FERN;
END;
  ZP=ABS(((1.0-BBP)*FP**0.333333-1.0+AAP)/SQRT(BBP*FP**0.666667+
  AAP));
FERN:
  IF BP<4 THEN ZP=ZP*(1.0+.8*ZP**4/BP**3);
  PROB=0.5/(1.0+ZP*(.196854+ZP*(.115194+ZP*(.000344+ZP*
  .019527))))**4;
  IF FR<1.0 THEN PROB=1.0-PROB;
  RETURN(PROB);
END PRBF;
END;

```

High-speed point plotting with computer-controlled displays*

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A method is described for the high-speed plotting of many independently generated elements in a computer-controlled display.

We shall describe a method of high-speed point plotting which we have found to be extremely useful. We are aware that special-purpose hardware-software display systems, e.g., character generators, vector plotters, are available to simplify certain display programs and to gain speed. The present method may be regarded as a supplement to, rather than a replacement of, such special-purpose display systems. The method may be expected to be useful when an extremely large number of independently generated points must be plotted within a short period of time.

The specific problem we faced was to paint four 32 x 32 element binary-coded displays upon a high-speed display scope (Tektronics 602 with P15 phosphor) without noticeable flicker. The two binary states—dot and no-dot—were equiprobable. We thus sought to plot the positions of approximately 0.5 x 4 x 32 x 32, or 2,048, independently calculated dots. In order to avoid flicker upon successive paintings, we aimed to paint successive displays within 25, and preferably within 20, msec.

Several schemes are obvious candidates for consideration. Each may be the solution for a particular display system, depending upon the tradeoffs available for speed and for storage.

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Scheme A is the most economical of machine storage, but is the slowest for displays with a large number of points. Displays are painted point by point as each point is calculated. No intermediate storage is required. This method of "on-line" plotting is excellent for displays with a very small number of display elements. The method is, however, hampered by the intermediate computational time required, especially for displays with a large number of display elements.

Schemes B and C are more costly of machine storage and are slower than Scheme A in terms of the delay from the beginning of computation to the display of the first point. The display cycle, however, is faster than for Scheme A, especially with a large number of display elements. Instead of computing and painting individual dots as in Scheme A, Schemes B and C first compute the binary state of each point and store the results of the successive computations. If the same display is presented over and over for successive paintings, the computation need be performed only once. In its most economical form for storage, Scheme B stores the binary code. The plotting routine scans the stored binary code, and in a plotting routine with a fixed geographical sequence, plots each binary element "1" and does not plot each binary element "0." This system is relatively economical of storage since the binary coding is stored, and the individual words can be "unpacked" bit by bit. In our laboratories, Lon Radin has successfully used this method to plot four 60 x 60 element matrices with

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the PDP-8I computer (4,096 12-bit words). A storage display scope must be employed, however, because the conversion from binary code to geographical position takes too long for flickerless displays.

Scheme C, like Scheme B, first performs all of the necessary computations to determine the binary state of each display element. Scheme C carries the computations one step further. Instead of storing the binary code, Scheme C computes and stores the coordinates of each to-be-plotted point. Scheme C is more wasteful of storage than Scheme B, since the coordinate, rather than the binary state, information must be stored. On the other hand, because only the coordinates of the to-be-plotted locations are stored, the no-dot elements may be ignored. The resulting display routine can be made extremely tight.

In our first use of Scheme C, we picked up the x coordinate of the to-be-plotted point and instructed the x digital-to-analog converter to generate and hold the respective x voltage; we then picked up the y coordinate of the to-be-plotted point and instructed the y digital-to-analog converter to generate and hold the respective y voltage. Then we fired the z axis of the display scope. The steps were: fetch x, generate and hold x, fetch y, generate and hold y, fire z. The display cycle alone took 16 microsec, and, with housekeeping, took 22 microsec per point on the PDP-9 computer. The result was unsatisfactorily long—2,048 x 22 microsec, or 44 msec—for the displays we planned to use.

We had exhausted the three schemes without solving our initial goal and became discouraged. Then we got lucky. We noted that we were not using the full 12 bits of our digital-to-analog converters. We figured that we could get away with only 9 bits' resolution upon each coordinate, since plotting took place upon only 64 lines. We decided to store both the x and y coordinates of each point within a single word of the