Scaling subjective velocity, distance, and duration¹

HOWARD C. RACHLIN
HARVARD UNIVERSITY

By magnitude estimation, the relation between the subjective and physical values of linear velocities, durations, and distances were found to be power functions with exponents of about 0.75, 0.90, and 0.90. When subjective values are substituted for their physical correlates in the equation, $\text{velocity} = \text{distance}/\text{duration}$, we obtain:

$$\text{subjective velocity} = k \left(\frac{\text{subjective distance}}{\text{subjective duration}} \right)^{0.8}$$

This relation depends on no physical measurements for its confirmation.

The influence of stimulus range on velocity judgments was also examined. Although the shorter ranges tended to yield power functions with higher exponents, the constraint due to range did not make itself fully felt in the first few judgments.

In the scaling of subjective velocity by means of visual stimuli that undergo linear motion, several problems arise. When different velocities are used as stimuli, either one or both of the variables, distance and duration, must change. If duration is held constant, distance must vary directly with velocity. If distance is constant, duration must vary inversely with velocity. The mutual dependence of the three physical variables raises a question concerning the extent to which the corresponding subjective variables are independent.

The experiments described below were designed to test the consistency of a velocity scale for several distances and durations, and to compare the velocity scale with scales of distance and duration.

Previous Attempts to Scale Subjective Velocity

Brown (1931) and Wallach (1939) determined subjectively equal velocities at various distances and for various field sizes, but these points of subjective equality create only a nominal scale. The same is true of the more recent work of Bonnet (1964).

Ekman and Dahlbäck (1956) were probably the first to construct a velocity scale by a ratio-scaling technique (method of fractionation). The stimulus was a band of stripes moving behind a stationary window, which subtended 5.7 deg of the visual field. The observer was required to set the velocity of a variable stimulus at half the velocity of a standard. The time of exposure was the time taken to make the setting. The angular velocities of the stripes ranged from 0.70 deg/sec to 5.7 deg/sec, a range of less than 10:1. Ekman and Dahlbäck found that subjective velocity grew as a power function of physical velocity with an exponent of about 1.8.

Mashhour (1964 pp. 80ff) constructed a scale of subjective velocity with the aid of a spike voltage traveling as a spot across the face of an oscilloscope.

The stimulus was viewed through a stationary aperture whose length could be varied from 0.27 to 2.5 deg. The velocities varied from 0.32 deg/sec to 5.0 deg/sec, a range of about 15:1. A single standard and one of several variable stimuli were shown to the observer, who was asked to estimate the ratio between the two stimuli (or assign numbers to the two stimuli). Mashhour found that subjective velocity grew as a power function of physical velocity, but the exponent was around 1.0 (for various distances). When Mashhour (1964 pp. 33ff) averaged results for several standard stimuli, he obtained a still lower exponent (about 0.8).

Eisler and Ottander (1963) used as a stimulus a spot of light moving in a 24 cm circle with a range of angular velocities of 10:1 and, as a method of scaling, a modified form of magnitude estimation. They found results similar to those of Mashhour (a power function with an exponent of about 1.0).

The principal difference between the present experiments and those described above is in the range of velocities and distances scaled. The length of path in these experiments could vary from about 2 deg to 120 deg. The wide angle of exposure allows greater velocities than those used by Ekman and Dahlbäck or Mashhour. The use of a larger range may give a better measure of the exponent. Künnapas (1960) found that the exponent of the power function relating subjective to physical distance increased by about 50% when the range was halved.

The method of scaling used in most of the present experiments was magnitude estimation (Stevens, 1958).

Apparatus

Figure 1 is a schematic diagram of the apparatus. A spot of light is projected onto the rotating mirror

Fig. 1. A schematic diagram of the apparatus.

Table 1. Six Sets of Stimuli

SET 1		2		3		4		5		6		
V	D	T	D	T	D	T	D	T	D	T	D	T
deg/sec	deg	sec	deg	sec	deg	sec	deg	sec	deg	sec	deg	sec
0.94	--*	--	--	--	1.9	2.0	--	--	--	--	--	--
2.3	--	--	2.3	1.0	4.7	2.0	--	--	23	10	--	--
4.7	2.3	0.50	4.7	1.0	9.4	2.0	47	10	23	5.0	12	2.5
9.4	4.7	0.50	9.4	1.0	19	2.0	47	5.0	23	2.5	12	1.3
23	12	0.50	23	1.0	47	2.0	47	23	23	1.0	12	0.50
47	23	0.50	47	1.0	94	2.0	47	1.0	23	0.50	12	0.25
94	47	0.50	94	1.0	--	--	47	0.50	23	0.25	12	0.13
230	120	0.50	--	--	--	--	47	0.20	23	0.10	12	0.05

* The blank spaces in the table are due to limitations of the apparatus.

and, from there, onto the curved stationary screen, which is 39 inches from the observer. When the mirror rotates, the spot (3/8 inches in diameter on the screen) moves across the screen. The speed of the mirror is controlled by the variable-speed motor and the gear changer. The motor runs throughout the experiment. The stimulus is presented by the experimenter's pressing the control button which operates the clutch, engaging the shaft of the mirror to the already turning shaft leading from the gear changer. The mirror rotates through one complete rotation and is then automatically disengaged until the control button is pressed again. The mirror rotates in only one direction. All stimuli move from left to right. The length of path is controlled by changing the opening in the adjustable aperture.

In later experiments a cam (not shown in Fig. 1) was added to the shaft of the mirror. This cam controlled a switch that operated a timer. The timer turned the projector light on and off. When the mirror reached a certain point (50 deg to the left of center), the projector light was turned on. It was turned off after a period of time determined by the setting of the timer. By this means the duration of the exposure could be controlled. In the first series of experiments, however, various durations were obtained by changing the setting of the aperture and the speed control.

Procedure

Each of the 18 experiments in series I and series II employed 10 observers. Observers in series I were drawn more or less at random from a pool of 30 people. In series II, one group of 10 observers was used for

Fig. 2. Psychophysical functions relating subjective to physical velocity. Distance and duration are the two parameters. The data are fairly well described by straight lines in the log-log plot, which indicates that subjective velocity grows as a power function of physical velocity. The Pearson r ranges from 0.993 to 0.999 for the various lines. The slopes of the lines, as determined by the method of least squares, are, from top to bottom, 0.78, 0.78, 0.77 at constant distance, and 0.77, 0.72, 0.70 at constant duration. Each point is the geometric mean of 20 judgments. The interquartile ranges, after correction for intercept differences, are indicated by the vertical lines through the points.

Fig. 3. Psychophysical functions relating subjective to physical duration. Distance is the parameter. These are the same physical stimuli as in the left half of Fig. 2. As in Fig. 2, the data are fairly well described by straight lines. The Pearson r ranges from 0.994 to 0.998. The slopes of two of the three lines (0.76, 0.91, 0.94), as determined by the method of least squares, are steeper than those of Fig. 2. Each point is the geometric mean of 20 judgments. The interquartile ranges after correction for intercept differences, are indicated by the vertical lines through the points.

all experiments. Of the pool of observers, about a fourth had never before made psychophysical judgments.

The observers judged the attributes velocity, distance, or duration. Each observer judged each stimulus twice. They received the following instructions with appropriate modifications for the other attributes.

I am going to present a series of white spots of light moving across a screen in front of you. Your task is to tell how fast they appear to move by assigning numbers to them. Call the first stimulus any number that seems to you appropriate. Assign numbers proportional to your subjective impression. For example, if a stimulus seems three times as fast, assign a number three times as large as the first. If it seems $1/5$ as fast, assign a number $1/5$ as large, and so forth. Ignore the distance moved and the time of exposure of the spot. Just judge speed.

After the stimulus disappeared behind the aperture, the gear setting was changed to slew the mirror quickly around to its starting position. The time between the end of one stimulus and the beginning of the next averaged around 10 sec.

Series I: Constant Duration or Distance

The present series of 12 experiments consists of six sets of stimuli, three at constant distances and three at constant durations. Subjects made magnitude estimations of the velocity of all six sets of stimuli. For the three sets with constant duration, subjects also made magnitude estimations of distance. For the three sets with constant distance, subjects also made magnitude estimations of duration. Table 1 shows

the velocities, distances, and durations of the six sets of stimuli.

Figure 2 is a log-log plot of the geometric means of the 20 velocity judgments, two by each of 10 subjects for the various experimental conditions. Figure 3 is a log-log plot of the duration judgments, and Fig. 4 is a log-log plot of the distance judgments. The slopes of these functions are equivalent to the exponent of a power function relating physical to subjective values. The slopes of the velocity functions are consistently around 0.75. The slopes for distance and duration are both around 0.90. The low exponent, 0.76, for duration appeared with the shortest durations and distances. It may have been caused by judgments of the afterimage added to judgments of the stimulus. The afterimage of shorter stimuli would last proportionally longer.

The fact that the duration and distance judgments of a given set of stimuli were different from the velocity judgments of that same set of stimuli indicates that the observers could judge velocity directly, without strict dependence on a more basic judgment of duration or distance. Whether velocity judgments are influenced at all by duration and distance is another question.

Series II: No Constant Parameter

In series I, one parameter, either distance or duration, was constant for each set of stimuli. In series II, no parameter was constant throughout, but all three parameters had various values within each set of stimuli.

Fig. 4. Psychophysical functions relating subjective to physical distance. Duration is the parameter. These are the same physical stimuli as in the right half of Fig. 2. As in Fig. 2, the data are fairly well described by straight lines. The Pearson r ranges from 0.997 to 0.999. The slopes, of the lines (0.96, 0.87, 0.87), as determined by the method of least squares, are steeper than those of Fig. 2. Each point is the geometric mean of 20 judgments. The interquartile ranges after correction for intercept differences are indicated by the vertical lines through the points.

Table 2. Three Sets of Nine Stimuli

Set 1			Set 2			Set 3		
V	D*	T*	V*	D	T*	V*	D*	T
deg/sec	deg	sec	deg/sec	deg	sec	deg/sec	deg	sec
10	10	1	1	2	2	1	10	10
50	50	1	1	5	5	10	10	1
100	100	1	1	10	10	100	10	0.1
2	10	5	5	10	2	1	50	50
10	50	5	5	25	5	10	50	5
20	100	5	5	50	10	100	50	0.5
1	10	10	10	20	2	1	100	100
5	50	10	10	50	5	10	100	10
10	100	10	10	100	10	100	100	1

* Magnitude estimations were made of the two starred parameters in each group.

li judged. The purpose of series II was to measure the influence, on each dimension, of the variation of the other two.

Three sets of nine stimuli were made up of three values of one parameter combined with three values of a second parameter, yielding nine values of the third parameter. Table 2 shows the three sets of stimuli.

A total of six experiments were run in this series. Magnitude estimations of velocity were made for sets 2 and 3. Magnitude estimations of duration were made for sets 1 and 2. Magnitude estimations of distance were made for sets 1 and 3. Each subject judged each stimulus twice. The same 10 observers served in all six experiments. Each observer was exposed to the ex-

periments in a different order, randomly chosen.

Each experiment contained only three values of the parameter being judged. From judgments of these values a rough estimate of the slope of the power functions may be obtained. Figure 5 is a log-log plot of the judgments for these three values, compared with the average values of series I (dashed lines). The functions of series II are slightly steeper than the corresponding functions of series I. This steepness may be due to the shorter range of stimuli in series II.

Within each set of nine stimuli, a single value of the parameter to be judged appeared three times, once with each of three values of another parameter. It is reasonable to ask whether a group of equal velocities, for instance, will be judged as equal if their distances and durations are different. More generally, we can ask what is the effect of varying one parameter on judgments of a second parameter when the second parameter is held constant.

Table 3 shows how the judgment of a given parameter is affected by variations in the other two parameters. Judgments of duration are affected most by variations in the other two parameters. With real duration held constant, a tenfold increase in distance (and velocity) increases apparent duration by about 45%. Judgments of velocity are not as consistently or as strongly affected as judgments of duration. A tenfold increase in distance (and duration) decreases apparent velocity by about 30%. These results generally agree with those of Mashhour (1964). Apparent distance is relatively unaffected by variations in the other parameters. This may be because all the stimuli started from the same point. Although the experimental room was kept dark, the stimulus itself gave some illumination to the surroundings. The subjects may have been judging distance in terms of the position in the field at which the stimuli disappeared. The apparent position at which a stimulus disappears may be independent of its velocity, whereas the distance travelled, with no position cues available, may be quite dependent on velocity. A better procedure might have been to start the stimulus from a different point in the field each time. However, the stimuli of longer length would have extended beyond

Fig. 5. Psychophysical functions relating subjective to physical velocity, duration and distance. The two sets of connected points on each curve represent judgments of two sets of stimuli (See Table 2). The vertical positions of the curves have been adjusted so that the bottom points overlap. This was done so that the slopes of the functions could be compared easily with the average slopes of series I, indicated by dotted lines. Each point is the geometric mean of 20 judgments.

Table 3. Percentage Change In The Subjective Judgment Of A Single Physical Value Of One Parameter As The Other Parameters Are Varied(Each item in the table is calculated from the geometric mean of 20 judgments)

Percentage Change in Judgments of Constant Duration				
Duration (sec)	Change of Distance		Change of Velocity	
	10 → 50	10 → 100	1 → 5	1 → 10
1	+52.3	+36.8	--	--
2	--	--	+38.4	+26.9
5	+66.6	+53.6	+49.3	+56.4
10	+24.3	+62.8	+42.2	+42.8
Avg.	+47.7	+51.1	+43.3	+43.0
Percentage Change in Judgments of Constant Velocity				
Velocity (deg/sec)	Change of Distance		Change of Duration	
	10 → 50	10 → 100	2 → 5	2 → 10
1	-15.8	-42.2	-25.0	-37.5
5	--	--	-27.4	-10.6
10	-18.6	-9.8	+7.7	+2.6
100	-39.1	-38.3	--	--
Avg.	-24.5	-30.1	-14.9	-15.1
Percentage Change in Judgments of Constant Distance				
Distance (deg)	Change of Velocity		Change of Duration	
	1 → 10	1 → 100	1 → 5	1 → 10
10	-9.3	+18.6	+2.6	+33.3
50	-5.8	+8.0	-8.6	-8.6
100	-1.0	-5.0	-4.2	-23.4
Avg.	-5.7	+7.3	-3.4	+0.4

the screen had they not always started at the extreme left-hand portion of the screen.

Influence of Range

In order to test the effect of stimulus range on the exponent for velocity, two additional experiments were performed with circular rather than linear motion. Circular motion was used for two reasons:

1. The experiment of Eisler and Ottander (1963), which yielded an exponent of about 1.0 for circular motion, with a stimulus range of 10:1, was one of the few experiments to use magnitude estimation, the method of scaling used in the present experiments.

2. The function relating subjective angular velocity to a wide range of physical angular velocities in this experiment can be compared to the function relating subjective linear velocity to the same range of physical linear velocities in series I.

Both experiments used magnitude estimations of the velocity of a rotating object (a half-inch wide black stripe along a diameter of a 6-inch white disk). The duration of the exposure was constant at 2 sec. The observers sat 8 feet from the disk. Ten Ss, drawn at random from a pool of 30, were used for each experiment. Each subject judged each stimulus twice. The instructions paralleled those previously presented.

In one experiment, four stimuli ranged from 15.3 to 153 rpm (92 to 920 deg/sec). This was approximately the range used by Eisler and Ottander (20 to 200 rpm).

In the other experiment, eight stimuli ranged from 3.8 to 770 rpm. Figure 6 shows the results of the two experiments.

For both exposures the straight lines, fitted to the data by the method of least squares, are steeper in slope for the short range (dotted lines) than for the long range. The difference is greater for the second exposure than for the first. The principal effect of the second exposure was to increase the slope of the short range function. As was also found in brightness scaling (Stevens & Stevens, 1960), the effects of a constricted range do not show up in the first few judgments, but become more strongly evident as the experiment proceeds.

The difference between the long and short range exponents in Fig. 6 suggests that stimulus range may be responsible (along with scaling method) for some of the differences in the power function exponents obtained by various investigators.

There appears to be general agreement between estimations of circular and linear motion. This agreement, if confirmed, can lead to a simplification of the experimental procedure. Circular motion is not limited

Fig. 6. Psychophysical functions relating subjective to physical velocity, for a short range (dotted lines) and a long range. The first and second presentations are plotted separately. The data are fairly well described by straight lines in the log-log plot. The lines fitted by least squares to the points for short range are steeper in slope than those for long range. Each point is the geometric mean of 10 judgments. The interquartile ranges are indicated by the vertical lines through the points.

in duration or distance, whereas a linearly moving stimulus is so limited. With a circularly moving stimulus, high speeds can be exposed for long durations. Adaptation and contrast effects can probably be more easily measured.

The Relation of Subjective Velocity to Subjective Distance and Duration

These, and previous, experiments have shown that subjective velocity, distance, and duration can all be related to their corresponding physical variable by equations of the form:

$$\begin{aligned}\psi_V &= k_1 (\phi_V + k_2) \beta_V \\ \psi_D &= k_3 (\phi_D + k_4) \beta_D \\ \psi_T &= k_5 (\phi_T + k_6) \beta_T\end{aligned}$$

where ψ = subjective value and ϕ = physical value. We know also the relation among the physical values:

$$\phi_V = \frac{\phi_D}{\phi_T}$$

Then, if k_2 , k_4 , and k_6 are small,

$$\psi_V^{1/\beta_V} = k_7 \frac{\psi_D^{1/\beta_D}}{\psi_T^{1/\beta_T}}$$

This is equivalent to an equation derived by Mashhour (1964, p. 83). In the present experiment and in other experiments unrelated to velocity (Stevens & Galanter, 1957), β_D and β_T have been found to be close to 0.9. If β_V is 0.75, then,

$$\psi_V = k_7 \left(\frac{\psi_D}{\psi_T} \right)^{\frac{0.75}{0.9}} = k_7 \left(\frac{\psi_D}{\psi_T} \right)^{0.8}$$

If k_7 is known, then we can predict the subjective value of any one of the three parameters from the subjective

values of the other two. Even if k_7 is unknown, the ratio of the subjective velocities of two stimuli could be predicted from the subjective durations and distances of those stimuli. Similarly, ratios of durations and distances could be predicted from subjective values of the other two parameters.

This kind of relation is noteworthy because it deals only with subjective values and depends on no physical measurements for its confirmation. In other words, without knowing the physical velocities, distances, or durations of a group of stimuli, we can nevertheless predict the judgments of one of these parameters of the stimuli from judgments of the other two.

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Note

1. Supported in part by grants from the National Science Foundation and the National Institutes of Health (Laboratory of Psychophysics Report PPR 312).