Individual brightness functions¹

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A total of 34 individual brightness functions were measured for 18 observers by two different methods. In one method the observer set various luminance levels of a white target and assigned numbers proportional to the apparent brightness of the levels set. In the other method the observer adjusted the loudness of a white noise and the luminance of a white target in order to achieve a series of cross-modality matches between loudness and brightness. Both methods gave good approximations to power functions, showing that the psychophysical power law holds for the individual perceiver.

The relation between subjective brightness ψ and the luminance of the stimulus L has been shown to have the form of the power function:

$$\psi = k (L - L_0)^\beta \quad (1)$$

where k is a proportionality constant whose value depends on the choice of units of measurement, β is the exponent, and L_0 approximates the absolute threshold (J. C. Stevens & S. S. Stevens, 1963). The power law seems to hold for any constant level of light adaptation, although the parameters of the brightness function, k , L_0 , and β , all depend upon the level of adaptation. For the dark-adapted eye the exponent is about 0.33 (cube-root relation), but as the level of adaptation increases the size of the exponent increases slightly to about 0.5 for an eye steadily adapted to 1 lambert. The threshold L_0 also rises with light adaptation, but k decreases, reflecting the fact that more stimulus energy is needed to produce a given level of subjective brightness.

Important changes in the brightness function result from other changes in the conditions of stimulation. For example, simultaneous contrast causes a sharp increase in the exponent (S. S. Stevens, 1961a), and the exponent for a very brief (e.g. 1 msec.) exposure is larger (about 0.5) than that for a longer (e.g. 1 sec.) exposure (about 0.33) (Aiba & Stevens, 1964).

The question has arisen whether the psychophysical law expressed in Equation (1) is able to describe the sensory function for the individual perceiver, as well as the average of the judgments from a group of observers. From the fact that some individual functions they obtained failed "to follow Stevens' law although averaging over observers does yield a power function," Pradhan & Hoffman (1963) concluded that the power function comes about as an "artifact" of averaging. According to their view, although the power law may provide a valid statistical description of the behavior of a group, its relation to the sensory processes of the individual organism is still in question.

Despite the importance of this question, until recently relatively few studies have made a point of reporting the behavior of the individual observer, although Onley (1961) and Onley & Boynton (1962) displayed separate plots of brightness estimates from a few observers under several conditions of stimulation. In the present investigation, interindividual differences in the judgment of brightness were the main concern. Two methods were used to measure a total of 34 individual brightness functions. One of the methods, used previously in a study of loudness (J. C. Stevens & Guirao, 1964), was essentially a combination of the methods of magnitude estimation and magnitude production. The other was a variation of the method of cross-modality matching, in which the task was to obtain matches between subjective brightness and subjective loudness.

Earlier studies

For several sensory continua there has existed published evidence that the power function holds for the individual observer.

Handgrip.

J. C. Stevens & Mack (1959) obtained 39 individual functions (published by S. S. Stevens, 1961b) that relate the magnitude of subjective effort to the force exerted on a precision hand dynamometer. Two methods were used: magnitude production, in which the observer produced forces whose subjective magnitude seemed proportional to numbers called out by the experimenter, and magnitude estimation, in which the observer assigned numbers in proportion to the subjective magnitude produced by exerting forces of various degrees. Both methods gave good approximations to power functions for almost every subject. The method of magnitude production tended to give slightly larger exponents than the method of magnitude estimation, and the exponent also varied in size from one observer to another.

Loudness of a vowel.

Using the method of magnitude estimation, Cross & Lane (1962) obtained 23 separate loudness functions, almost all of which seem to obey the power law, as judged by visual inspection. The largest exponent was about twice the size of the smallest.

Loudness of white noise.

The method of magnitude production was used by Schneider & Lane (1963) in a study in which 10 functions were generated by a single observer. In five sessions the instructions were to judge loudness, and in another five sessions to judge the inverse, softness. Good fits to power functions were obtained, with a slight variation in the size of the exponent from session to session. The scale of softness was shown to be the reciprocal of the loudness scale.

Pure tone.

J. C. Stevens & Guirao (1964) measured 22 loudness functions, two from each of 11 observers. The method used combined features of magnitude estimation and magnitude production, in that the task was to set various levels of the stimulus and to assign numbers in proportion to the subjective magnitudes of the levels set. Since the estimates were plotted as a function of the sound pressure levels set, without any averaging of either the estimates or the levels, the form of the function was independent of averaging. Power functions were obtained on virtually every series, and the variability within the judgments of a given observer was fairly low, although there was substantial variation in the size of the exponent from one person to another. Approximately the same results were obtained from an additional five observers when the method of magnitude production was used.

The production-estimation procedure, which was first used in this laboratory by Lee McMahon, offers the advantages of speed and relative lack of constraints on the observer, who is free to choose both the stimulus level and the number to stand for its apparent intensity. The apparent success of this method in measuring the loudness function provided an incentive to use it in the present study for the scaling of brightness.

Roughness.

In a study by Ekman, Hosman, & Lindström (1964) on the relation between tactual roughness and the coefficients of friction of a set of paper surfaces, the observer's task was to estimate the ratio of subjective roughness (or smoothness) for each of the possible pairs of seven surfaces. For 10 observers individual scales were obtained for roughness and for smoothness. Each of the 20 scales approximated a power function, although the exponent was said to vary greatly from one observer to another. Smoothness was found to be the inverse of roughness for the individual observer.

Taste.

Eight individual functions for the saltiness of sodium chloride solutions and seven individual functions for the sweetness of sucrose solutions were measured by Ekman & Åkesson (1964). Of the two equations that have been proposed to describe the downturn that occurs near threshold when the psychophysical function is plotted in log-log coordinates, $y = a + cx^\beta$ apparently fitted the data of each observer better than did the equation $y = c(x - a)^\beta$. The exponent varied from 1.11 to 1.97 for saltiness and from 1.43 to 1.98 for sweetness.

Apparent force.

Of eight observers studied intensively by Bernyer (1962), most apparently did, though some apparently did not, yield good approximations to power functions when the task was to estimate the force exerted on a lever. It is worth noting that the more prominent deviations usually occurred in the vicinity of the standard stimulus, which was at the top, bottom, or in the middle of the stimulus range. The potentially disruptive in-

Fig. 1. The principal parts of the visual apparatus (proportions not exact).

fluence of the location of the standard on the slope and the form of the loudness function has been investigated intensively in two laboratories (S. S. Stevens, 1956b; Hellman & Zwislowski, 1961).

Other studies.

Interindividual differences in the exponent were studied by McGill for loudness (1960), and by Jones & Marcus for taste, smell, and lifted weight (1961). Jones and Marcus reported that a given observer may give consistently high or low exponents on different sensory continua when judgments of all three continua were made in the same session. This finding suggests that the spread of the exponents among observers can perhaps best be understood as a reflection of differences in the way people assign numbers to sensory magnitudes, rather than differences in the way their sense organs function. According to this view, although the power function can be shown to hold for the individual perceiver, the fact that observers adopt various quantitative criteria when called upon to estimate sensory magnitude makes it advisable to average a group of observers in order to ascertain a reasonably representative value of the exponent.

Procedures

Figure 1 is a diagram of the main apparatus used in each of four experiments. The target was a white disk (diameter, 2 cm) made of milk Plexiglas and trans-illuminated by the light from a 300-watt projector. Between the observer's eye and the target were two circular 40-db neutral density wedges, which were mounted on the same shaft. Rotation of the shaft permitted a continuous variation in the luminance of approximately 80 db. This range was occasionally extended by means of neutral density filters. Calibra-

tions were made with a Pritchard photometer and a MacBeth illuminometer.

A timer triggered a shutter once every 8 sec. for a brief (0.3 or 0.75 sec.) exposure of the target. The purpose was to help maintain a fairly uniform state of dark adaptation by allowing the eye to recover sensitivity after each exposure. Before a session the observer wore red goggles for 10 min. and then spent the first 1 or 2 min. of the session observing several stimuli before beginning the actual task. The observer was instructed to view the target foveally with the right eye and between exposures to look at a red fixation spot that appeared near the center of the target location. The fixation light was off during exposures.

Experiment 1.

The distance from the observer to the target was 47 cm, and the angle of subtense was 2.5° . The exposure time was 0.3 sec. The observer read the following instructions:

The task is to assign numbers in proportion to the subjective brightness of a light, whose level is controlled by a knob that is turned with the right hand. The light will be on briefly, then off for a longer period, then on again briefly, etc., in a continuous cycle. During the off-phase, a tiny red light will appear, which is to be looked at directly all the time. The chin is kept in the chinrest.

For a minute or two at the start, turn the knob up and down a couple of times to observe the change in brightness and to get acquainted with the apparatus.

When you are ready, adjust the brightness to some level and assign a number to stand for its subjective brightness. Thereafter, adjust the brightness to various levels and assign numbers proportional to the subjective brightness.

It is inadvisable to turn the knob *directly* from the highest to the lowest positions.

The final admonition was an attempt, not altogether successful, to avoid large changes in the absolute threshold and in the brightness of low levels of luminance.

The production-estimation procedure, in which the observer both adjusts the stimulus levels and emits the numerical estimates, permits a variety of strategies on the part of the observer. Some observers seem to adopt an "estimation" strategy: they tend to set the luminance and then assign a number to its brightness; others prefer a "production" strategy: they tend to vary the luminance until the brightness matches a number previously selected. Most observers, however, appear to combine the two strategies. It is noteworthy, nevertheless, that the delay of about 8 sec. between target exposures may favor "estimation" over "production," because the task can be completed faster by estimating almost every level set, rather than by waiting out the several exposures necessary to adjust the brightness to the desired level.

Each of nine observers made 30 settings and estimates. Sessions rarely took longer than 20 min. (about 150 stimulus exposures).

Experiment 2.

The procedure was the same as Experiment 1, except that the exposure was 0.75 sec. instead of 0.3 sec. This difference in exposure duration turned out to have little effect. Nine observers took part.

Experiment 3.

The procedure was the same as Experiment 2, except that an attempt was made to lower the absolute threshold by increasing the visual angle of the target from 2.5° to 4° . This was done by placing a magnifying lens of 8 diopters at a distance of 6.5 cm in front of the eye. Six observers took part.

Experiment 4.

The task was cross-modality matching of loudness to brightness and vice versa. The visual apparatus was modified slightly: because of its noisiness, the shutter was eliminated and the lamp was lighted by applying the voltage for 0.75 sec. every 10 sec. The angular subtense was 4° .

The auditory stimulus was a band of noise (75 to 4800 cps) whose level could be varied by the observer by means of a sone potentiometer (see J. C. Stevens & Guirao, 1964, for the circuit diagram and input-output characteristic of the sone potentiometer). The voltage across the earphones (PDR-8) was monitored by means of a Ballantine model 320 root-mean-square voltmeter. A voltage amplifier was occasionally needed for the measurement of very low settings of the noise. The sone potentiometer permitted an essentially continuous variation of about 100 db in the sound pressure level.

The observer adjusted the levels of the light and the noise in order to achieve 30 matching pairs (except for one observer who gave 26 pairs). The visual target appeared automatically every 10 sec., but the noise could be heard at any time and for any duration by pressing a switch. This difference may have influenced the outcome, because the easiest strategy was to adjust the noise, which could be heard at will, to match a given level of the light, and then quickly to reset the light to a new level and again adjust the noise to match the light. In effect, the observer may have tried to match the sound to the light, rather than vice versa. This introduces a possible bias, for the regression tendency, which acts to shorten the range of whatever stimulus is adjusted, occurs both in the method of cross-modality matching and in other psychophysical procedures (S. S. Stevens, 1956a, 1958). Despite the potential bias, the procedure was deemed suitable for the purpose of generating quickly the individual cross-modality functions. Sessions lasted from about 15 to 30 min., and 10 individual functions were obtained.

Observers.

No special criteria were used in the selection of the observers. Both authors (LM and JCS) participated as observers, and the other observers included graduate

Table 1. The constants of the individual brightness functions, $\psi = k(L - L_0)^\beta$. L_0 is in millilamberts; db_{10} is the luminance in decibels re 10^{-10} lambert for $\psi = 10$. For Experiment 4, β/a is the slope of the cross-modality function and β^* is the calculated exponent for brightness.

Observer	Experiment 1				Experiment 2				Experiment 3				Experiment 4		
	k	L_0	db_{10}	β	k	L_0	db_{10}	β	k	L_0	db_{10}	β	L_0	β/a	β^*
RB	16	0.00028	61	0.24											
LD	11	0.0012	68	0.25	15	0.0012	63	0.29							
JCS	7.7	0.00025	74	0.29									0.000090	1.18	0.40
DP	8.1	0.00035	73	0.30					7.0	0.000072	76	0.32			
PK	13	0.00012	63	0.31	2.5	0.00023	82	0.53	36	0.00010	46	0.24	0.000014	1.04	0.34
WS	7.5	0.0025	74	0.35	4.8	0.0072	78	0.38					0.0080	0.74	0.24
PH	13	0.00058	67	0.37									0.00010	0.80	0.26
SS	6.5	0.0020	74	0.37					8.0	0.00012	74	0.33			
LM	5.5	0.0075	77	0.37	12	0.00014	68	0.34	4.4	0.000050	71	0.34			
SW					9.4	0.0052	72	0.21							
TI					17	0.012	62	0.31							
SHC					16	0.00010	64	0.33					0.00012	0.70	0.23
RF					12	0.00025	67	0.38	10.5	-- ^a	80	0.33			
AN					9.3	0.00040	71	0.38	4.3	0.00040	78	0.38	0.00050	0.84	0.28
WD													0.00035	0.72	0.24
GZ													0.00010	0.78	0.26
JS													-- ^a	0.94	0.31
HR													-- ^a	1.00	0.33
Mean			70	0.32			70	0.35			71	0.32			0.29
Standard deviation			5.2	0.048			6.4	0.082			11.5	0.042			0.052

^a Unable to estimate

and undergraduate students, secretaries, and two staff members of the laboratory (TI and LD). Of the observers, 7 served in only one experiment, 7 served in two, 3 served in three, and 1 in all four experiments.

Results

The brightness functions obtained by the production-estimation procedure are plotted in Fig. 2 (2.5° field, 0.3 sec. duration), Fig. 3 (2.5° field, 0.75 sec. duration), and Fig. 4 (4° field, 0.75 sec. duration). The numerical estimates of brightness are plotted on the logarithmic ordinate, the luminances on the logarithmic abscissa, where each subdivision stands for 20 db. On each curve the arrow marks the value corresponding to 70 db re 10^{-10} lambert, which is equivalent to 1 millilambert.

Every judgment made has been plotted in Figs. 2-4, and no averaging has been performed, either of the

luminance settings or of the estimates of apparent brightness.

The functions fitted to the data have the equation:

$$\log \psi = \beta \log (L - L_0) + \log k \quad (2)$$

which is the logarithmic form of Equation (1). These functions approximate straight lines in log-log coordinates but depart sharply from linearity as the threshold value L_0 is approached. (If luminance were measured beginning at threshold, the functions would be straight throughout their entire length.) Most of the observers made settings near enough to the threshold for the need for the constant L_0 to become clearly evident. The same constant is needed in the power functions for the other sense modalities (J. C. Stevens & S. S. Stevens, 1961), although in practice the need does not usually manifest

Fig. 2. Nine individual brightness functions for a 2.5° target exposed for 0.3 sec. The abscissa is a decibel scale on which each subdivision stands for a 20-db interval of luminance. The arrow stands for 70 db re 10^{-10} lambert.

Fig. 3. Nine individual brightness functions for a 2.5° target exposed for 0.75 sec. Abscissa and arrow same as in Fig. 2.

itself unless the stimulus range extends to the vicinity of the threshold.

Table 1 lists for each experiment the values of the exponent β , the threshold constant L_0 , and the proportionality constant k when L is measured in millilamberts. These three constants completely define the power function fitted to each observer's data.

The power functions themselves were fitted by eye. The slope (exponent) and intercept (defined here as the ordinate corresponding to 1 millilambert) are usually fairly easy to determine visually to the required degree of precision. The values of L_0 , on the other hand, involve considerable guesswork and should be considered only approximations.

Variation in the threshold.

The absolute threshold was not actually measured in these experiments, and it is worth noting that the actual threshold undoubtedly shifted continually in the course

of a session as the eye was exposed to various levels of the stimulus. The delay between exposures, designed to stabilize the sensitivity, cannot eliminate all fluctuations of the threshold. Small changes in the level of adaptation of the eye may cause a substantial threshold shift without appreciable influence on moderate and high levels of brightness (J. C. Stevens & S. S. Stevens, 1963). It is thus not surprising that the functions are often unstable at the low end, or that it is difficult to know exactly how to measure the "effective" or "average" threshold in these experiments. Our chief interest in the constant L_0 is to show that Equation (1) can describe the data reasonably well.

The estimated values of L_0 ranged from 31 to 49 db in Experiment 1, from 30 to 51 db in Experiment 2, and from 27 to 36 db in Experiment 3 (not including observer RF, who failed to show any departure from linearity down to the lowest level set, 29 db). This rather large

Fig. 4. Six individual brightness functions for a 4° target exposed for 0.75 sec. Abscissa and arrow same as in Fig. 2.

variation probably stems not so much from physiological differences in sensitivity as from the various ways in which the observers could approach the task. Some observers tended to adjust the luminance in large steps and occasionally from high levels directly to low ones. More typically, however, the observer proceeded more systematically up and down the continuum in smaller steps. Another potential source of variation in the threshold was the failure to maintain a constant fixation. When working at low levels it is almost impossible always to resist viewing the target slightly off center. A 2° shift in fixation, for example, could lower the threshold by 10 db or more (Hecht, Haig, & Wald, 1935).

The size of the stimulus also influences the threshold. In Experiment 3, which used a 4° target, the functions tend to turn down at lower levels than do the functions for the 2.5° target. No systematic differences in any of the constants of the brightness function seemed to result from changing the stimulus duration from 0.75 to 0.3 sec.

Variation in the intercept.

The variations in the position or intercept of the functions are reflected in the values of k in Table 1, which, except for the minute contribution of L_0 , are equal to the ordinate values that correspond to 1 millilambert. Table 1 also gives the luminance in decibels that is necessary to produce an estimate of "10." Although intercept differences do not by themselves reveal anything about the growth of brightness with stimulus intensity, it is interesting to note that medium levels of brightness (in the vicinity of 65 to 75 db) tend to evoke roughly the same numerical response; in other words, observers tend to choose approximately the same "modulus" for judging subjective magnitudes when they are left free to do so. This suggests that the imposition of an arbitrary standard on the observers is a potential source of bias, and that a standard chosen by the experimenter may hinder more than help the observer. A decade of experience with magnitude estimation suggests that the experimenter should studiously avoid constraining the observer in any way other than to direct his attention to the property of the stimulus that is of interest. The estimation-production procedure represents an effort to reduce the extraneous constraints.

Variation in the exponent.

The variability in the exponents in Table 1 is fairly typical of the spread encountered in earlier studies of individual psychophysical functions. The variability among the exponents is a more salient feature than either the random or systematic deviations from the power functions. The mean and median exponent of the 24 functions of Figs. 2-4 is 0.33, that is to say, for the average observer the cube-root relation is confirmed, with a standard deviation of 0.064. This outcome emphasizes the need to employ a group of observers in psychophysical research, since there is a good chance that a given observer's results, however closely they may

conform to a power function, may not be representative of the average exponent. On the other hand, the average exponent from a group of about 10 observers has proved to be quite stable and reproducible.

There are at least two ways to interpret the variation among the exponents. The first is to assume that there exist large functional differences in the sensory systems of individual organisms. The second is to assume that people adopt different quantitative criteria when they attempt to judge the relative magnitudes of sensory impressions. Observers may disagree considerably when asked to judge the ratio of subjective magnitudes produced by a given pair of stimuli, but any single observer in a given experiment may apply his own criterion consistently. On the other hand, an observer may or may not change his criterion from one session to another. Thus, of the two observers who served in Experiments 1-3, LM gave practically the same exponent each time, but PK gave exponents that differ almost as much from one another as do the exponents of any two observers. It is easier to believe that what changed from one session to another was PK's quantitative criterion, rather than the physiological components of his brightness vision.

Similar changes in the exponent of the individual observer occurred in a study of loudness (J. C. Stevens & Guirao, 1964). Each of 11 observers was tested twice, with an interval of 1 to 6 months between the tests. The product-moment coefficient of correlation between the two sets of exponents was 0.53, which indicates that sizable changes occur quite commonly, although some observers may continue to use the same criterion for a long period of time.

Cross-Modality Functions

Cross-modality matching has proved a useful technique for demonstrating that sensory magnitude ψ grows as a power function of stimulus magnitude ϕ . For any pair of sensory continua a and b the psychophysical equations may be written as $\psi_a = k_a \phi_a^\alpha$ and $\psi_b = k_b \phi_b^\beta$. The physical magnitudes that produce matching sensations ($\psi_a = \psi_b$) at various levels should be related by the cross-modality function $k_a \phi_a^\alpha = k_b \phi_b^\beta$, which, given the appropriate choice of units, can also be written $\phi_a = k \phi_b^{\beta/\alpha}$. Thus the psychophysical power law predicts that the equal-sensation function is a power function whose exponent (slope in log-log coordinates) equals the ratio of the exponents governing the growth of sensory magnitude on the two continua that are matched. For the mathematics of the argument, see Luce & Galanter (1963). The predicted result has been closely approximated in many experiments (examples can be found in S. S. Stevens, 1959; J. C. Stevens, Mack, and S. S. Stevens, 1960; S. S. Stevens & Harris, 1962; S. S. Stevens & Guirao, 1963).

The equal-sensation function for matches between loudness and brightness was shown to have an exponent (slope) of approximately 1.0 (J. C. Stevens & Marks, 1965). This result would be expected if the exponents of the brightness function and the loudness function were

Fig. 5. Ten cross-modality functions for the matching of brightness and loudness. The abscissa is a decibel scale on which each subdivision stands for 20 db. The arrow stands for 70 db re 10^{-10} lambert. The ordinate is a decibel scale of sound pressure level. This scale is directly proportional to a decibel scale of sound energy.

both 0.33. Although the exponent for the loudness of a noise band is not known with precision, its value seems to lie between 0.3 and 0.35 when the stimulus is measured in terms of sound energy² (Harper & S. S. Stevens, 1964). Thus, for group data the cross-modality functions were in essential agreement with the power functions for brightness and loudness determined independently. The present experiment was an attempt to show that the predicted cross-modality functions can also be generated by the individual observer.

The results appear in Fig. 5 and in Table 1. The slopes (β/a) ranged from 0.7 to 1.2 with an average of 0.9—a little smaller than the earlier average of nearly 1.0. This discrepancy may have resulted from the somewhat asymmetrical procedure, which tempted the observer to adjust the sound to match the light and not vice versa. For the group results reported earlier, the exponent was slightly smaller than 1.0 when the sound was adjusted, and slightly larger than 1.0 when the light was adjusted. Thus an exponent slightly smaller than 1.0 is probably to be expected under a procedure that permits the observer to hear the noise at will but to view the light only once every 10 sec.

On the assumption that the exponent for the loudness of the white noise is 0.33, it is possible to compute the individual values of the exponents β^* for brightness. They are given in Table 1. The average, 0.29, is a little lower than the average (0.33) obtained by the production-estimation procedure.

The ten cross-modality functions of Fig. 5 tell very

much the same story as do the brightness functions of Figs. 2-4. Each appears to obey the power law and to have approximately (but not precisely) the slope found for results averaged for a group of observers. The influence of the visual threshold is also apparent in the sharp downturn near the low end of some of the functions. **Is the Power Function an "Artifact"?**

The recent assertion of Pradhan & Hoffman (1963) that the power function does not hold for the individual observer, but in some way manages again and again to appear in the averaged results, seems to be no more true for brightness than for the several continua studied earlier. The present results bring to more than 150 the number of published psychophysical functions for single observers, and the methods used include magnitude estimation, magnitude production, the combined production-estimation procedure, ratio estimation, and cross-modality matching. The over-all evidence suggests that the power function holds for a large percentage of the individual observers.

The generality of the power function raises the interesting possibility of searching for stable departures under unusual conditions, such as sensory pathology, by means of a battery of psychophysical procedures. When obtained by one procedure in a single session, apparent departures from the power law may be regarded with skepticism. It would seem desirable to try the other available psychophysical methods before concluding that a given individual function is a valid exception to the generality of the power law.

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Notes

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- The function for loudness could be written as $L = kE^\beta$ or as $L = kP^{2\beta}$, where L is loudness, E is sound energy, and P is sound pressure. This is true because $E = kP^2$. A decibel difference is likewise defined either as $10 \log E_1/E_2$ or as $20 \log P_1/P_2$. Thus the exponent for the loudness of 1000-cps tone (the so-called sone function) is sometimes written as 0.3 (sound energy) and sometimes as 0.6 (sound pressure).