RANCOVA: A Minitab macro for the calculation of nonparametric (ranked) analysis of covariance

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Although Minitab provides an excellent medium for teaching basic statistical procedures, it has a number of deficiencies. Recently, Brown (1984) and others (Dillard, 1984; Lehman, 1984; Lindsay, 1984; Papangelou, 1984) have attempted to fill these gaps with Minitab macro routines, but many deficiencies still remain.

Release 5.1 of Minitab is much enhanced, but one area that requires some support is the nonparametric section. Although Minitab supports the typical routines (e.g., Mann-Whitney, Kruskal-Wallis, etc.), more sophisticated nonparametric procedures are becoming more frequently needed (Conover & Iman, 1981). In this paper, I address the specific area of nonparametric analysis of covariance, using the popularized rank transformation procedures (Conover, 1980; Conover & Iman, 1981).

Although numerous procedures exist for the calculation of nonparametric analysis of covariance (see Olejnik & Algina, 1985, for an excellent review), the procedure proposed by Quade (1967) seems to be one of the most powerful, statistically speaking (Huitema, 1980). The Minitab macro presented here is based on Quade's (1967) procedure.

Procedures. Quade (1967) proposed the use of ordinary least squares regression on mean adjusted rank transformed data for the calculation of the covariance analysis. Let matrix R_{ij} be an $N \times 1$ corrected ranks (mean adjusted) matrix of responses, and C_{ij} be an $N \times p$ matrix of corrected ranks (mean adjusted) on concomitant variates (p = number of covariates). With a fitted ranks matrix,

$$\hat{R}_{ij} = C_{ij}[C'_{ij} C_{ij}]^{-1} C'_{ji} R_{ij}. \quad (1)$$

With the residuals from the ranked regression equal to:

$$Z_{ij} = R_{ij} - \hat{R}_{ij}. \quad (2)$$

A variance ratio (VR) approximating an F-distribution with $(K-1, N-K)$ degrees of freedom is as follows:

$$VR = \frac{(N-K) \sum_i \left(\sum_j Z_{ij} \right)^2 / n_i}{(K-1) \left[\sum_i \sum_j Z_{ij}^2 - \sum_i \left(\sum_j Z_{ij} \right)^2 / n_i \right]}. \quad (3)$$

A multiple comparison test proposed by Huitema (1980), analogous to the protected least significant difference (LSD) procedure used for parametric analysis of covariance, is also incorporated into the Minitab macro. This test evaluates all possible transformed mean rank differences. The test statistic is:

$$\delta_{ij} = (\bar{Y}_{R_i} - \bar{Y}_{R_j}) / \sqrt{[(1/n_i) + (1/n_j)]MSE}, \quad (4)$$

where δ_{ij} approximates a t-distribution with $(N-J)$ degrees of freedom.

Description. The Minitab macro outputs the mean adjusted rank transformations, the rank adjusted means, a variance ratio, and multiple ranked mean comparison tests. Because the user is operating in the Minitab environment, various tests for assumptions may easily be carried out. For example, the relationship between the dependent variable and the covariate(s) is assumed to be monotonic. This may be easily plotted in Minitab by referencing the rank transformation variables that are maintained from the interactive session. Also, the assumption of identical covariate distributions can be tested using Minitab's existing Kruskal-Wallis procedure.

The nonparametric RANCOVA macro is driven by a Digital command language (Digital Equipment Corporation, 1982) routine for VAX/VMS operating systems. This routine offers the user the options of either batch or interactive sessions, with permanent file storage of RANCOVA results for later printing. Although the Digital command language routine was designed for the macro, it is not necessary for its use.

Table 1
Example Dataset* with Example of Minitab Entry

Sample 1			Sample 2			Sample 3		
Y	X1	X2	Y	X1	X2	Y	X1	X2
16	26	12	44	21	17	17	1	8
60	10	21	67	28	2	28	19	1
82	42	24	87	5	40	105	41	9
126	49	29	100	12	38	149	48	28
137	55	34	142	58	36	160	35	16
			C1	C2	C3			
			16	26	12			
			60	10	21			
			82	42	24			
			126	49	29			
			137	55	34			
			44	21	17			
			67	28	2			
			87	5	40			
			100	12	38			
			142	58	36			
			17	1	8			
			28	19	1			
			105	41	9			
			149	48	28			
			160	35	16			

*Data from Quade (1967).

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Table 2
Example of RANCOVA Output

```
MTB > RETRIEVE 'DATAFILE'
MTB > LET K1=3
MTB > LET K2=2
MTB > EXEC 'RANCOVA'
MTB > #
MTB > #
MTB > #      | A N A L Y S I S   O F   C O V A R I A N C E |
MTB > #      | MULTIPLE COVARIATES           RANK PROCEDURE |
MTB > #      |-----|
MTB > #
MTB > #              << PLEASE WAIT >>
MTB > #
MTB > ((/25X,28(A1)//12X,8(A1),15X,4A1,2(6X,4A1),8X,2A1/3(10X,&
MTB >
MTB >
MTB > 17(A1),5X,F8.2,5X,F3.0,4X,F8.2,3X,F8.2/))
MTB >
```

Rank Analysis of Covariance

Source	SS	df	MS	F
Treatment	50.49	2.	25.24	4.73
Error term	64.03	12.	5.34	*****
Total residual	114.52	37.	*****	*****

```
MTB > #
MTB > #      MULTIPLE COMPARISON TEST MATRIX ON MEAN RANK DEVIATION
MTB > #      [USE T-DISTRIBUTION WITH N-J DEGREES OF FREEDOM]
MTB > #
MTB > PRINT C45 C40 M9
MATRIX M9
```

1.00000	0.00000	0.00000	0.00000	0.00000
2.00000	1.00000	1.26767	0.00000	0.00000
3.00000	2.00000	1.79335	1.00000	3.06102

ROW	T-GROUPS	MEAN DEV
1	1	-2.108
2	2	-0.256
3	3	2.364

δ test

```
MTB > #
MTB > #      RANK DEVIATION SCORES PER DEPENDENT VARIABLE AND COVARIATES
MTB > #
MTB > PRINT C1-CK45
ROW DEP VAR COVS: C2 C3
```

1	-7	-1	-3
2	-3	-5	0
3	-1	3	1
4	3	5	3
5	4	6	4
6	-4	-2	-1
7	-2	0	-6
8	0	-6	7
9	1	-4	6
10	5	7	5
11	-6	-7	-5
12	-5	-3	-7
13	2	2	-4
14	6	4	2
15	7	1	-2

Input. The macro requires the user to enter data directly into Minitab using either READ statements or file read or retrieval. The user only specifies two parameters: K1, the number of treatment levels, and K2, the number of covariates. Data must enter Minitab sequentially in columns C1 through Cn, with column C1 being the dependent variable. (Table 1 provides an example of data correctly entered into Minitab.)

Output. Depending upon the selection made in the Digital command language file, output is either directed to the display or to a file. Table 2 is an example of an interactive session with results directed to the display. This example uses the data from Table 1.

Limitations and Requirements. The Minitab macro provides for calculation of a maximum of 15 covariates with up to 20 treatment levels. Although the macro has been tested only on a VAX 11/780 system, it should work with little trouble on all other systems, including Minitab for the IBM/PC. Execution times will vary contingent upon the number of covariates, size of the sample, and size of the computing system. The VAX/VMS command file may be used only at installations supporting Digital VAX/VMS operating systems.

Availability. A source listing of the Minitab nonparametric RANCOVA macro, the VAX/VMS command file,

and a brief user's guide are available without charge from the author.

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