Notes and Comment

The method of transitions

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There are special cases where the experimenter cannot avoid using the method of constant stimuli: studies on short term memory of perceptual material are clear examples. For instance, pairs of sounds are presented successively, and the subject is asked to report whether or not the second sound (the variable stimulus, V) is louder than the first (the standard stimulus, S). In this kind of experiment, it is concluded that a memory effect has occurred when the point of subjective equality (PSE) is statistically different from the S. However, this conclusion will be unwarranted when the obtained difference is biased, due to, for example, adaptation level phenomena or the physiological habituation of sensory receptors (Guilford, 1954; Woodworth & Schlosberg, 1955). These phenomena produce biases whose magnitude increases as the number of stimulus presentations increases. In order to abolish or, at least, to reduce their magnitude, the number of stimuli shown to the observer must be minimized. This minimum may be a single response per V for each observer. In this case, it is necessary to calculate the individual PSEs in order to get the information essential for the application of statistical techniques, such as analysis of variance. This requirement may be satisfied by the use of the method of transitions (Masin & Cavedon, 1980; Masin & Vidotto, 1982; Vidotto, 1983).

A typical record of data obtained by the constant stimulus procedure is shown in Table 1. A "1" means that the perceptual correlate of the stimulus value was detected by the observer (in the case of a determination of a threshold), or that the perceptual correlate of V was judged to be greater than the perceptual correlate of S (in the case of a determination of a PSE). A "0" means a failure to detect V, or that V was judged to be less than S. In Table 1, there are 5 Vs (arbitrary unit), each of which was presented randomly once to each of 10 observers. At the bottom of Table 1, the obtained proportions of 1s are reported. (The terms "threshold" and "PSE" are considered interchangeable here.)

A transition is defined as the midpoint between two contiguous Vs, where a change of response from 0 to 1, or from 1 to 0, occurs. For example, the transitions for Observer 1 in Table 1 are 5.5, 6.5, and 7.5. It is assumed that, as in the method of limits, a transition is an estimate of a momentary threshold.

Let us now suppose that the Vs are infinite in number, and that the difference between any two contiguous Vs

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Table 1
Example of Record of Data from the Method of Constant Stimuli and Individual Thresholds by the Method of Transitions

Observer Number	Variable Stimuli (Arbitrary Unit)					Individual Threshold
	4	5	6	7	8	
1	0	0	1	0	1	6.54
2	0	0	0	1	1	6.63
3	0	0	0	1	1	6.63
4	0	0	0	1	0	7.51
5	0	0	1	0	1	6.54
6	0	0	1	1	0	7.18
7	0	0	1	0	1	6.54
8	0	1	0	1	0	6.51
9	0	0	0	1	1	6.63
10	0	0	0	0	1	7.58
p*	.0	.1	.4	.6	.7	
Mean						6.83

Note—A 1 indicates a detection, a 0 indicates a nondetection, and p* indicates the obtained proportion of 1s.

is a constant different from 0. Given two contiguous Vs, X_i and X_{i+1} , there are four possible ways in which 0s and 1s combine, that is, 00, 11, 01, and 10. Let the probability of a 1, for a given V, be p_i (the probability of a 0 is then $q_i = 1 - p_i$). A transition is defined as

$$T_i = \frac{X_i + X_{i+1}}{2}$$

A transition occurs only when a 1 combines with a 0. It follows that the probability of a transition is

$$p_i q_{i+1} + p_{i+1} q_i$$

The mean, τ , and the standard deviation, σ_T , of the theoretical frequency distribution of transitions are

$$\tau = \frac{\sum_{-\infty}^{+\infty} (p_i q_{i+1} + p_{i+1} q_i) T_i}{\sum_{-\infty}^{+\infty} (p_i q_{i+1} + p_{i+1} q_i)} \quad (1)$$

and

$$\sigma_T = \left[\frac{\sum_{-\infty}^{+\infty} (p_i q_{i+1} + p_{i+1} q_i) (T_i - \tau)^2}{\sum_{-\infty}^{+\infty} (p_i q_{i+1} + p_{i+1} q_i)} \right]^{1/2} \quad (2)$$

If the obtained proportions of 1s and 0s (Table 1) are substituted, respectively, for the p s and q s in Equations 1 and

2 and the summation is calculated over the five experimental Vs, the two equations give, respectively, the mean and standard deviation of the distribution of transitions relative to the observed ps.

Let us assume that the psychometric function is the integral of a normal distribution function $N(\lambda, \sigma_L^2)$, where λ and σ_L are the mean and the standard deviation: This is the so-called phi-gamma hypothesis (Bock & Jones, 1968; Urban, 1909/1910). The theoretical threshold is defined as the point on the stimulus continuum coinciding with λ . It may be shown that there exists one unique $N(\lambda, \sigma_L^2)$ for any given couple of τ and σ_T (Masin & Vidotto, 1982).

Let the experimental Vs be $X_1, X_2, \dots, X_n$. Let us suppose that dummy Vs extend, on the left ($\dots X_{-1}, X_0$) and on the right ($X_{n+1}, X_{n+2}, \dots$) of the experimental Vs, from $\lambda - n\sigma_L$ to $\lambda + n\sigma_L$ (a large n simulates an extension to the infinite). Let all Vs, experimental and dummy, be equally spaced. Defining initially an arbitrary $N(\lambda, \sigma_L^2)$, the ps and qs for each dummy and experimental V are obtained. Using these ps and qs, a provisional τ and σ_T may be computed, respectively, by Equations 1 and 2. Using these ps and qs for the dummy Vs, and the obtained proportions of 1s and 0s (Table 1) for the experimental Vs, Equations 1 and 2 give the estimates $\bar{T}$ and s_T . An iterative computer search may be performed to detect the $N(\lambda, \sigma_L^2)$ for which $\tau = \bar{T}$ and $\sigma_T = s_T$. The $\tau = \bar{T}$ so obtained is an estimate of the mean threshold for the 10 observers. The integral of the $N(\lambda, \sigma_L^2)$ so obtained is an estimate of the psychometric function.

Using the ps and qs obtained from the estimated $N(\lambda, \sigma_L^2)$ for the dummy Vs, and the 1s and 0s in Table 1 as individual estimates of ps and qs for the experimental Vs (1 means that the perceptual correlate of X_i was present with probability 1, and 0 means that the correlate was present with probability 0, that is, that it was not present), the individual thresholds may be obtained from Equation 1.¹

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NOTE

1. A copy of the listing of a computer program based on the algorithm described in the appendix is available from the authors on request. The program was written in FORTRAN IV and implemented on an IBM 370/158 computer.

APPENDIX

(1) Given the experimental Vs, $X_1, X_2, \dots, X_n$, and the observers' responses (the 0s and the 1s), the individual transitions are computed. For example, the transitions for Observer 6 are 5.5 and 7.5.

(2) The obtained proportions of the 1s, $p_1^*, p_2^*, \dots, p_n^*$, are computed.

(3) The mean, $\bar{T}^*$, and the standard deviation, s_T^* , of the sample of the individual transitions are computed. In our example (Table 1), $\bar{T}^* = 6.5$.

(4) A normal density function $N(\lambda, \sigma_L^2)$ is defined in which $\lambda = \bar{T}^*$ and $\sigma_L = s_T^*$.

(5) Given the equally spaced experimental Vs, $X_1, X_2, \dots, X_n$, such that $k = X_{i+1} - X_i$ with $X_{i+1} > X_i$, the dummy Vs are obtained by subtracting (adding) iteratively k from X_1 (to X_n) until the last-obtained dummy Vs is less (greater) than $\lambda - 5\sigma_L$ ($\lambda + 5\sigma_L$). Practically, the range $\lambda \pm 5\sigma_L$ is equivalent to the range $\pm \infty$.

(6) For each dummy and experimental V, the z scores $z_i = (\lambda - X_i)/\sigma_L$ are computed. Using the standardized normal density function $N(0, 1)$, each z_i is transformed into the theoretical probability p_i .

(7) Using these p_i s in Equations 1 and 2, τ and σ_T are computed over the dummy and experimental Vs in the range $\lambda \pm 5\sigma_L$.

(8) The obtained proportions, $p_1^*, p_2^*, \dots, p_n^*$, are substituted for the corresponding theoretical probabilities, $p_1, p_2, \dots, p_n$, and Equations 1 and 2 are again applied, obtaining $\bar{T}$ and s_T , respectively.

(9) The differences $\Delta T = |\tau - \bar{T}|$ and $\Delta S = |\sigma_T - s_T|$ are computed.

(10) If $\Delta T + \Delta S \leq \epsilon\sigma_L$, where ϵ is the desired approximation (which may be made equal to .00001), then the integral of $N(\lambda, \sigma_L^2)$ is an estimate of the theoretical psychometric function. If $\Delta T + \Delta S > \epsilon\sigma_L$, then the following iterative search procedure may be executed.

(11a) Let us consider nine normal distributions with the following parameters:

1. λ, σ_L	4. $\lambda(1+C_L), \sigma_L$	7. $\lambda/(1+C_L), \sigma_L$
2. $\lambda, \sigma_L(1+C_S)$	5. $\lambda(1+C_L), \sigma_L(1+C_S)$	8. $\lambda/(1+C_L), \sigma_L(1+C_S)$
3. $\lambda, \sigma_L/(1+C_S)$	6. $\lambda(1+C_L), \sigma_L/(1+C_S)$	9. $\lambda/(1+C_L), \sigma_L/(1+C_S)$

The constants C_L and C_S may initially be made equal to 1.

(11b) Steps 5 to 9 are executed once for each of the nine combinations of parameters. The combination that minimizes the sum $\Delta T + \Delta S$ is selected and used in next executions of Steps 5 to 9.

(11c) If $\Delta T + \Delta S \leq \epsilon\sigma_L$, then the selected parameters are those looked for. If $\Delta T + \Delta S > \epsilon\sigma_L$, then Steps 11a and 11b are again executed, substituting $C_L/2$ for C_L and $C_S/2$ for C_S , when one of the corresponding selected parameters remains unchanged.

(12) For each observer, the individual threshold is computed by Equation 1 using p_i s for the dummy Vs and the 0 or 1 produced by the observer for the experimental Vs.