Scaling of saturation and hue¹

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The study concerns the relation of saturation to the purity and luminance of aperture colors viewed in a dark surround. For the primary hues, red, yellow, green, and blue, and the intermediate hues, orange and yellowish green, the saturations increased as power functions of colorimetric purity. An 18-dB increase in luminance caused a threefold increase in the exponent for yellow, but luminance had little effect on the exponents of the other colors. The direct heterochromatic matching of saturation to saturation confirmed the validity of the scales determined by magnitude estimation and led to the construction of families of saturation scales based on a common unit called a crome. Equisection and jnd scales were also determined. Their nonlinearity suggests that saturation is a prothetic continuum. It was found that mixing red or green with yellow behaves much the same as mixing red or green with achromatic light. The changes in hue behave as prothetic continua, for the equisection and jnd scales are nonlinearly related to the power-function scales obtained by magnitude estimation and matching.

The progress made in previous efforts to erect ratio scales of visual saturation, analogous to the brill scale of brightness and the sone scale of loudness, has suggested, as the next step, the scaling of all the primary hues, and also the scaling of variations along the hue dimension itself. The ratio scaling of saturation was undertaken by Galifret (1959), who scaled redness in two separate experiments but obtained rather different results by his two procedures. Both redness and greenness were later scaled by Onley and collaborators (1963), who demonstrated that the saturations of both red and green increase as power functions of colorimetric purity. Panek and Stevens (1966) undertook to re-examine Galifret's suggestion that saturation of red behaves like a metathetic continuum on which the jnd scale is linearly related to the magnitude scale. A similar conjecture had been made by Stevens and Volkman (1940). Panek and Stevens found, however, that redness behaves like a prothetic continuum: both the jnd scale and the category scale are nonlinearly related to the magnitude scale of saturation. The prothetic nature of the saturation of red was confirmed by Indow (in press).

The present paper describes a series of scaling experiments on the saturation of aperture colors: the four primary hues, red, yellow, green, and blue, and two intermediate hues, orange and yellowish green. Similar scaling experiments were made on changes in hue, red through orange to yellow, and yellow through

greenish yellow to a slightly yellowish green. Most psychophysical studies of the color space have been based on judgments of color differences, especially small differences. In the present experiments the observers used two principal procedures: they matched numbers to saturation (magnitude estimation), and they matched saturation to saturation (heterochromatic matching). The matching of numbers to sensations (magnitude estimation) has been amply validated by other forms of cross-modality matches (Stevens, 1961, 1966a) and by direct matching within a sense modality (Stevens, 1966b). Similar validation is demonstrated by the present study.

The scales generated for aperture colors are not necessarily useful scales for practical purposes, such as the specification of color samples. Their purpose is to try to make explicit and quantitative a basic property of the human visual system.

PROCEDURE

In this report, most analyses are made on the pooled data of groups of observers. A few observers participated in almost all the experiments, however, and analyses of the individual data will be reported elsewhere.

The experiments are referred to by the numbers in Table 1. The hues are referred to by abbreviations: red R, yellow Y, a slightly reddish yellow Y', green G, blue B, orange OR, and a slightly yellowish green G'. Luminance is given in terms of the decibel system, in dB re 10⁻¹⁰ L. The decibel, a tenth of a log unit, is a convenient size, and the reference level 10⁻¹⁰ L is approximately the absolute threshold of the dark-adapted eye (Stevens, 1955). A level of 70 dB corresponds to 1 millilambert.

Apparatus

By means of a rotating prism, lights are mixed and presented through five apertures (Ap) arranged in a row. In each aperture the mixing ratio of two colors is varied continuously and independently without affecting the luminance. The principle of the apparatus is schematized in Fig. 1. A rotating prism, 4.1 x 4.1 x 20.8 cm, deflects the observer's line of sight through an aperture so that an area about 3 cm long is repeatedly scanned. Pairs of contiguous filters, mounted on glass microscope slides, are placed in the scanning area. The mixing ratio of the lights through a pair of filters depends merely on the location in the scanning area of

Table 1. List of Experiments

Experiment	Description	No. of Observers	Observer Group
Series 1. Saturation of the four primary hues, R at 71 dB, Y at 79, G at 78, and B at 65.			
1.1	Magnitude estimation with hue as the parameter	14	
1.2	Magnitude estimation with stimuli varied in both hue and purity	10	
1.3	Equisection	14	as in 1.1
1.4	Absolute threshold	14	as in 1.1
1.5	Heterochromatic matching, pairwise	10-12	
1.6	Heterochromatic matching, four hues at a time	10	as in 1.2
1.7	Heterochromatic matching of saturation intervals	10-12	as in 1.5
Series 2. Saturation at various levels of luminance, R at 63 to 81 dB, Y at 72 to 90, G at 67 to 85, and B at 62 to 78.			
2.1	Magnitude estimation with hue and luminance as parameters	11	
2.2	Heterochromatic and heteroluminous matching	11	as in 2.1
2.3	Heteroluminous matching	8	
2.4	Absolute thresholds	10	as in 2.1
Series 3. Saturation of intermediate hues, OR and G' at 81 dB.			
3.1	Magnitude estimation with hue as the parameter	10	
3.2	Heterochromatic matching	10	as in 3.1
3.3	Absolute threshold	8	as in 3.1
Series 4. Changes along the hue dimensions, R to Y' and Y to G' at 81 dB.			
4.1	Magnitude estimation of a component hue in a mixture	11	
4.2	Matching between saturation and a component hue in a mixture	6-11	as in 4.1
4.3	Equisection of the interval between two hues	11	as in 4.1
4.4	The end for hue	11	as in 4.1

the dividing line between the two filters. The location of the dividing line can be controlled by either the experimenter or the observer. Thus, when a chromatic and an achromatic filter of the same luminance transparency are mounted on a glass filter slide, movement of the slide changes the colorimetric purity of the mixture.

After the experiments in the first series, Experiment 1.1 to 1.7, had been completed, the apparatus was modified, and all subsequent experiments were made with the improved apparatus. Both versions of the apparatus were constructed by R. Gerbrands. Notable features were as follows:

1. The single slide projector (300 W., 3200°K) used in the first apparatus to illuminate the white mat surface was replaced by four projectors of the same type. The maximum luminance was thereby increased from 86 dB to 93 dB re 10^{-10} L.

2. The filter holders, moved by a threaded screw in the first apparatus, were attached to rods which could be more easily pushed or pulled by the observer.

3. In the second apparatus the comfort of the observer

was improved by a front-silvered mirror that enabled him to observe the row of apertures through a horizontal tube. By this arrangement the distance from the apertures to the eye was increased from 38 cm to 79 cm. In the first apparatus, each aperture was about 1° long and 0.8° high, and the five apertures were spread over a subtense of 19.4° . In the second apparatus, each aperture was about 30 min. long and 22 min. high and the subtense of the five apertures was 9° . As will be seen, the reduction in size did not appear to affect the results.

In the first apparatus, when all five stimuli were set physically equal and were presented simultaneously, the end colors tended to appear less saturated than the three in the middle. Therefore, in Experiment 1.1 to 1.7, simultaneous presentation was limited to four stimuli. In the second apparatus, because of the reduction in size, a row of five stimuli could be presented simultaneously without any appreciable bias due to position.

4. Between presentations, the eye was exposed to a gray paper illuminated by the slide projector. The luminance produced was 64 dB in the first apparatus and 68 dB in the second apparatus.

5. By means of a Corning Glass filter (No. 5900) in front of each projector, the color temperature of the beam was converted to about 6500°K. This is fairly close to the standard illuminant C. Hence specification

Fig. 1. Schema of color mixer. Light from a 300-watt projector, filtered by Corning Glass 5900, illuminates a white diffusing surface. The observer sees this surface through an aperture and through two contiguous filters that are scanned by the rotating prism. The position, relative to the scanned area, of the boundary between the two contiguous filters is changed by moving the filter slide back and forth. The light through the two filters is thereby mixed in any desired proportion. The complete apparatus contains 5 apertures and 5 corresponding filter slides.

Table 2. Characteristics of chromatic filters

Hue	Wratten No.	Wavelength in nm	Attenuation in dB	Pc in per cent
R	29	631.6	12.0	100.0
Y	15	579.2	1.8	99.3
Y'	90	583.2	10.1	99.9
G	61	536.8	7.7	93.1
B	45A	477.6	15.5	74.1
OR	22	595.1	4.5	99.8
G'	99	553.6	11.5	99.2

of the stimuli is made according to the stated characteristics of Wratten filters with respect to illuminant C.

6. The relation between the position of a filter slide and the mixture ratio of two light rays was calibrated by means of a slide with a transparent part and an opaque part. The luminance of fused light was measured by a spot photometer (Spectra, Pritchard), and a calibration chart was thus obtained. When a chromatic filter of a given colorimetric purity P_c is used with an achromatic filter of the same luminance transparency, the colorimetric purity of the mixture is given by the product of P_c and the mixture ratio. In the experiments on saturation, unless it is otherwise stated, the purity of the stimuli ϕ , is given in terms of colorimetric purity, i.e., the percentage of the luminance of the chromatic component.

7. The apertures from left to right will be referred to as Ap1 to Ap5. During the stimulus presentation, the surround was dark. Each aperture could be separately occluded by a black cover. The observer viewed the field through a rectangular slit 3 mm high and 15 mm wide at the end of the viewing tube. He could move his eye horizontally to fixate any of the apertures, but eye movement perpendicular to the row of stimuli was strictly limited. The purpose of this limitation was to maintain a constant mixture ratio between the two components in the stimulus. This ratio depends critically upon the viewing angle.

8. The possibility of stray light between neighboring stimuli was checked by visual matching. Various standard red stimuli were presented in Ap3, and red stimuli in Ap2 and Ap4 were matched to the standard by five observers. There were red stimuli in Ap1 and Ap5, but they were covered and not visible. The matches were made with an error of less than 1%, and there was no shift in the matching values when the purity of the stimuli in Ap1 and Ap5 was changed.

Filters

Wratten gelatin filters were mounted on microscope glass slides. Of the chromatic filters used, the catalogue number, dominant wave-length λ_d , attenuation factors in decibels, and colorimetric purity P_c for the illuminant C are listed in Table 2. Neutral filters of various attenuation factors (densities) were combined with the chromatic filters as needed to equate the luminous transparencies of the two parts of the filter slide.

The general luminance level of each aperture was changed by one or more neutral density filters inserted between the prism and the filter slide.

Observers and Procedure

Except for the obvious requirement that the observers have normal color vision, they were selected at random from among the graduate students, secretaries, and other people employed by the laboratory. Most of them had no technical knowledge of color.

Using his favored eye, the observer viewed the gray adapting field, 64 dB in the first experiments, 68 dB in the others. As soon as the adapting field was turned off, a stimulus or a row of stimuli was presented in a dark surround. The observer was asked to fixate single stimuli and to scan when more than one stimulus appeared. When the task required the observer to adjust the stimuli, he was allowed to change the stimuli in whatever way he liked. Once the adjustment was made, the adapting field was presented again for about 5 sec. (or for a longer period when afterimages were apparent). Then the stimuli were presented again and the observer was allowed to make whatever adjustments seem necessary. This procedure was continued until the observer was satisfied. When the task was to determine thresholds by the method of limits, the observer was asked to change the stimulus in a prescribed direction only, and the stimulus was not presented again.

In magnitude estimation, in which the observer assigns numbers proportional to the apparent strength or magni-

Fig. 2. Relation of saturation to purity as determined by magnitude estimation for aperture colors in a dark field (Experiment 1.1). Each point is the geometric mean of 28 estimations, 2 by each of 14 observers. Since the hues were scaled one at a time, the vertical positions of the four power functions are arbitrary. The slopes (exponents) are R 1.7, Y 2.9, G 1.7, B 1.4.

Fig. 3. Relation of saturation to purity when both hue and purity were varied (Experiment 1.2). Each point is the geometric mean of 40 magnitude estimations, 4 by each of 10 observers. Since both hue and purity were varied, the relative positions of the power functions are not arbitrary. The slopes (exponents) are R 1.4, Y 2.8, G 1.3, B 1.1.

tude of the stimuli, the stimuli were presented for about 1.5 sec., separated by a short period of darkness of about 2 to 4 sec. After a certain number of stimuli had been presented in this way, the eye was re-exposed to the gray adapting field.

SERIES 1. SATURATION OF THE FOUR PRIMARY HUES

The first four experiments, 1.1 to 1.4, concerned the question whether saturation behaves as a prothetic continuum for all primary hues. Experiments 1.5 to 1.7 concerned the construction of magnitude scales of saturation with a common unit for the four primary hues. The luminances were fixed at the following levels: R 71, Y 79, G 78, and B 65 dB re 10^{-10} L. According to Purdy (1931), those are approximately the levels of luminance at which the primary aperture colors give the maximum degree of saturation.

The seven experiments were divided into three groups. In a given session, Experiment 1.4, 1.1, and 1.3 were carried out in that order with a single hue. The same 14 observers repeated this group of experiments in a total of four sessions, with the order of the hues randomized. Experiments 1.5 and 1.7 were conducted in that order in six sessions with one pair of hues per session. For each pair of hues, 10 to 12 observers were used, and most of them participated with four of the six possible combinations of hues. Experiments 1.2 and 1.6 were conducted in that order in a session lasting about 50 min., with the four hues in a fixed spatial arrangement. The arrangement of the hues was made different in each of the two 50-min. sessions, with the restriction that the members of the two complementary pairs, R, G, and Y, B, were never placed side by side.

Experiment 1.1. Magnitude Estimation, One Hue at a Time

A given hue was presented at five levels of purity in irregular order, a different order for each of the 14 observers. The instructions, with appropriate modifications for the other hues, were as follows:

I will present a series of reddish colors one at a time. Your task is to tell how red the colors look by assigning numbers to them. Call the first color any number that seems to you appropriate.

Try to assign numbers proportional to your impression of the saturation, the relative amount of red. For example, if a color looks 3 times as red, assign a number 3 times as large as the first. If a color looks 1/5 as red, assign a number 1/5 as large. Use any numbers that seem appropriate, decimals, fractions, or whole numbers.

The field was dark between the 1.5-sec. stimulus presentations. After the five levels of purity had been presented, the gray adapting field was turned on for about 2 min., after which the five purities were presented again in a different order. The geometric means are plotted in log-log coordinates in Fig. 2.

Experiment 1.2. Magnitude Estimation with Stimuli Varied in Both Hue and Purity

For each hue, five levels of purity were used, making a total of 20 stimuli. The stimuli were presented one by one in semirandom order. The instructions were like those for Experiment 1.1, except that the observer was told that he would see different hues as well as different saturations. He was asked to assign numbers proportional to his impression of the saturation, regardless of the hue. Each stimulus was presented for about 1.5 sec. Following every fourth stimulus, the adapting field was turned on for about 1 min., during which time the stimuli were changed for the next four presentations. In order to help the observer maintain the same modulus for assigning numbers, the fourth stimulus was presented as the first of the next series of four, and the observer was reminded of the number he had assigned to that stimulus. The observer rested periodically and then began a new series with a new modulus. There were four sets of magnitude estimations spread over two sessions. Geometric means of the judgments are plotted in Fig. 3.

Magnitude Scales Derived from Experiment 1.1 and 1.2

The data points in the log-log coordinates of Fig. 1 and 2 are fairly well represented by straight lines, fitted by least squares. This suggests that for each hue a power function

$$\psi = a\phi^b \quad (1)$$

represents the relation of saturation ψ to colorimetric purity ϕ . The exponent is given by the slope of the straight line in the log-log coordinates. The exponent is not affected by the fact that each ob-

Fig. 4. The results of magnitude estimation shown in Fig. 2 are plotted here as functions of excitation purity P_e on the XYZ system and an analogous measure $P'e$ on the RGB system. The ordinate is the same as that in Fig. 2 (Experiment 1.1). The exponents in terms of P_e are R 1.8, Y 2.2, G 1.1, and B 2.3. The exponents in terms of $P'e$ are R 2.2, Y 2.4, G 1.1, and B 1.9.

server used a different modulus for matching numbers to saturation, or that a particular observer may have changed his modulus from one series of magnitude estimations to the next. The choice of modulus affects only the value a , the intercept in the log-log coordinates. Because the four hues were used in separate series in Experiment 1.1, the vertical locations of the four lines in Fig. 2 are arbitrary. On the other hand, in Experiment 1.2 all four hues were used in a given series of judgments, which fixes the relative locations of the four lines in Fig. 3. These locations suggest that B is the most saturated and Y the least saturated hue.

The values of the exponent b are systematically lower in Experiment 1.2. The ratios of b in the two experiments are R .84, Y .99, G .79, and B .76. The same tendency was evident even with the five observers who participated in both Experiment 1.1 and Experiment 1.2. Two considerations suggest that the slopes (exponents) in Fig. 2 are more representative than those in Fig. 3. First, the results in Fig. 2 are more consistent with the results of other experiments. Second, it has often been observed in this laboratory, with the visual as well as with other modalities, that the exponent determined by magnitude estimation tends to decrease when the observer's task is made more difficult (for example, see J. C. Stevens and S. S. Stevens, 1963). In Experiment 1.2 the observer was required to abstract saturation from a constellation of stimuli that also changed in hue.

Alternative Stimulus Measures

Colorimetric purity P_c is not the only form in which the purity of stimuli can be specified. Excitation purity P_e is also widely used. Given three points in the chromaticity diagram of the XYZ system—W, a point corresponding to 0%; M, a point corresponding to 100%; and P, a point corresponding to a given value of the mixture ratio—then P_e is defined as the ratio of the two distances $\overline{WP}/\overline{WM}$. It is possible to define an analo-

gous purity with respect to the RGB system, which we may designate by $P'e$. This is the ratio of the two distances $\overline{WP'}/\overline{WM}$ defined in the plane of a constant luminance in the RGB system that is analogous to the chromaticity diagram of the XYZ system. The results of Experiment 1.1 are plotted against P_e and also $P'e$ in log-log coordinates in Fig. 4. The three stimulus specifications, P_c , P_e , and $P'e$, are not linearly related. When power functions are fitted to the data, the exponents change considerably, depending on the definition of the stimulus. The sums of squares of the deviations of the points from the best fitting power functions in Fig. 2 and 4 are shown in Table 3. It is interesting to note that the use of colorimetric purity P_c gives the smallest residual deviation in Table 3. Furthermore, since colorimetric purity has the advantage of being independent of the coordinate system of colorimetry, further analyses will be made here only in terms of P_c .

Experiment 1.3. Equisection

Experiment 1.4. Absolute Threshold

The instructions for equisection were as follows, with appropriate changes for the other colors.

You will see a row of four colors. Your task is to divide the total range into three equal-appearing intervals of redness by adjusting the two middle colors.

During the course of the adjustment, take care to look at each color one by one and to scan the whole series.

You may start with either of the two adjustable colors. Try to work rapidly. Once the adjustments are made, you will be asked to look at a homogeneous surface for a while, and then the four colors will be presented again so that you may correct your adjustments if you like. The adjustment will be continued in this way until you are fully satisfied.

For each color, two different purity ranges were equisectioned, as follows.

- R: 0 to 100%, and 20 to 100%
- Y: 0 to 99%, and 50 to 99%
- G: 0 to 94%, and 23 to 94%
- B: 0 to 75%, and 18 to 75%

The lowest purity was presented in Ap1 and the highest in Ap4. The equisection settings were made twice in each series, starting once from a purity level close to the purity of Ap4 and once from a purity level close to the purity of Ap1.

The absolute thresholds were determined by the method of limits with a single stimulus presented in Ap3. The observer was asked to approach the threshold twice by increasing the purity from 0% (ascending series), and twice by decreasing the purity from a level giving moderately saturated color (descending series). Between one setting and the next, the eye was exposed to the adapting field.

The equisection results are plotted as circles in Fig. 5. For each hue, the major part of the data can be described by a straight line intersecting the abscissa at a point between 0% and the threshold. This result accords with the finding in a previous study of the saturation of red by a more elaborate equisection procedure (Indow, in press).

Saturation as a Prosthetic Continuum

The question of the prosthetic nature of saturation depends on whether the equisection scale and the jnd are nonlinearly related to the magnitude scale. As a jnd measure, we can use the length of the radius of the MacAdam ellipses (MacAdam, 1942, 1943) along a line connecting the point C and a point representing the dominant wavelength λ_d of a hue. This length represents the standard deviation of color matching when the variable stimulus varies only in purity. Since the jnd

is essentially a measure of variability, this standard deviation will serve the present purpose. The radii were calculated from Fig. 1 to 3 of MacAdam's paper (1943) in terms of P_c and plotted as a function of P_c . In the case of Y ($\lambda_d=579$ nm), the trend thus obtained is not in full accord with the experimental data for the color ($\lambda_d=576$ nm) shown in Fig. 9 of MacAdam's earlier article (1942). Hence, the 1942 data have been used to determine the jnd scale for Y. For each hue the variability increases approximately as a linear function of purity. From these functions, summated jnd scales were constructed (Luce & Galanter, 1963).

The relations among the three scales are shown in Fig. 6. As is characteristic of prosthetic continua (Stevens, 1957; Stevens & Galanter, 1957), both the equisection (partition) scales and the jnd scales in Fig. 6 are concave downward, and, as is nearly always the case, the jnd scales are more strongly curved than the equisection scales. The evidence in Fig. 6 makes it possible to generalize the findings previously made with red by Panek and Stevens (1966) and by Indow (in press) and to state that saturation is a prosthetic continuum in each primary hue.

Experiment 1.5. Heterochromatic Matching, Pairwise

Experiment 1.6. Four Hues at a Time

Pairwise heterochromatic matching of saturation

Table 3. Sums of squares of the deviations (in log units) of the points from the fitted lines in Fig. 2 and Fig. 4.

	P_c	P_e on XYZ	P'_e on RGB
R	.0029	.0048	.0102
Y	.0078	.0074	.0107
G	.0001	.0020	.0070
B	.0019	.0082	.0060
Sum	.0127	.0224	.0339

Fig. 5. Results of two series of equisections (Experiment 1.3). Four purities were presented in a row and the observer adjusted the two middle values to produce three equal-appearing intervals. The fixed end stimuli are represented by squares, except for those end stimuli that were of zero purity. The adjusted stimuli for the two series are represented by filled and unfilled circles. Each circle is the average of 28 adjustments, 2 by each of 14 observers. The results of the two series were graphically superimposed. The arrows represent two absolute thresholds, one determined by descending and one by an ascending method of limits (Experiment 1.4). Note that both coordinates are linear.

Fig. 6. Relations among the three different scales of saturation. The abscissa represents saturation ψ obtained by magnitude estimation (Experiment 1.1). The ordinate represents both equisession (Experiment 1.3) and the summation of jnd (MacAdam's data). The ordinate values have been tied together at the two ends of the scales. Approximate P_c values at the lower ends are R 20, Y 50, G 23, and B 20%. These are the lower ends of the linear parts of the functions in Fig. 5.

was carried out with all combinations of the four primary hues. Most of the 10 to 12 observers participated in the experiments with four out of six possible pairs of hues, and the order was randomized. The spatial arrangement of the two hues was reversed for half the observers. An example of the instructions is as follows.

You will be presented with two colors, a standard color and an adjustable color. Your task is to adjust the adjustable color to make it appear equal to the standard color in saturation. For example, if the standard color is red and the adjustable color is yellow, please make the yellowness of the yellow match the redness of the red.

In making the adjustment, try to produce colors that are plainly more saturated and less saturated than the desired one.

If you find that you cannot change the adjustable color to the degree you want and cannot make it match the standard color, please tell the experimenter.

Each hue served as both the standard and the adjustable color. A standard was set at three levels of purity, and matching was repeated twice for each level. The adjustment was started once from far below and once from far above the supposed matching point. The three levels of the standard purity were so chosen that a

match was nearly always realizable by all the observers. In only 8 out of 816 attempts were the observers unable to achieve a satisfactory match. In these eight cases, the value of P_c would be larger than 100%. The value that would give a match was easily estimated by extrapolation, and the extrapolated value was adopted in the averaging process.

In Experiment 1.6, heterochromatic matchings of saturation were made among the four hues presented simultaneously in a row. The first part of the instructions was as follows.

You will be presented with a row of four colors of different hue (red, yellow, green, and blue). The one on the farthest right or left is a standard color and the remaining ones are adjustable colors. Your task is to adjust the adjustable colors to make them appear equal to the standard color in saturation. For example, if the standard color is red, please make the yellowness, greenness, and blueness of the adjustable colors match the redness of the red standard color.

The remaining part of the instructions was the same as in Experiment 1.5. The hues were so arranged that two complementary pairs, R, G and Y, B, were not placed in two adjacent apertures, and hues that had been placed in Ap2 and Ap3 in the first session were placed in Ap1 and Ap4 in the second session. In a given session, the two stimuli in Ap1 and Ap4 were in turn designated as the standard. Before the observer made his adjustment, the variable stimulus was set once to a low and once to a high purity. The average of the observer's two adjustments was taken as the matching value.

There were a few cases in which the observer could not make one of the three adjustable colors saturated enough to match the others. In these cases, a value larger than 100% was estimated by extrapolation from the other data.

The four hues were not the same luminance. Hence, the pairs of colors in Experiment 1.5 and the four colors in Experiment 1.6 were not the same in brightness. When asked, the observers were unanimous in pointing out that Y looked brighter than B. It appears, however, that in saturation matches a difference in brightness is not particularly disturbing.

Matching Functions

When the saturation ψ of two colors, i and j , is made equal, we can write the equation

$$c_{ij}\psi_i = \psi_j \quad (2)$$

where c_{ij} is a constant to adjust the difference in unit between ψ_i and ψ_j . If both scales ψ_i and ψ_j are directly comparable, as in Fig. 3, c_{ij} is unity. By putting Equation (1) into Equation (2) and arranging, we have a matching function:

$$\phi_i = (c_{ij} \frac{a_i}{a_j})^{1/b_i} \phi_j^{b_i/b_j} \quad (3)$$

Thus, ϕ_i and ϕ_j are related by a power function and

Fig. 7. Heterochromatic matching functions for six pairs of hues (Experiment 1.5). The bars through the points show the direction of the adjustable stimulus. Each point is the geometric mean of matches repeated twice by each of 10 to 12 observers.

should be linear in log-log coordinates. Furthermore, the slope is given by the ratio of the exponents.

The geometric means of the pairwise matchings (Experiment 1.5) are plotted in log-log coordinates in Fig. 7. For fitting the data by a straight line, a number of different methods are possible. In the present instance, two regression lines had been fitted separately to two groups of points: the regression line of $\log \phi_i$ on $\log \phi_j$ to the points with the vertical bars, and the regression line of $\log \phi_j$ on $\log \phi_i$ to the points with the horizontal bars. The former is less steep than the latter in four out of six cases, providing another example of what has been called the regression effect by Stevens (1961). To represent the data in Fig. 7, a line

was adopted that passes through the intersection of the two regression lines and has a slope equal to the geometric mean of the slopes of two regression lines. The reason for taking the geometric mean rather than the arithmetic mean of the two slopes is as follows. Let us assume two regression lines:

$$y = \beta_{xy}x + a_{xy}$$

$$x = \beta_{yx}y + a_{yx}$$

If both are expressed as functions of x , they are

$$y = \beta_{xy}x + a_{xy}$$

$$y = \left(\frac{1}{\beta_{yx}}\right)x - \frac{a_{yx}}{\beta_{yx}} \quad (4)$$

Fig. 8. Four samples of data from the heterochromatic matching of four hues at a time (Experiment 1.6). Each point represents a single match. The scatter is due mainly to individual differences among the ten observers. The dashed lines are the perpendicular regression lines based on the least-square distances of the points from the lines. The thin continuous lines are the predicted matching functions based on the results of magnitude estimation shown in Fig. 3. The heavy lines are the double-regression functions based on the geometric mean of the slopes of the two regression lines.

Table 4. Comparison of the predicted values (b_i, b_j) from Experiment 1.1 and Experiment 1.2 with the obtained exponents (β_{ij}) for the heterochromatic matching functions in Experiment 1.5 and 1.6.

Pair		b_i, b_j		β_{ij}	
i	j	Exp. 1.1	Exp. 1.2	Exp. 1.5	Exp. 1.6
R	Y	.59	.50	.58	.59
R	G	1.01	1.07	.73	.94
R	B	1.21	1.33	1.30	1.15
Y	G	1.72	2.15	1.84	1.60
Y	B	2.06	2.68	1.95	1.96
G	B	1.20	1.25	1.09	1.22

If both are expressed as function of y , they are

$$x = \left(\frac{1}{\beta_{xy}} \right) y - \frac{a_{xy}}{\beta_{xy}} \quad (5)$$

$$x = \beta_{yxy} + a_{yx}$$

A representative line synthesizing the two regression lines must be the same no matter which set, Equation (4) or (5), we start from. The geometric mean of the two slopes is the same for Equation (4) and (5) but the arithmetic mean is not.

In Experiment 1.6, the observer matched four colors at a time. The results were plotted pairwise as in Fig. 7. Four of the resulting plots are shown in Fig. 8, where each point represents a match. In these plots, it is not clear which is the independent variable, for the observer made his matches by considering all four colors. He tried to produce a row of four colors that matched each other in saturation. Even in the case where one of the coordinates was designated as the standard stimulus, it is not certain that it acted as the sole reference for the matching. For this reason, straight lines were fitted to each plot in Fig. 8 in two ways. One method, which gives the dashed lines in Fig. 8, minimizes the sum

of the squares of the perpendicular distances from the points to the line. The other method, which gives the heavy line in Fig. 8, is to compute the two conventional regression lines fitted to all the points, and take the geometric mean of their slopes in a common coordinate system. This is the double-regression procedure outlined above in connection with Fig. 7.

Relation of Heterochromatic Matching to Magnitude Scales

The matching function between two stimulus continua ϕ_i and ϕ_j can be written

$$\log \phi_i = \beta_{ij} \log \phi_j + a_{ij} \quad (6)$$

The values of β_{ij} from Experiments 1.5 and 1.6 are given in Table 4.

If heterochromatic matchings were made in accordance with the magnitude scales ϕ , β_{ij} should be equal to b_i/b_j . The ratios between the two in all possible combinations are shown in Table 4. The following conclusions appear warranted.

1. In general, the exponents of the power functions from Experiment 1.1 (slopes in Fig. 2) give closer agreement with the matching results than do the exponents from Experiment 1.2. In Fig. 8 the matching functions predicted from the results of Experiment 1.2 (Fig. 3) are given by the thin continuous lines. Since the results in Fig. 2 give only the slopes and not the intercepts of the predicted matching functions, the predicted lines are not shown in Fig. 8. Nevertheless, it should be noted that the relative slopes in Fig. 2 fit the data in Fig. 8 better than do the thin lines derived from Fig. 3.

2. As shown in Table 4, the best agreement between magnitude estimation and heterochromatic matching is

Table 5. Results of heterochromatic matching of saturation intervals (Experiment 1.7). Column N gives the number of observers. The exponents β of the heterochromatic matching functions defined by these data are given in column β_{ij} , and the ratios of these exponents to those predicted from magnitude estimation (Experiment 1.1) are shown in the next to last column. The ratios of the obtained to the predicted exponent based on the equisection data are shown in the last column.

Color pair	N	Standard	Match	Standard	Match	Obtained exponent β_{ij}	Obtained from magnitude estimation	Obtained predicted from equisection
R Y	12	R 33 57	Y 65 86	Y 68 92	R 36 58	.57	.97	.74
R G	12	R 27 66	G 52 80	G 53 83	R 31 65	.56	.55	.70
R B	12	R 30 69	B 16 56	B 22 66	R 33 71	1.48	1.21	1.65
Y G	10	Y 68 87	G 41 73	G 36 70	Y 62 88	2.12	1.23	1.64
Y B	10	Y 68 99	B 24 53	B 19 49	Y 63 87	2.50	1.21	2.06
G B	12	G 43 71	B 27 46	B 25 48	G 40 68	1.16	.97	.83

shown by Experiment 1.1 and Experiment 1.6. The slopes of the matching functions in Experiment 1.6, fitted by the double-regression method, are within 7% of the slopes predicted by the ratios (b_i/b_j) of the exponents obtained by magnitude estimation in Experiment 1.1. The next best agreement is that between the exponents from Experiment 1.1 and the slopes of the matching functions in Experiment 1.5, provided the one exception (R and G) is disregarded. The reason for the disagreement in the case of the pair R and G is clear phenomenally, for G, the stimulus appearing brighter, had a strong inducing effect on R. Even G at a low purity made an adjacent achromatic color appear definitely reddish to most observers. This contrast effect tends to reduce the R-G slope in Fig. 7. This induction effect provided the reason, in Experiment 1.6, for the precautions about the arrangement of the hues.

3. The best method for summarizing the data of heterochromatic matching appears to be the double-regression method, which computes the geometric mean of the slopes of the two regression lines. This is shown not only by the results given in Table 4, but also by the goodness of fit as judged visually. The distributions of points from heterochromatic matchings are apt to be truncated at 100% purity of either color. In Fig. 8 the truncation affects the perpendicular regression (dashed lines) more than the double regression (heavy lines).

4. The approximately linear functions in Fig. 5 obtained by equisection (Experiment 1.3) can also be used to predict a set of matching functions. These predicted matching functions would themselves be approximately linear. If purity is measured as distance from the abscissa intercepts of the straight lines in Fig. 5, the slopes of the matching functions would all be unity in log-log coordinates. Such a comparison was carried out and the results were found to depart rather widely from the value 1.0 predicted by equisection. The obtained slopes ranged from 0.73 to 1.92. When compared with the results of the direct matching procedure, the results of magnitude estimation agree better than those of equisection.

Experiment 1.7. Heterochromatic Matching of Saturation Intervals

This experiment followed Experiment 1.5 in each session. The instructions, with R and Y as the example, were as follows.

You will be presented with two pairs of colors: two red colors and two yellow colors. One will be designated as a standard pair, and the other as an adjustable pair. Your task is to adjust the adjustable pair in the following way.

First adjust the left-hand color of the adjustable pair to make it appear equal to the left-hand color of the standard pair in saturation. For example, if the standard color is red, make the

yellowness of the left-hand yellow match the redness of the red on the left.

Then, adjust the right-hand color of the adjustable pair to make that pair appear equal to the standard pair with regard to size of color difference. For example, if the standard pair is red, make the difference in yellowness of the adjustable pair match the difference in redness of the standard pair.

In making the adjustment, try to produce colors that are plainly more saturated and less saturated than the desired one.

Each of the two pairs consisted of two colors of the same hue and each pair served in turn as the standard pair. Of a given pair of hues, e.g., R and Y, the two reds were presented on the left of the two yellows for half the observers, and on the right for the other half. Within a pair, the left-hand member was always the less saturated color.

Standard pairs and averages of 20 to 24 settings (2 repetitions by 10 to 12 observers) are given in Table 5 in terms of colorimetric purity P_c . The standard pair was set for each observer in a way that permitted a match to be made. Hence, the mean values of standard pairs are given in Table 5. The spatial arrangement of the two pairs was found to make a difference, but the spatial effects have to a large extent been balanced by averaging the results of two spatial arrangements.

If the observers were in fact matching intervals in this experiment, the results ought to accord with those obtained by equisection, Experiment 1.3. A study of the results shows, however, that the magnitude estimations of Experiment 1.1 do a better job of predicting the results of interval matching. This probably means that the experiment did not succeed in its purpose of eliciting from the observers an interval match. It may well have proved easier to match saturations than to match intervals between saturations.

Individual Observers

In the present paper, the analysis is based on the pooled data of groups of observers. Although an analysis

Table 6. Intercorrelations of exponents of power functions over 14 individual observers (Experiment 1.1). The values in the principal diagonal were estimated on the assumption that the matrix is of rank one. The loadings are the result of a factor analysis. They represent the part of the variance of the exponents that is accounted for by a common factor. The table suggests that a given observer tends to be consistent in giving exponents that are relatively large or small.

	R	Y	G	B
R	(.66)			
Y	.72	(.67)		
G	.66	.65	(.74)	
B	.62	.64	.77	(.68)
Loadings	.79	.80	.85	.82

of the data of the individual observers will be published elsewhere, a few points may be noted here.

The magnitude estimations of each observer in Experiments 1.1 and 1.2 could be fitted by a power function with about the same goodness of fit as for the pooled data in Fig. 2 and 3 if the difference in the numbers on which the points are based is taken into account. Individual observers have also been shown to give power functions in judgments of brightness (Marks & J. C. Stevens, 1966). In Experiment 1.1 the root-mean-square deviations (in log units) of the points around the lines of least-square fit were R .02, Y .04, G .005, and B .02 for the pooled data, and R .06, Y .08, G .07, and B .06, on the average, for the individual data. Individual observers show differences in the values of the parameters but not in the form of the function. For example, the exponents of the individual power functions in Experiment 1.1 range as follows: for R, 1.11 to 2.37; for Y, 1.24 to 4.59; for G, 1.10 to 2.34; and for B, .89 to 2.33. A given observer tends to be consistent in giving a higher or lower value of the exponent for all four hues. This fact is clear in Table 6, which gives the product-moment correlations of the exponents and estimates of the communalities based on the assumption that the matrix is of rank one. In fact, the mean absolute value of the residuals after the first factor had been extracted was .03. The so-called factor loadings (see Harman, 1960) given in Table 6 suggest that about 80% of the variance of the exponent values over all 14 observers for each hue is ascribable to the variance (individual difference) in a common variable operating throughout all the magnitude estimations of saturation of the four hues.

Individual consistency is also found in the matching experiments. Thus the wide scatter of the points in Fig. 8 does not mean that the matchings by each individual scatter as widely. As a matter of fact, the individual matching data tend to fall close to a straight line in a log-log plot, but the slopes and intercepts of the lines vary from observer to observer to produce the scatter in Fig. 8. For a given observer, the matching functions tend to reflect the individual characteristics of the power functions obtained by magnitude estimation. Thus the agreement between matching and magnitude estimation does not appear to be an artifact of averaging. These relations will be analyzed further in another paper.

Summary of Series 1

1. The saturation of each of the four primary hues was shown to be a prothetic continuum.

2. Magnitude estimations of saturation, both with hue constant (Experiment 1.1) and with hue varied (Experiment 1.2), showed that saturation increases as a power function of colorimetric purity.

3. The heterochromatic matching of saturation with all four hues in view (Experiment 1.6) gave matching functions that agree well with the results of magnitude estimation (Experiment 1.1).

4. Based on the foregoing experiments, the best estimate of saturation exponents is: R 1.7, Y 2.9, G 1.7, and B 1.4.

5. For both the magnitude estimations and the heterochromatic matchings, the data of individual observers show the same functional form as the pooled data.

SERIES 2. SATURATION AT VARIOUS LEVELS OF LUMINANCE

The purpose of this series of experiments was to construct magnitude scales of saturation for the four primary hues at various levels of luminance. The luminance levels used will be designated by a number following a hue. Thus R81 means red at 81 dB re 10^{-10} L. The ranges of luminance used were those that could be presented by the apparatus and those for which each hue could keep its identity despite the change in luminance. Slight shifts of hue are unavoidable with changes in luminance, but they were not disturbing in the ranges used. The shifts were not necessarily in the directions that would be expected from the Bezold-Brücke effect.

In a given one of the five sessions, Experiments 2.2, 2.1, and 2.4 were conducted in that order.

Experiment 2.1. Magnitude Estimation, One Hue and One Luminance Level at a Time

To reduce the experiment to a reasonable size, magnitude estimations were made only with R81, R71, R68, Y90, Y80, G85, G78, B65, and B61. The stimulus was presented in the middle aperture Ap3, and the magnitude estimations were continued until each of five purity levels had been presented twice. The instructions were the same as in Experiment 1.1. Only stimuli of the same hue and luminance were presented in one set. In a given session, two sets of magnitude estimations for two luminance levels of the same hue were made, with an intermission. The order with respect to hue and luminance was different for each observer.

Geometric means of the 22 estimations (2 by each of 11 observers) were plotted against colorimetric purity Pc in log-log coordinates. In every case, a power function described the data very satisfactorily. The values of the exponents are given in Table 7, together with the values obtained in Experiments 1.1, 1.2, and 4.1.

Table 7. Exponents b of the power functions obtained by the method of magnitude estimation (Experiments 1.1, 1.2, 2.1, and 4.1)

Hue	Luminance in dB	Exponent b			
		Exp. 2.1	Exp. 1.1	Exp. 1.2	Exp. 4.1
R	81	1.47			1.78
R	71	1.69	1.68	1.41	
R	67	1.48			
Y	90	4.12			
Y	80	2.86	2.87	2.84	
G	85	1.73			
G	78	1.39	1.67	1.32	
B	65	1.15	1.39	1.06	
B	61	1.03

Fig. 9. The values of the exponents of the power functions of saturation obtained from the two matching experiments (Experiment 2.2 and 2.3, each combined with Experiment 1.6). Matching experiments provide only relative values of the exponent, but the relative values are given absolute meaning on the assumption that the exponent for red at 71 dB is 1.7.

Experiment 2.2. Heterochromatic Matching and Heteroluminous Matching

In the revised apparatus, a row of five colors could be presented without any appreciable positional effect on their appearance. The following five sets of stimuli were used.

I	R81	Y90	Y80	G81	B77
II	R76	Y85	G85	G78	B73
III	R71	R63	Y76	G73	B69
IV	R67	Y72	G67	B65	B61
V	R71	Y80	G78	B65	

One set of stimuli was used in each of five sessions. Stimuli were assigned to apertures under the same restrictions as in Experiment 1.6, plus an additional restriction prohibiting the same hue from occupying adjacent apertures. The arrangement of the stimuli was different from observer to observer. One stimulus was set to a given purity and designated the standard. The observer adjusted the other stimuli in the same way as in Experiment 1.6. Each stimulus was given the role of standard at least twice. Hence, as a rule, 10 matchings were made for the sets I to IV and 8 matchings for V. A stimulus designated as the standard was set once at a relatively low purity and once at a relatively high purity.

In general the scatter of the results was about the same as in the examples shown in Fig. 8. Matching functions were fitted by the double-regression method based on the geometric mean of the slopes of the two regression lines.

Experiment 2.3. Heteroluminous Matching

In this experiment, the five colors presented in a row were of the same hue but different luminance. The five levels of luminance were randomly placed, with a different arrangement for each of the eight observers. The extremes of the luminance levels were not placed in adjacent apertures. In a given session, two experiments on two hues were carried out with an intermission. Four of the observers had participated in Experiment 2.3, but the remaining four had had no experience in an experiment of this sort. Plots of the matching data showed a degree of scatter that, for pairs of the largest separation in luminance level, was about the same as in the examples shown in Fig. 8. Pairs of adjacent luminance levels gave much smaller scatter,

Fig. 10. The magnitudes of saturation ψ in logarithms for stimuli of 100% purity obtained from the two matching experiments (Experiment 2.2 and 2.3, each combined with Experiment 1.6). The bottom graph shows the absolute thresholds of saturation (Experiment 2.4) for the four primary hues at five luminance levels each. The saturation magnitude for 100% purity is based on the assumption that the exponent is 1.7 and that $\log \psi$ is 2.0 when the stimulus is a red at 71 dB re 10^{-10} L.

as would be expected. The matching functions were fitted by the double-regression method.

As described below, the data of Experiment 2.2 and Experiment 2.3 were used to determine the exponent functions in Fig. 9.

Experiment 2.4. Absolute Thresholds

The values of threshold plotted in Fig. 10 were obtained by the method of limits, as in Experiment 1.4. Stimuli of the same hue with different luminance were randomly located in the five apertures, and thresholds were determined by the ascending method of limits until two determinations were made for each stimulus. Between one setting and the next, the eye was exposed to the adapting field. Averages were made of the 22 determinations (2 by each of 11 observers).

The results in Fig. 10 suggest that for each hue there is a luminance at which the threshold is a minimum. These levels are not necessarily the same as the luminance levels at which saturation ψ is a maximum

for a stimulus of 100% purity.

Saturation Scales in Units of Cromes

Magnitude scales with a common unit can be constructed from the interrelated functions of Experiments 1.6, 2.2, and 2.3, provided the exponent for one of the colors is either known or assumed. A reasonable exponent value for R at 71 dB appears to be 1.7. On the basis of that exponent, plus the definition that the saturation of R71 at 100% purity is 100, the exponents and intercepts for the other colors were determined from set V, i.e., from matches among R71, Y80, G78, and B65. The exponents and intercepts were also determined from the results of Experiment 1.6, and an average, weighted in terms of the number of matches, was made of the two determinations. The values thus obtained were used as a basis for determining the exponents and intercepts for the other matching data from Experiments 2.2 and 2.3. All the values are plotted in Fig. 9 and Fig. 10.

Fig. 11. Magnitude scales of saturation in terms of a common unit, called a crome, for the four primary hues. These power functions were constructed on the basis of the two matching experiments, Experiment 2.2 and 2.3, each combined with Experiment 1.6. The luminance in dB re 10⁻¹⁰ L, and the exponent b are indicated for each function. The value of the power function at 100% purity is noted at the top end of each line. The abscissa value of each function is terminated at the approximate threshold determined by the data in Fig. 10 (Experiment 2.4). The saturation scale is based on the assumption that for a red at 70 dB (1 millilambert) the exponent is 1.7 and the saturation at 100% purity is 100 cromes.

The two sets of results in Fig. 9 and 10 are both referred to the red exponent 1.7, and the unit is so defined that 100% purity of red equals 100 units of saturation. Figures 9 and 10 are fairly similar in their general features. Since the averages of the exponents and intercepts showed a simple, regular relation to luminance, an attempt was made to achieve a consensus by smoothing slight irregularities and extrapolating the data in both directions. It was also thought desirable to specify the luminance level of the red reference standard as 70 rather than 71 dB, because 70 dB is exactly one millilambert. The word *crome* is suggested as the name for the common unit of saturation.

Figure 11 presents the resulting power functions, giving magnitude of saturation in terms of a common scale in cromes for five luminance levels of the four primary hues. The luminance in decibels and the exponent b are indicated for each function. The bar and number at the top of each function shows the saturation at 100% purity. It will be noted that, for a given purity, the luminance that produces the most saturated color of a hue depends on level. The lower ends of the functions have been terminated at the values of threshold estimated from the data shown in Fig. 10 (Experiment 2.4). At the estimated threshold values of purity, the power functions in Fig. 11 do not all give the same magnitude value of saturation. It is also evident that the saturation functions, unlike the brightness function, do not turn down and become asymptotic to the threshold value (J. C. Stevens & S. S. Stevens, 1963; Marks & J. C. Stevens, 1966). Perhaps, as suggested by Panek and Stevens (1966), judgments of saturation near threshold are biased by the fact that as purity decreases the total visual sensation does not vanish, as it does in the case of brightness.

In the event that further experiments lead to a revised value of the exponent for red at 70 dB, the families of functions in Fig. 11 would need to be altered. The exponents listed in Fig. 11 would then be multiplied by the ratio r of the new exponent to the old. The new saturation ψ of a given color at 100% purity would be given by

$$\log \psi = 2 + r \log (\psi_o / 100)$$

where ψ_o is the old value.

Summary of Series 2

1. Two sets of matching experiments, heterochromatic matchings of saturation among five colors differing in hue and saturation (Experiment 2.2) and matching of saturation among five luminance levels of the same hue (Experiment 2.3) gave 86 matching functions to which the six matching functions from Experiment 1.6 could be added. From the 92 matching functions the relative values of the two parameters, exponent and unit, of the power functions for saturation were obtained for five luminance levels of the four hues.

2. The resulting relative values were given absolute meaning by assuming that for red at 70 dB the exponent

is 1.7 and the unit is such that a purity of 100% gives a saturation of 100 units, or 100 cromes. Four families of power functions, one for each hue, are shown in Fig. 11. From these families, magnitude of saturation for any purity of a primary hue at any luminance can be approximated for a wide range of values.

3. Magnitude estimations of saturation (Experiment 2.1) were made for 9 of the 20 colors used in Experiments 2.2 and 2.3. The exponents obtained are shown in Table 7.

SERIES 3. SATURATION OF ORANGE AND YELLOWISH GREEN

The purpose was to see whether the saturations of the two intermediate hues, orange OR and yellowish green G', both at 81 dB, behave in the same way as the saturations of the four primary hues and, if so, how the parameters of the saturation scale of an intermediate hue are related to the parameters of the saturation scales of the two adjacent primary hues. The experiments were carried out in the order 3.2, 3.1, 3.3. Ten observers participated in Experiments 3.2 and 3.1 and eight in Experiment 3.3. Of the ten, eight had served in Experiments 2.1, 2.2, and 2.4.

Experiment 3.1. Magnitude Estimation, One Hue at a Time

The instructions and the procedure were the same as in Experiment 2.1. Geometric means of the magnitude estimations are plotted against colorimetric purity in log-log coordinates in Fig. 12. The data are well described by power functions. The exponent for OR is 1.6, for G' is 1.4. The series of points for G' were shifted upward in Fig. 12 by 0.4 log unit.

Experiment 3.2. Heterochromatic Matching among Five Colors

A set of five stimuli were used, R, OR, Y', G', and G, all at 81 dB. The instructions, the procedure, and the number of matchings were the same as in Experiment 2.2. The spatial arrangement of the five hues was different for each observer, but R was never placed adjacent to G or G'. The resulting matching functions are well described by power functions, and the scatter is especially small for those pairs of hues that are adjacent on the spectrum, such as R and OR, or OR and Y'. Instead of Y, a slightly reddish yellow Y' was used, because it was the stimulus that had to be used in Experiment 4.3, and information concerning it needed to be obtained in this experiment.

From the matching functions fitted by the double-regression method, a magnitude scale of saturation with a common unit was constructed for the five hues by the general procedure used earlier to construct the crome scale. This scale was tied to the scale of Fig. 11 by using once the exponent and the unit constant for R at 81 dB and once the exponent and the unit constant for G at 81 dB, as determined from Fig. 11. As shown in Fig. 13, the resulting values of the exponents were almost the same, but there was a systematic change in the intercept values, $\log a$, depending on which hue

Fig. 12. Magnitude estimations of saturation (Experiment 3.1) as functions of colorimetric purity for orange and yellowish green at a luminance of 81 dB. Each point is the geometric mean of 20 estimates, two by each of 10 observers. The slopes (exponents) are OR 1.6 and G' 1.4. The series of points for G' are shifted upward by 0.4 log unit.

was used to link the results of Experiment 3.2 to the general crome scale. The upper circles in the top part of Fig. 13 were obtained with G as the link. The values of the exponent b and the unit constant $\log a$ for R, Y, and G at 81 dB from Fig. 11 are plotted as triangles in Fig. 13.

Experiment 3.3. Absolute Thresholds

In Fig. 13, the values of the thresholds obtained from Experiment 3.3 are also plotted. The procedure was the same as in Experiment 2.4. Averages of 16 settings (2 by each of 8 observers) by the ascending method of limits are shown by circles. For comparison, the thresholds for R at 81 dB, Y at 80 dB, and G at 81 dB from Experiment 2.4 are also shown (triangles).

Summary of Series 3

1. For each of the two intermediate hues, orange OR and yellowish green G' at 81 dB, the scale of saturation ψ obtained by magnitude estimation (Experiment 3.1) is a power function of colorimetric purity.

2. For each intermediate hue, the magnitude scale ψ by heterochromatic matching (Experiment 3.2) has an exponent value that lies midway between the values of the exponents of the adjacent primary hues of the same luminance. A similar relation obtains for the saturations produced by stimuli of 100% purity.

3. The absolute thresholds for the intermediate hues

(Experiment 3.3) lie between the absolute thresholds for the adjacent primary hues.

SERIES 4. SCALING OF CHANGES ALONG HUE DIMENSIONS

The results of the experiments on saturation posed the interesting question whether a change along a hue dimension behaves like a change from an achromatic to a monochromatic color. By change along a hue dimension is meant a continuous change of the stimulus along the spectrum locus in the chromaticity diagram between two adjacent primary hues. It was possible to present such a change by placing a pair of contiguous filters, R and Y', or Y and G', at the appropriate place in the apparatus. Since these filters have an excitation purity of almost 100% and the spectrum locus is linear in this region, the position of the mixture along the spectrum locus changes as a function of the mixture ratio.

Both pairs of filters were matched in luminance transparency so that the luminance of the mixture was kept constant at 81 dB. For mixing with R, a slightly reddish yellow Y' was used, but Y was used with G'. The purpose was to limit the change to a segment

Fig. 13. The values of the intercepts $\log a$ (ψ at 100%) and the exponents b of the power functions for saturation, and the absolute thresholds of saturation for six hues at 81 dB. Circles in the two upper diagrams represent the results obtained in heterochromatic matchings (Experiment 3.2) linked with the ordinate scale in Fig. 11 through two different reference hues, once through R and once through G. The triangles represent the values taken from Fig. 11. Hence all the values of the exponent b and the intercept $\log a$ are referred to the magnitude scale of saturation in cromes shown in Fig. 11.

defined by two adjacent primary hues and to make sure that the change did not go beyond a corner corresponding to a primary hue. The color changed in one case from a red having a very slight tint of yellow, through orange, to a slightly reddish yellow; and in the other case from yellow, through greenish yellow, to a slightly yellowish green. The slight tint of redness in Y' and the tint of yellowness in G' was apparent, but the colors were well within the domains that are categorized as yellow and green in color naming. Consequently these stimuli were referred to as yellow and green in the instructions.

As a function of mixture ratio, the effective wavelength of the mixture follows a rather smooth function that is slightly concave downward for $G'-Y$ and slightly concave upward for $R-Y'$.

The same 11 observers served in all four experiments, and the order in one session was Experiment 4.4, 4.1, 4.3; Experiment 4.2 was conducted in a separate session.

The continuous change along the spectrum locus from R to Y' or from Y to G' does not necessarily imply that saturation or depth of color is also kept constant. The top diagram of Fig. 13 suggests that saturation may change in cromes by about 1.5 decilogs from R to Y' , although the change seems to be much smaller from Y to G' . It should be noted, however, that when a color changed from one primary hue to another, the change in hue was so obvious that it was practically impossible for the observer to direct his attention to the saturation attribute.

Experiment 4.1. Magnitude Estimation of a Component Hue in a Mixture

Six magnitude-estimation experiments were conducted by the procedure used in Experiments 2.1 and 3.1. The judgments were on redness and yellowness in orange, on yellowness and greenness in greenish yellow, and on the saturation of R and G' . The order of the experiments was different for each observer. The two magnitude estimations of saturation, made for the purpose of comparison, followed the instructions of Experiment 2.1. For the hue judgments, the first part of the instructions was different, as shown by the following example for redness in orange.

You will see a series of colors one at a time. They are orange varying between yellow and red. Your task is to tell how red the colors look by assigning numbers to them. Call the first color any number that seems to you appropriate. What is to be assessed is not the depth of orange itself, but the degree of redness in the orange.

The 11 observers found it easy to make judgments of redness in the $R-Y'$ series and greenness in the $Y-G'$ series. On the other hand, they found it difficult to assess the degree of yellowness in the orange and in the greenish yellow. They all become confused and had to think carefully about the task before assigning numbers. In estimating the yellowness in orange, one observer assigned almost the same number to all

mixture ratios, though she herself was clearly aware that the colors were different and that they looked sometimes reddish and sometimes yellowish. Two observers gave similar results for yellowness in the greenish yellow. These results were not included in the analysis. Both observers were experienced, and both gave excellent results in all other experiments. These extreme difficulties raise a question whether the remaining observers succeeded in assigning numbers proportional to the degree of yellowness. Perhaps, instead of abstracting the yellowness, they assigned numbers inversely proportional to the degree of redness or of greenness. Change in the degree of redness and of greenness is intuitively clear and palpable, but change in yellowness certainly is not. Furthermore, a decrease in redness is not necessarily accompanied by an increase in yellowness. A change in redness seems rather to play the leading part "in front" of the background of yellowness, a background that may seem to remain more or less constant. Yellowness behaves similarly in the $Y-G'$ series. As we shall see, the same phenomenon occurred in Experiment 4.2.

Fig. 14. Magnitude estimation (Experiment 4.1) of the primary hue component in a mixture of two hues as a function of the mixture ratio (circles and crosses), and magnitude estimation of saturation as a function of colorimetric purity (triangles). Each point is the geometric mean of the estimates, 2 by each observer. The number of observers varied from 9 to 11, because it proved too difficult for some observers to estimate the degree of yellowness. The ordinate scale gives relative values only.

Geometric means of the magnitude estimations are plotted against the mixture ratio in Fig. 14. The mixture ratio is given in terms of the hue that was being judged. The points lie on straight lines in log-log coordinates, and the values of the exponents b are given in Fig. 14. They are about the same for the two primary hue components, and they are not far from the exponent values for saturation. The results in Fig. 14 suggest that, when yellow is mixed with R or G', it acts much the way achromatic light does when mixed with the same colors.

Experiment 4.2. Matching between Saturation and a Hue Component

Three filter slides were presented in three adjacent apertures. The one in the middle consisted of two chromatic filters—R and Y', or Y and G'—and each of the others consisted of an achromatic and a chromatic filter—R, Y, or G'. One color was designated the standard, and the task was to match the others to it with respect to the degree or depth of the hue at issue, e.g., saturation of red was matched to redness in orange. The main part of the instructions for this example read as follows.

You will see three colors. The orange in the middle is the standard color, and the red and the yellow on either side are the adjustable colors. Each adjustable color can be changed in depth (from gray to deep red, for example).

Your task is to adjust the adjustable colors to make the redness and the yellowness of the adjustable colors match the redness and the yellowness in the orange. What is to be matched is not the depth of the orange itself, but the degree of redness and yellowness in the orange.

In another series, the R on one side was also used as a standard with the orange adjusted. The Y on the other side was not present in the field on the latter occasion. The procedure was similar for the greenish yellow in the middle. In both cases, Y was at 87 dB, whereas all the other stimuli were set at 81 dB. This is because the apparent yellowness in the orange or in the greenish yellow greatly exceeded the range that could be matched by Y of the same luminance.

Of the 11 observers, 10 participated in the matches between orange and R, and 8 in the matches between greenish yellow and G'. Of the 10, 6 could make a satisfactory match between Y and the yellowness of the orange. But even for these observers it proved impossible to use Y as a standard and to adjust the yellowness of the orange to achieve a match. Eight observers could establish a match between Y and the greenish yellow, but, again, to use Y as a standard and to adjust the greenish yellow was impossible.

Geometric means of the settings, two by each observer, are plotted in log-log coordinates in Fig. 15. The values of the exponent β in Fig. 15 can be combined with the saturation exponents b in Fig. 14 to yield, as the exponents determined by matching, the

values 1.58 for redness in orange, and 1.84 for greenness in greenish yellow. The corresponding values obtained by direct magnitude estimation were, as Fig. 14 shows, 1.95 and 1.79. For comparison with the results of equisection, the average values, 1.77 for redness in orange and 1.82 for greenness in greenish yellow, were taken as the exponents of the power functions of the mixture ratios. If we combine the data for yellow in Fig. 15 with the value 4.5 for the exponent estimated from Fig. 14 for Y at 87 dB, we obtain, as the exponents of the power functions determined by matching, 1.36 for yellowness in orange, and 1.76 for yellowness in greenish yellow. The corresponding values obtained by direct magnitude estimation were, as Fig. 14 shows, 1.88 and 1.64.

Experiment 4.3. Equisection of Hue Interval

Equisection along the hue dimension was made in essentially the same manner as in Experiment 1.3. The key part of the instructions read as follows.

You will see a row of five colors. One end is yellow, and the other end is red (green). Your task is to adjust the three middle colors and to construct a series of colors that appear to mark off equal steps from one color to the next.

In the R-Y' series, R of 100% and Y' of 100% were used as the fixed stimuli, and R was set once in Ap1 and once in Ap5. In the Y-G' series, G' of 100% and a mixture (82% of Y and 18% of G') were used as the fixed stimuli. When Y of 100% was used, it appeared appreciably reddish to all observers because of the

Fig. 15. Functions obtained when the primary hue components in mixtures of two hues are matched with the saturations of the respective hues (Experiment 4.2). The bars through the points show the direction of the adjustable stimulus. The ordinate is colorimetric purity and the abscissa is the analogous mixture ratio for a pair of colors. Each point represents the geometric mean of the matches. The lines were fitted by the double regression method, as in Fig. 7. The values of the exponents β of the matching functions are noted on the figure. All luminances were 81 dB, except Y when mixed with achromatic light, which was 87 dB.

Fig. 16. Results of equisection (Experiment 4.3). The three middle colors in a row of five were adjusted to produce four equal-appearing intervals between the fixed end stimuli indicated by filled squares. Each point represents the average of two settings by each of 11 observers. Circles and crosses represent equisections of the same stimulus range but with the spatial order reversed. In the Y-G' series, a mixture of 18% of G' with 82% of Y was used as one of the end stimuli. Hence there are two filled squares at the lower end of the upper curve.

induction from the greenish colors in the row. The Y color appeared, so to speak, to be around the corner from the green colors, and the interval between Y and its neighbor did not appear homogeneous with the intervals between the remaining adjacent pairs. It was found that adding 18% of G' was just enough to cancel the reddish tint in the yellow. In the Y-G' series also, Y was set once in Ap1 and once in Ap5.

As in Fig. 5, the data in Fig. 16 fall close to lines that are almost straight in linear coordinates. With the Y-G' series it was not clear where the yellow fixed stimulus should be located on the abscissa. Physically it is at 18% but phenomenally it is almost equivalent to 0%. The straight line intersects the abscissa about midway between those two values (filled squares).

Experiment 4.4. Jnd for Hue

Just noticeable differences along the hue dimension were determined by two methods: variability of matching and method of limits. Two similar filter slides, both made up of R and Y', for example, were presented in two adjacent apertures. One was set to a given mixture ratio, and the observer was asked to change the other stimulus by first matching it to the standard and then changing it slowly in a designated direction until it appeared just noticeably different from the standard. Then the eye was exposed to the gray adapting field, and the standard was set at another mixture ratio.

Two matches and one difference setting for each of the two directions of change were made by each of the 11 observers for each mixture ratio with the standard stimulus on the left. The same procedure was repeated with the standard on the right.

Two jnd measures were computed, as shown in Fig. 17. The sizes of the limit intervals tend to increase with the percentage of R or G' in the standard, except for the standard of 81%. On the other hand, the value of SD increases over the whole range. Wright and Pitt (1934) measured jnds with the Wright colorimeter, using the method of limits. When converted into mixture ratio, their data for five observers (corresponding spectrum region, luminance 72 dB) show an increasing trend up to a certain point, as in Fig. 17. The increase was from 10 to 90% in the R-Y' series and from 10 to 70% in the Y-G' series. MacAdam (1942) used a measure equivalent to the SD. When converted into the mixture ratio, the data of his observer (PGN, 82 dB luminance) show a slightly increasing trend over the entire range, as in Fig. 17.

Straight lines were fitted to the SD points in Fig. 17 by the method of least squares.

Primary Hue Component as a Prosthetic Continuum

As a result of Experiments 4.1 to 4.4, two facts have become clear. As regards changes along two segments

Fig. 17. Two measures of jnd for change along hue dimensions (Experiment 4.4). The lower diagram shows averages of 22 intra-individual standard deviations SD of color matches. The upper diagram shows the intervals bounded by the mean settings in the increasing and in the decreasing directions by the method of limits.

Fig. 18. Relation among the three different scales for changes along the hue dimension. The abscissa represents the magnitude scale ψ obtained by magnitude estimation (Experiment 4.1) combined with matching results (Experiment 4.2). The dashed curve represents the results of equisection (Experiment 4.3). The continuous curves represent the summated jnd determined by the intra-individual standard deviations for color matching in Experiment 4.4.

of the spectrum locus, R to Y' and Y to G', changes in the opposite direction, R to Y' and Y' to R, are not symmetrical. The changes toward increasing redness and increasing greenness are both dominant over the change toward increasing yellowness. In addition, the strength or degree of the dominant primary hue component in the intermediate hue behaves as a prothetic continuum, just as saturation does in each primary hue. The evidence for the prothetic nature of redness and greenness in mixtures with yellow is shown in Fig. 18. The magnitude scales (abscissa) are power functions of the mixture ratio, with exponents of about 1.8 for both the R-Y' and Y-G' series. These values were obtained from the direct magnitude estimations (Experiment 4.1) combined with the matching results (Experiment 4.2). The equisection scales determined by Experiment 4.3 and the jnd scale are plotted on the ordinate with the unit and origin for the two scales equated at two points: 100 and 0% of R in one case, and 100 and 9% of G' in the other. The value 9% is the intercept of the straight line in Fig. 16. As is characteristic of prothetic continua, the two curves are concave downward, and the jnd scale is more strongly curved than the partition scale obtained by equisection.

Summary of Series 4

1. The change in the degree of redness or greenness that accompanies a shift from R to Y' or from Y to G' along the spectrum locus in the chromaticity diagram behaves as a prothetic continuum. Scales based on judgments of liminal or supraliminal differences are nonlinearly related to the magnitude scales obtained

by magnitude estimation and by direct matches between the degree of redness in orange, or of greenness in greenish yellow, and the saturation of red or of green.

2. Magnitude scales for redness and greenness are power functions of the percentage of R and G' in the mixture. The exponents are both about 1.8.

3. The jnd increases as the percentage of R or G' increases, both in terms of the stimulus mixture ratio and in terms of the corresponding subjective value on the magnitude scales.

4. The foregoing facts suggest that mixing Y' with R, or Y with G', is functionally equivalent to diluting R or G' with achromatic light.

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Notes

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