Loudness and reaction time: II Identification of detection components at different intensities and frequencies

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Equal-loudness contours were first obtained for five stimulus frequencies at four stimulus intensities. These 20 stimuli were then presented as reaction-time signals in a Donders C paradigm. The Z-transform method of convolution, as applied in linear systems identification, was used to deconvolve an empirically generated response (or "residual") distribution (T_R) from each of the 20 reaction-time (RT) distributions obtained at different intensities and frequencies. The resulting sensory-detection (t_d) models formed exponential densities at strong intensities (60 and 80 phons), but their shapes were either gamma or normal at relatively weak intensities (20 and 40 phons). Our analyses support the idea that the simple reaction-time process (RT) is a convolution (or sum) of two component stages: stimulus detection (t_d), followed by response evocation (t_r). Based on the shapes of t_d , a neural-impulse theory is offered to account for the detection of simple auditory RT signals.

In two previous papers (Kohfeld, Santee, & Wallace, 1981; Santee & Kohfeld, 1977), we reported that equally loud stimuli across five frequencies produced equal reaction times at 80- and 60-phon intensities, but at the 40 and 20-phon levels the latencies were longer at 1,000 Hz than at the higher and lower stimulus frequencies. In order to provide a rigorous evaluation of these results, our present intent is to analyze the observed RT distributions generated from 100-, 500-, 1,000-, 5,000-, and 10,000-Hz signals at intensities of 20, 40, 60, and 80 phons. Any dissimilarities among the shapes of these RT distributions, especially at the 20- and 40-phon levels, should provide some insights as to whether the components of the RT process are different across stimulus frequencies when equivalently weak signals are employed.

One basic premise in this paper is that signal detection and response initiation are the two component stages in the simple reaction time (RT) process. This notion is not new, as Green and Luce (1971), Hohle (1965), and McGill (1963) have proposed models of

the RT process which begin with the assumption that observed RT distributions represent a convolution of the densities of two component random variables, one of which reflects the detection, or decision process (t_d), and the other having to do with the response or "residual" latency (t_r). The forms of these component densities have been open to controversy; for example, McGill (1963) assumed that t_d is normally distributed and t_r is exponentially distributed, whereas Hohle (1965) argued for the exact opposite interpretation. More recently, Green and Luce (1971) attempted to find the shape of t_r by constructing t_d theoretically (based on a Poisson pulse model of sensory detection), and then deconvolving t_d from the empirical RT distribution. Although somewhat successful, Green and Luce were unable to identify a completely realistic model of t_r , thus leading them to conclude that their initial model of t_d would require further revision.

In view of the difficulties encountered by Green and Luce in their attempt to derive a model of t_r , we decided to try two different approaches to the analysis of the RT components. First, instead of constructing a theoretical model of t_d and then solving for t_r , we thought it might work to obtain an estimate of t_r and subsequently to generate models of t_d . Essentially, this is the same strategy employed by Green and Luce (1971), except that they started with a model of t_d , whereas we began with an estimate of t_r . Our rationale was based in part on Luce and Green's (1974) suggestion that the RT distribution that is generated from very strong signals should be identical to t_r , the residual distribution. This view reflects their belief that the time for sensory detection (t_d) gets progressively shorter as signal intensity increases, until at

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some strong intensity level, the time required by t_d in the overall RT process reaches zero. Accordingly, we reasoned that if a 110-dB, SPL, tone served as the RT signal, then RT should approximate t_r , especially if a highly practiced subject were employed. This empirical estimate, T_R , of the response component, t_r , could then be deconvolved from other RT distributions at weaker intensities, across different frequencies, in order to generate a "family" of t_d models.

The second major difference in our approach is the use of linear systems identification (Hsia, 1977; Sage & Melsea, 1971) instead of Fourier analysis to identify and separate t_d and t_r from the RT distributions. Since this may be the first attempt to use systems identification in a RT study, we will devote an entire section in this paper to a description of this analytic procedure.

In summary, we believe that some degree of success in the identification of the component densities, t_d and t_r , is necessary in order to accomplish our research goals. Specifically, we reasoned that reaction times to weak-intensity signals (e.g., 20 phons) involve longer detection times than RTs to strong-intensity signals (e.g., 80 phons). This would imply that the models of t_d at 20 phons should have different shapes (larger means and standard deviations) than those derived at 80 phons. Moreover, the models of t_d for 20-phon signals across stimulus frequencies should also differ, thus providing one way to account for the finding that equally loud stimuli do not yield the same RTs at weak intensity-levels. Correspondingly, the models of t_d at 80 phons, across frequencies, are more likely to be similar, since at least the mean RTs are equal across the stimulus frequencies. In the main, we expected that signal detection (t_d), an essential ingredient in the overall RT process, would take longer and be more variable for 20- and 40-phon signals, thus allowing for the effects of signal frequency on t_d to be more apparent than when 60- and 80-phon signals were utilized.

METHOD

Subject

The subject in this study was J.S., the second author of this paper. J.S. was a 25-year-old graduate student with a normal hearing threshold ($20 \mu\text{N/m}^2$ at 1,000 Hz). He was thoroughly practiced in both the equal-loudness matching techniques and the Donders Type C reaction-time procedures that were required in this research. J.S. had already participated as a subject in our previous work (Kohfeld et al., 1981, Experiment 1), and his previous equal-loudness data, along with his RT measures, provided the data for the present analysis.

Apparatus

A detailed description of the apparatus and procedures that were used for obtaining equal-loudness curves, and subsequent presentation of these signals in the RT paradigm, has been provided elsewhere (Kohfeld et al., 1981). In brief, during the loudness-matching sessions, the subject was seated in a sound-attenuated chamber. The stimuli were generated by two audio oscillators and were presented monaurally through a set of calibrated earphones. Stimulus control consisted of appropriate signal attenuation, by

means of two decade attenuators, and switching between the standard and comparison tones. Specifically, the subject was presented with alternating bursts of a 1,000-Hz tone (left ear) at a standard intensity and the comparison tone (right ear) at a variable intensity, and was asked to adjust the comparison tone to match subjectively the 1,000-Hz reference tone. An electronic switch was used to alternate the stimuli at 2.5-sec durations with rise and decay times of 10 msec. There was no interval between the standard and comparison tones. Two adjustable attenuators, controlled by the experimenter, were located outside the chamber. The attenuation of the comparison tone was adjusted by the experimenter based on the subject's verbal instructions (i.e., "up" or "down"). An intercom system provided means for subject-experimenter communications.

The RT sessions were also conducted while the subject was seated in the sound-attenuated chamber. Auditory signals were generated by an audio oscillator and, after appropriate attenuation, were presented to the subject binaurally through the same set of earphones that was used in the equal-loudness, matching sessions. The rise and decay times of the RT signals were 10 msec. On each trial, a 1-sec ready signal consisting of a dim white light was followed by a 1-sec foreperiod interval. The response signal was terminated when the subject pressed a response button. The intertrial interval was set at 1 sec. The appropriate RT signal intensities and frequencies (based upon the subject's equal-loudness judgments) were selected prior to each RT session. The timing and switching of events were controlled by a tape reader, a system of shielded relays, and appropriate solid-state logic. Reaction time was recorded in milliseconds by a digital printout counter.

Procedure

Initially, the subject's absolute threshold for a 1,000-Hz tone was determined, using the method of limits. Subsequently, equal-loudness matches were conducted over four daily experimental sessions, one for each of the 20-, 40-, 60-, and 80-phon intensity levels. During each session, matches for each of the stimulus frequencies were obtained in the following order: 5,000, 100, 10,000, and 500 Hz. After this procedure, response latencies were measured in each of the 20 intensity-frequency combinations that were obtained in the equal-loudness matching (including the 1,000-Hz standard). One RT session was devoted to each of the 20 stimulus conditions, and the sessions were presented randomly. The exact intensity of each RT signal was determined by the subject's equal-loudness judgments. The method of signal presentation followed a Donders Type C RT procedure; that is, the subject responded as quickly as possible upon detection of the RT signal (a "go" trial) and withheld a response when no signal appeared (a "catch" trial). The RT sessions were divided into blocks of trials, 32 trials per block, and each block could be initiated by the subject from within the sound chamber. In order to prevent the subject from initiating a response before the RT signal was presented, the order of presentation of the 24 go trials and 8 catch trials was randomized within each block. A total of 288 RT measures, in 12 blocks, were obtained in each of the stimulus conditions. If the subject committed an error by responding on a catch trial, that block of trials was terminated and he proceeded to the next programmed block. The incidence of errors was virtually nonexistent; that is, it was less than 1% (see also Kohfeld et al., 1981).

After measuring the response latencies to each of the 20 tones, 12 more blocks of trials were obtained in response to a 1,000-Hz signal at 110 dB, SPL. The distribution of these RT measures, T_R , served as the model of the subject's response (residual) density, t_r , that was later deconvolved from each of the RT distributions obtained in the 20 stimulus conditions.

RESULTS

Table 1 presents the results of the equal-loudness matches. Each entry represents the mean value (in dB,

Table 1
Means and Standard Deviations of the Subject's Equal-Loudness Judgments (in Decibels, SPL)

Stimulus Frequency (in Hertz)	Stimulus Intensity (in Phons)							
	20		40		60		80	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
100	79.70	1.06	89.60	.97	92.70	1.34	95.30	.67
500	36.90	.99	43.30	1.25	62.80	1.93	78.70	.95
5,000	40.80	1.40	49.70	1.25	64.10	.74	80.30	1.77
10,000	49.00	1.76	59.50	1.84	72.30	1.06	84.00	1.25

Note—1,000 Hz was the standard frequency for the equal-loudness matches.

SPL) of 10 loudness matches between the 1,000-Hz standard and the various comparison tones. From Table 1, it is evident that (1) much greater levels of signal energy were required at the high and low frequencies for loudness to remain constant, (2) the effects of frequency became less pronounced as stimulus intensity increased, and (3) the subject produced very consistent equal-loudness judgments, as indicated by the low standard deviations. An extended discussion of the effects of stimulus intensity and frequency on judgments of equal loudness is provided elsewhere (Kohfeld et al., 1981).

After the tones that differed in frequency were judged to be equally loud, they were presented as RT signals at specific intensities, depending on the particular values obtained from the equal-loudness matches. The observed RT distributions that resulted are shown in Figure 1, as denoted by the solid lines. Each distribution is based on 288 observations, and the bin size is 5 msec. The ordinate of each distribution indicates the proportion of responses falling into each of the successive 5-msec intervals. Also indicated are the mean and standard deviation of each distribution. The densities represented by the dashed lines are described subsequently in the section of this paper called "Reconvolutions."

Reading down a column in Figure 1 provides a comparison across intensities at a particular frequency, while reading across a row allows a comparison across frequencies at a given intensity level. The former comparison reveals that RT is inversely related to signal intensity, as indicated by the progressive increase in mean RT and the corresponding shift of the observed RT distributions along the time axis. Moreover, if equal loudness were to generate equal RT, then a comparison at each intensity across the five frequencies should yield similarly shaped distributions (e.g., the means and standard deviations should be about the same). Although the equal-loudness, equal-RT relationship was upheld at 80 and 60 phons, equally-loud tones did not yield equivalent RTs at 40 and 20 phons across the signal frequencies. Specifically, as found in two previous investigations (Kohfeld et al., 1981; Santee & Kohfeld, 1977), RTs at 40 and 20 phons to 1,000-Hz signals were longer than those to the other frequency values at these in-

tensities. Moreover, the overall shapes of the RT distributions appeared to differ across frequencies, especially at 20 phons.

Figure 2 presents a smoothed version of T_R , or the RT distribution obtained in response to the 1,000-Hz tone at 110 phons, along with its mean of 136 msec and standard deviation of 9.1 msec. The distribution is based on 288 RT measures, and the bin size is 5 msec. This response distribution, T_R , was smoothed once by a method of running averages known as the "hanning" technique.¹ We think that smoothing T_R , which later served as the model of t_r , resulted in a more reliable application of the deconvolution process without sacrificing too much detail in the original response distribution.

DECONVOLVING THE REACTION-TIME DISTRIBUTIONS

Background Considerations

The main assumption which underlies our analyses is that the overall RT distribution is a convolution of two independent, component densities representing the detection stage, t_d , and the response (or residual) stage, t_r . As shown in Figure 2, we obtained an empirical estimate of t_r , called T_R , by generating its distribution from RTs to a very strong signal in which the t_d component was assumed to be negligible. Our next task was to develop analytic methods which were suited to the identification of t_d .

The point of departure was to consider the basic equation

$$x = y + z, \quad (1)$$

where x , y , and z are continuous random variables with probability density functions v , h , and u , respectively. It is well known (see, e.g., Hoel, Port, & Stone, 1971) that if y and z are statistically independent, then

$$v(x) = \int_{-\infty}^{\infty} h(y)u(x-y)dy. \quad (2)$$

Equation 2 is called a convolution integral, and therefore the process of finding h , given u and v , is some-

Figure 1. Reaction-time distributions (solid lines) generated from 20 signals at different intensities (columns) and frequencies (rows). Each distribution is based on 288 reaction-time measures, and the bin size is 5 msec. The dashed lines represent a convolution of the response density (t_r) with the detection density (t_d)—see Figure 3—which was identified from each respective distribution. (See text for detailed discussion.)

Figure 2. Reaction-time distribution generated from a 110-phon, 1,000-Hz signal. The distribution contains 288 reaction-time measures, and the bins are 5 msec wide. This distribution served as the response density, t_r , which was deconvolved from each of the 20 distributions shown in Figure 1.

times called "deconvolution." We will usually call this process "identification," a term which is more common in the vocabulary of linear systems identification (Hsia, 1977; Sage & Melsea, 1971).

The Fourier transform of Equation 2 is given by

$$V(\omega) = H(\omega)U(\omega), \quad (3)$$

where the capital letters V , H , and U are the Fourier transforms of v , h , and u , respectively. It is known that if one has the Fourier transforms of u and v , the transform of h can be obtained, and the inverse transform yields the identification of h . Since this solution is now quite practical to derive (Green & Luce, 1971), we utilized a fast Fourier transform (FFT) program in order to compute transforms of RT and T_R , and thus obtain the transform of the desired component, t_d . What resulted was a chaotic estimate of t_d , which was similar to the initially poor results obtained by Green and Luce (see their Figure 5, p. 261; we also obtained extreme oscillations and negative values in the estimated densities of t_d).

It could be suggested that our difficulty with the Fourier transform arose from the relatively small number of RT measures ($N=288$) in each of our distributions. There are at least two reasons why we do not think that sample sizes were the primary source of our problems with Fourier analysis. First, the data were taken from a well-trained subject who had more than a year of prior experience with the same Donders C paradigm. Based on block-by-block feedback of both his RT means and standard deviations, our subject achieved remarkably consistent RT performance across trial blocks, as evidenced by the generally small standard deviations reported in Figure 1. Moreover, his RT data are not characterized by marked irregularities in the frequency distributions. Second, we had previously attempted to apply Fourier

analysis to RT distributions comprising 1,440 measures, taken from the same highly practiced subject in another experiment. Even with these larger sample sizes of similarly consistent RT measures, we were unsuccessful with the Fourier transform.

Perhaps a basic reason for the initial failures of Fourier analysis of RT distributions is that v and u , in Equation 2, are not generated as continuous functions, but are empirically obtained samples at discrete intervals (i.e., they are histograms with prescribed bin widths in milliseconds). Thus, Equation 2 can appropriately be written as

$$v_k = \sum_{i=0}^k h_i u_{k-i}, \quad (4)$$

where the subscripts indicate the number of the sample at some implied sampling interval Δ , the bin width; for example, v_k is $v(k\Delta)$. One could still take the discrete Fourier transform of Equation 4 and solve for the samples of h . However, since the record length from which the estimates of u and v are obtained is finite, numerical instability occurs when computing the required Fourier transforms. As noted by Green and Luce, some of the numerical problems can be alleviated by heavily smoothing the estimates of u_k and v_k , but at the risk of destroying vital information about the actual forms of the obtained distributions. In this vein, we tried Fourier analysis after smoothing RT and T_R three times, using the hanning technique. Even then, highly unstable and unrealistic estimates of t_d were obtained. Because of the problems associated with Fourier analysis of RT data, we decided to evaluate another approach to the identification of t_d .

System Identification of Detection Models

System identification is concerned with a technique which enables us to determine a mathematical model for a process whose input-output relationships are already known (Hsia, 1977). In terms of our present analysis, a systems-identification approach implies that the observed RT is the output of a linear system comprising a known input process (T_R) and another, hypothetical, input process (t_d) for which we seek to identify a model. What follows is a description of how we applied linear systems identification to our RT data.

Linear difference equations. Consider the Z-transform (Jury, 1964) of a discrete function, f :

$$F(Z) = \sum_{k=0}^{\infty} f_k Z^{-k}. \quad (5)$$

Equation 5 may be regarded as a generating function for the samples f_k . According to the definition of the Z transform, as shown by Jury (1964), it can be shown that the Z transform of Equation 4 is

$$V(Z) = H(Z)U(Z). \quad (6)$$

It may be seen that there is a direct relationship between Equation 2 and Equation 3 for the continuous case, and between Equation 4 and Equation 6 for the discrete case.

We may now draw an analogy between the problem at hand and the identification problem in linear systems identification (Jenkins & Watts, 1969). Equation 2 can be visualized as the result of passing u through a linear system with impulse response h . It is well known that the output of a continuous linear system can be characterized by a linear differential equation. In the discrete case, Equation 4 can be visualized as the result of passing samples of u through a discrete linear system with an impulse response given by the samples of h . The output of this discrete linear system can be characterized by the linear difference equation

$$\begin{aligned} v_k - q_1 v_{k-1} - q_2 v_{k-2} - \dots - q_m v_{k-m} \\ = b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} + \dots + b_n u_{k-n}. \end{aligned} \quad (7)$$

By repeated use of the shift theorem for Z transforms (Jury, 1964), the Z transform of Equation 7 is

$$\begin{aligned} V(Z) - q_1 Z^{-1} V(Z) - q_2 Z^{-2} V(Z) - \dots - q_m Z^{-m} V(Z) \\ = b_0 U(Z) + b_1 Z^{-1} U(Z) + \dots + b_n Z^{-n} U(Z). \end{aligned} \quad (8)$$

Accordingly,

$$\begin{aligned} V(Z)(1 - q_1 Z^{-1} - q_2 Z^{-2} - \dots - q_m Z^{-m}) \\ = U(Z)(b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_n Z^{-n}), \end{aligned} \quad (9)$$

or

$$V(Z) = \frac{(b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_n Z^{-n})}{(1 - q_1 Z^{-1} - q_2 Z^{-2} - \dots - q_m Z^{-m})} U(Z). \quad (10)$$

Since v_k is the output and u_k is the input in the application of a linear system, the polynomial in Z in Equation 10 is the transfer function of the system that was identified as $H(Z)$ in Equation 6. It follows that

$$\begin{aligned} H(Z) &= \sum_{i=0}^{\infty} h_i Z^{-i} \\ &= \frac{(b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_n Z^{-n})}{(1 - q_1 Z^{-1} - q_2 Z^{-2} - \dots - q_m Z^{-m})}. \end{aligned} \quad (11)$$

In Equation 11, if the values of b_i ($i=0, \dots, n$) and q_i ($i=1, \dots, m$) are known, then the values of h_i

($i=1, 2, \dots$) can be computed. Thus, the problem of finding h has been reduced to obtaining the parameters in the difference equation given by Equation 7. This is the reason why this analytic method is sometimes called the "parametric" approach.

Estimating the parameters. There are recursive routines which have been developed by control systems analysts for estimating the parameters in Equation 11 (Hsia, 1977; Sage & Melsa, 1971). However, we used a procedure which is likely to be more acceptable to those who are interested in the analysis of, for example, RT distributions.

Our general approach can be described as a three-step process: (1) Choose reasonable numbers for m and n in Equation 7, which relates the output values to the input values; (2) compute a stepwise, multiple-regression analysis to determine the values of q_i and b_i which yield the best, least squares fit for Equation 7; and (3) from knowledge of the values q_i and b_i , use Equation 11 to determine the values of h_i .

Initially, we decided to execute Step 1 by choosing a reasonably large number for both m and n (e.g., 6 or 8), and to let the stepwise multiple-regression routine do the work of eliminating those variables that were not significant in the regression at a chosen level of significance. It became obvious that this "best of all possible models" approach had limitations, mainly because of the variability that is inherent in all RT data. Accordingly, it was to our advantage to have some prior knowledge about the probability density to be identified in order to help us select reasonable values for m and n . For example, it was possible for us to estimate the mean value of t_d , since $t_d = RT - T_R$, where RT and T_R had known values. As discussed subsequently, this information about known mean values proved useful in the selection of the most appropriate models for the entire shapes of t_d .

The computations in Step 2 are straightforward because of the availability of a stepwise, multiple-regression routine in the most recent version of SPSS. In our early applications of Step 2, we allowed several nonzero values of b_i in the regression analyses. After deconvolving a wide variety of unimodal distributions, it became obvious that the use of more than one significant b_i in the regression analysis did not provide additional useful information about the shapes of the component distributions that were to be identified. Consequently, the outcomes of the regression analyses reported in our research involved only one nonzero value of b_i . While it is clear that more than one significant b_i in Equation 7 would obtain if it were suspected that the unknown distributions were multimodal, this simply does not follow because of the unimodal shapes of our obtained RT distributions. Thus, by using only one b_i , we have limited our general class of models to unimodal dis-

tributions, a restriction which seems reasonable for at least two reasons: (1) All of the empirical RT distributions (the T_R input and the RT outputs) from which our models were derived are unimodal; and (2) each of our component models, when reconvolved on the input distribution, T_R , provided a very satisfactory estimation of the particular output distribution, RT, from which the model was originally identified (see the dashed lines in Figure 1 and appropriate discussion). It appears that Box and Jenkins' (1976) discussion of the principle of parsimony in the construction of mathematical models describes our procedure rather well. In particular, they note that it is "important to employ the smallest possible number of parameters for adequate representation" (p. 17). Considering the experimental variation inherent in RT data, if we were to employ, say, six values of b_i when one would suffice, we might needlessly obtain a somewhat more complex model when a more simple and adequate one would suffice.

Finally, Step 3 could be performed by the process of long division. However, a more expedient method is to multiply both sides of Equation 11 by the denominator of $H(Z)$, and to equate coefficients of Z^{-i} on both sides of the equation. When this was done, the following, recursive relation was derived:

$$h_i = b_i + \sum_{j=1}^{\min(i,m)} h_{m-j} q_j \quad (12)$$

Equation 12 can be readily programmed for solution by a computer. In fact, we have developed a program which not only computes h_i , but also determines the mean value for the desired distribution we seek to identify.

In view of the foregoing considerations, we modified our original, three-step approach for estimating the parameters of the densities we sought to identify, as follows: (1) In Equation 7, choose m equal to some relatively large number (we used $m=6$ in our analyses), and subsequently test individual values of u_{k-i} , as determined by the significance of each in the regression analysis; (2) compute a stepwise, multiple-regression analysis to determine the coefficients of the significant variables in Equation 7 for each choice in Step 1; and (3) determine the values of h_i for the best result obtained in Step 2. While the word "best" implies a rather subjective interpretation, we followed three specific guidelines in our completion of Step 3: (1) a high coefficient of correlation (.96 or higher) for the specific choice selected in Step 2; (2) a mean value which approximates most closely the known mean value for the density to be identified; and (3) a convolution of the derived, best models of t_d (usually one or two) with T_R which most closely approximates the original, composite RT distribution. This last criterion was the most important test of the final version of each t_d that was accepted.

THEORETICAL ANALYSIS OF THE COMPONENT DENSITIES

Detection Densities

An important assumption in our analysis was that the response component, t_r , remained constant at all levels of intensity and frequency employed in our design. This is because the same Donders C paradigm was used throughout experimentation; that is, the foreperiod interval was 1 sec, and the ratio of "go" trials to "catch" trials was always 3:1. Furthermore, the same, highly practiced subject was used in all of the conditions, including T_R . When the variables of expectancy and set are controlled in this manner (particularly the constant foreperiod duration), Hohle (1965) has also argued for a relatively stable response component. Assuming that t_r does remain the same, the variations in the observed RT distributions, as seen in Figure 1, should reflect corresponding changes in t_d that result from fluctuations due to signal intensity and frequency.

The process of identification (deconvolution) followed the procedures outlined previously, thus yielding the most appropriate density for the particular detection process which underlies each of the observed RT distributions shown in Figure 1. These 20 models of t_d are shown in Figure 3. A view down each column permits a comparison across intensities, while a view across each row allows one to compare the models of t_d across frequencies. The ordinate of each detection density indicates the proportion of times a "detection response" occurred within each of the 5-msec bins.

It is evident from Figure 3 that detection at 80 and 60 phons is characterized by a family of exponential densities, whereas detection at 40 and 20 phons appears to range from gamma to near normal in form. The two exceptions are the 60-phon, 1,000-Hz density which looks either gamma or normal, and the somewhat irregular model at 40 phons, 500 Hz which appears "borderline" between gamma and exponential. In this respect, it should be noted that the exponential can be viewed as a special case of the gamma, so clear-cut distinctions between these two densities are not always appropriate. Furthermore, a lower order gamma and a normal distribution are similar in form; we retain these distinctions simply for clarity of discussion. Since the shapes of our detection models can range from sharply exponential to normal, the selection of the appropriate model (gamma, exponential, or normal) can sometimes be arbitrary, particularly if this selection is based solely on the appearance of the densities, that is, as shown in Figure 3.

In order to provide additional rationale for the selection of appropriate models, Table 2 gives detailed information about each density shown in Figure 3. The first two columns identify the particular intensity-

Figure 3. Sensory-detection densities (t_d) obtained by deconvolving t_r (Figure 2) from each of the corresponding reaction time distributions shown in Figure 1. The data are plotted in 5-msec bins. (The convolution of t_d with each t_r model above yields the corresponding density, at that particular intensity and frequency, indicated by the dashed lines in Figure 1.)

frequency value, and the next two columns give the mean detection latency (DL) and the standard deviation of the detection latency (SD, DL), respectively; these two statistics are also shown in Figure 3. The next column indicates the time delay (t) between zero and the first bin in which detection responses were observed. In the case of an exponential density, the value t is the fixed time delay. The criterion column (n) lists the estimated number of events (or "impulses") that underlie each density; this estimate is based on the a priori assumption that all the densities are gamma in form.² The relation between gamma densities and underlying "neural impulse" counts will be discussed subsequently.

Although it seems obvious from Figure 3 that about half of the densities are exponential in form, we thought the calculation of the gamma parameter n would also prove useful in helping to determine whether the parent density was exponential, gamma, or perhaps

normal in form. This was because an exponential is known to have one underlying event, whereas gamma and normals represent the convolution of two or more exponentials, and thus are characterized as having two or more underlying events. In this vein, we have found that the convolution of, for example, four or five exponential densities usually causes the shape of gamma to resemble the normal distribution. That is, as n becomes greater than two, the gamma distribution becomes less skewed and progressively more symmetrical. An excellent discussion of the transition from exponential to gamma to normal may be found in Benjamin and Cornell (1970).

Returning to Table 2, the number of impulses depicted in the criterion column was indicative of whether the various detection densities were either exponential or gamma. Thus, if the number of criterion impulses was one, the density was regarded as exponential, whereas if two or more impulses were obtained, either

Table 2
Exponential and Gamma Models Applied to the
Detection Densities Shown in Figure 3

Stimulus Intensity	Stimulus Frequency	DL		Delay (t)	Criterion (n)
		Mean	SD		
Gamma Distributions					
20	100	39.2	9.4	20	4
20	500	62.1	15.0	30	5
20	1,000	80.8	15.0	50	4
20	5,000	38.1	8.6	20	4
20	10,000	41.9	10.8	20	4
40	100	25.9	9.2	5	5
40	500	41.9	8.1	30	2
40	1,000	44.3	9.0	30	3
40	5,000	33.2	8.9	20	2
40	10,000	36.0	9.4	15	5
Exponential Distributions					
60	100	28.7	8.3	20	1
60	500	32.0	6.6	25	1
60	1,000	27.9	7.9	10	5*
60	5,000	27.7	7.2	20	1
60	10,000	27.1	11.6	15	1
80	100	19.3	3.8	15	1
80	500	19.9	4.2	15	1
80	1,000	25.2	9.7	15	1
80	5,000	23.1	7.7	15	1
80	10,000	20.4	4.8	15	1

Note—Stimulus intensity is given in phons; stimulus frequency is given in hertz. Detection latency (DL) and delay are given in milliseconds. *The 60-phon distribution at 1,000 Hz is gamma.

a higher order (near exponential) or a lower order (near normal) gamma was indicated. This information is listed in the last column of Table 2. If one compares the kind of distribution selected in Table 2 with the actual forms of these densities that are shown in Figure 3, no obvious discrepancy exists between these two sources of information about the shapes of our detection densities.³

The exponential models of t_d imply that the detection process operates much like a constant, conditional probability mechanism. A lower bound of an exponential distribution represents, in this case, a fixed time delay from signal onset to when the detector receives enough neural information for signal detection. The probability that detection will occur prior to the constant delay is essentially zero. An upper bound represents the point at which the decay function reaches zero. Obviously, the probability that detection will occur between the lower and upper bounds of an exponential distribution is 1.0. More importantly, it can be shown that the conditional probability of a temporal event described within the upper and lower bounds of an exponential distribution is fixed, and independent of any arbitrary time delay (see McGill, 1963). For instance, if the detection of an 80-phon, 1,000-Hz tone (see Figure 3) has not occurred by 20 msec, then the probability that detection will occur beyond, for example, 25 msec, is equal to the probability that detection will occur beyond 30 msec, given that it has not already occurred by 25 msec. In more

general terms, if the detector fails to respond to a signal up to time t , for any t above the lower bound, this failure does not alter one's prediction of the length of time (from t) before the detector will respond. Incidentally, this description also characterizes, for example, the time between arrivals to a queue when the arrivals are characterized by a Poisson probability law.

As noted previously, the relation between exponential and gamma distributions (including gammas with normal or near-normal shapes) is straightforward; that is, a gamma random variable is simply the sum of two or more exponential random variables. One way to conceptualize the events that underlie our gamma and/or normal densities at 20 and 40 phons is to assume that signal detection did not occur in response to only one impulse, as was the case with an exponential process; instead, the detector required several impulses before it was satisfied that a signal was present. In this vein, let us assume that signal detection in the RT process involves the continuous monitoring of neural activity in the detection mechanism. The onset of a RT signal initiates an abrupt change in the level of spontaneous neural activity (or "neural noise") in the auditory sensory system. This change in the signal-to-noise ratio might be described as a series of neural events or "impulses" which is processed by the detector. If a signal is sufficiently strong (i.e., 60 or 80 phons), only one impulse is necessary to meet the detection criterion (n). This can be inferred from the fact that an exponential density ($n = 1$) best describes the detection process for strong signals (see Figure 3 and Table 2). However, for relatively weak RT signals (i.e., 20 or 40 phons), the detection criterion requires more impulses before it is satisfied that a signal is present. The basis for this latter deduction is the observed, gamma-like densities that characterize detection at weak intensity levels. Of course, even when the criterion count is fixed (e.g., $n = 1$), detection times for a given RT signal will vary from trial to trial. This variability is described by the value of λ , the rate of decay in an exponential density. For weaker signals, the trial-to-trial variability can be described by the value of λ in a gamma density. Assuming gamma, λ refers to the average rate at which impulses are accumulated in the detector, whereas n specifies the criterion number that must accrue before the detector will fire.⁴

Returning to Figure 3, it is noteworthy that all of the exponential densities at 80 phons are very sharply peaked. For example, at 100 Hz, 80 phons, the detection process exhibits little variability, almost to the point of being a fixed time delay. If this were true, the detection density would be represented by a delta function (or "spike") at some point on the time axis, with virtually no variability. Indeed, almost all of the variability in the overall RT distributions at 80 phons (see Figure 1) is based on t_r (see Figure 2), the response

process. This supports our belief that the much more intense, 110-phon signal did, in fact, generate a RT distribution based almost entirely on t_r .

Finally, it may be observed that the shapes of the detection components differed across stimulus frequencies as well as stimulus intensities. For example, there are subtle differences in the shapes of the exponentials at 80 phons (see Figure 3). Since the time delay and the criterion number are the same across all five frequencies at 80 phons (see Table 2), any differences in the density shapes are attributable to variation in λ . Moreover, at 20 phons, the criterion counts (n) are nearly identical across frequencies, but the five densities have different, lower order gamma shapes. This again suggests that any effect of stimulus frequency on detection latency is reflected in λ . In other words, for stimulus intensities that were initially judged to be equivalently weak across five stimulus frequencies, the onset of these stimuli, when used as RT signals, generated different impulse rates (gamma λ) at these five different frequencies. However, at equivalently strong stimulus intensities, the frequency (in Hz) of the RT signal influenced the temporal precision with which a single impulse is detected (exponential λ).

Response Density

The distribution of T_R presented in Figure 2 resembles a Laplace distribution. McGill (1963) has described a Laplace distribution as being "essentially two exponential distributions turned back to back, resulting in a density function that is more sharply peaked than a normal variable" (p. 335). Kristofferson (1976) has also obtained similar highly peaked, symmetrical RT distributions from a practiced subject (see his Figure 9, p. 94). A Laplace distribution suggests the operation of a highly periodic latency mechanism that is well timed, practically invariant, and probably not based on a random, underlying variable. Furthermore, the Laplace often describes a mechanism in which excitations are input at regular intervals, but responses to the input can be subject to minor delays. This description is consistent with our general view of simple reaction time; once the detection mechanism is "satisfied" that a signal has occurred, its output directs the response mechanism to execute. Delays in responding to this output may result from minor fluctuations in the subject's attention or readiness to respond from trial to trial. Whatever the source of variability in t_r may be, it is clear from its Laplace form that the delays between excitation and response are minimal.

Reconvolutions

The most important test of the adequacy of our detection components is the reconvolution of each of the estimated densities in Figure 3 with the response distribution shown in Figure 2. Thus, each distribu-

tion of t_d in Figure 3, when reconvolved with t_r , should approximate the corresponding, composite probability distribution from which it was originally identified. In Figure 1, the distributions comprising the dashed lines represent the distributions obtained by means of reconvolution. While it appears obvious that the reconvolutions (predicted densities) closely approximate the observed RT distributions, we applied the chi-square tests for goodness of fit to each of the 20 pairs of distributions shown in Figure 1. In each case, the null hypothesis was that the predicted (reconvolutions) and the obtained distributions were the same. We decided to adopt the .05 level of significance. The number of degrees of freedom for each chi-square test was determined by (1) the number of RT bins, or samples, in which comparisons were made, minus (2) one, for the estimate of u_k , minus (3) the number of estimates of v_k that were utilized, minus (4) one, for the constraining number of RT measures in each distribution ($N = 288$). The number of estimates of v_k varied from a minimum of one (e.g., at 100 Hz, 80 phons) to a maximum of three (e.g., at 500 Hz, 20 phons).

Table 3 shows the obtained chi-square values, the appropriate degrees of freedom, and the p values for the 20 chi-square tests. In all cases, the chi-square values were not large enough to reject the hypothesis at the .05 level. For example, the comparison at 20 phons, 100 Hz involves 16 df, for which a chi-square value equal to or greater than 26.296 is necessary to reject the null hypothesis that the predicted and observed distributions are the same. In Table 3, the column marked " $p >$ " indicates the p value from

Table 3
Chi-Square Goodness-of-Fit Tests Applied to the 20 Pairs
of Distributions Shown in Figure 1

Stimulus Intensity	Stimulus Frequency	χ^2	df	$p >$
20	100	19.87	16	.30
20	500	30.20	21	.05
20	1,000	15.42	25	.90
20	5,000	16.09	18	.50
20	10,000	20.06	22	.50
40	100	19.00	17	.30
40	500	14.13	16	.50
40	1,000	11.38	18	.80
40	5,000	13.23	14	.50
40	10,000	23.52	19	.20
60	100	5.13	17	.99
60	500	17.48	15	.20
60	1,000	20.25	15	.10
60	5,000	21.45	17	.20
60	10,000	6.92	19	.99
80	100	25.79	18	.10
80	500	13.68	14	.30
80	1,000	18.14	21	.50
80	5,000	25.71	19	.10
80	10,000	23.53	18	.10

Note—Stimulus intensity is given in phons; stimulus frequency is given in hertz.

the table of chi-square which the corresponding obtained chi-square value did not quite reach. For example, at 20 phons, 500 Hz, a $\chi^2(21) = 30.20$ has a *p* value greater than .05 but less than .10; this particular comparison came closest to rejecting the hypothesis that the observed and predicted distributions were the same. In general, we think that the chi-square tests were a rather conservative (sensitive) test of the goodness of fits between the predicted and obtained distributions in Figure 1. Consequently, the hypothesis that the reconvolutions were good estimates of the observed distributions is clearly acceptable. Moreover, the success of the convolutions provides strong support for our view that the simple RT process is the composite of a uniform response density (t_r) and a unique detection density (t_d).

GENERAL REMARKS

One important assumption in this paper is that the response component in simple reaction time "holds still" when the same, highly trained subject is employed in a fixed RT paradigm. That is, for a given subject, the residual density (t_r) retains its shape, but the form of the detection density (t_d) is labile. In order to challenge this assumption, let us argue for the moment that t_r does change, particularly as a function of stimulus intensity. If so, perhaps our models of t_d at 80 and 60 phons are reasonably accurate representations of the detection process, but the models of t_d at 40 phons, and especially at 20 phons, are convolutions of the "true" t_d with some unknown component. Such an "unknown component" could be represented by the difference between the shape of t_r at 110 phons and the shape of t_r at, say, 20 phons. If this were the case, the family of t_d models obtained in the present study would be rather tentative, at best.

There are at least three reasons why we believe that t_r did, in fact, remain approximately the same across all of the stimulus conditions. First, as noted earlier, the same Donders C RT task was used in each condition. Second, the same subject was used throughout, and the variables of practice and incentive were carefully controlled. Third, and perhaps most important, it is generally accepted that changes in the response component in a RT paradigm result from variations in the foreperiod interval (e.g., Hohle, 1965; Sanders, 1977) and not from manipulations of the RT signal itself, that is, its intensity and frequency. In view of the fact that a fixed foreperiod of 1 sec was used throughout our research, we believe that the changes in the observed RT distributions across stimulus intensity and frequency reflect changes in the detection component, t_d .

Despite our confidence in the stability of t_r , the design of the present study does not permit us to reach decisive conclusions about this matter. In particular,

our current procedures do not rule out the possibility that T_R , even at 110 dB, may contain a small detection component whose shape is almost surely exponential, but whose time constant is unknown. Accordingly, a necessary problem for future research is to construct a priori models of t_d at different intensities and frequencies, and then to deconvolve these models from the empirical RT distributions to determine whether the same t_r density will result in each case; this is an extension of the procedure outlined by Green and Luce (1971). In this vein, a more conservative approach is to deconvolve t_d and t_r from RT without making any prior assumptions about the shapes of these component densities. Recent work in our laboratory indicates considerable initial success in this latter endeavor. Specifically, we have found that t_r tends to remain constant across different RT tasks involving signal detection (t_d), stimulus discrimination (t_D), and attention switching (t_{sw}); see Kohfeld, Note 2.

As a final note, we are well aware that the successful application of linear systems identification requires that the component processes that underlie the RT distribution must operate in series. Some investigators (e.g., Vorberg, Note 3) contend, based on rather compelling logic, that selection of a serial or a parallel model of the RT process should not be made at the outset, and may be virtually impossible to determine, even after careful data analysis. It is not our intent to make a decisive case for serial vs. parallel processing. However, the fact remains that linear systems identification worked satisfactorily (i.e., the reconvolutions shown in Figure 1 are quite acceptable), which provides evidence for our belief that the component stages, t_d and t_r , operated successively. What is less clear, strictly from a linear systems point of view, is the order in which the two stages occur. However, it seems intuitively obvious that a signal must be detected before a "true" response can be initiated in the RT paradigm. Future work in our laboratory, in which multiple stages in the RT process are to be identified within the framework of linear systems, should provide additional evidence pertaining to the consecutive nature of RT components.

SUMMARY AND CONCLUSIONS

In 1963, McGill proposed that the shape of the reaction-time distribution would provide a kind of "signature" that reflected the nature of the underlying events that comprised this distribution. Since then, Green and Luce (1971) have demonstrated that simple auditory RT is the convolution of two main components: sensory detection (t_d) and response execution (t_r). Based on these considerations, the main goal of the present research was first to identify the components that comprise the overall RT distribution, and

subsequently to infer the kind of hypothetical, "neural" events that give rise to the particular shapes of these components.

In order to deconvolve the overall RT distribution into its component stages, we obtained reaction times from a highly practiced subject to equally loud signals (across five frequencies) at 20-, 40-, 60-, and 80-phon intensity levels. These 20 RT distributions were treated as "outputs" within the framework of a systems-identification analysis of the RT process. The empirical "input" density was obtained by generating a RT distribution from responses to a very intense, 110-phon signal. Based on its sheer intensity, this distribution of responses was regarded as equivalent to the residual, or response, process (t_r), free of any sensory-detection latency. When t_r was deconvolved from each of the 20 RT distributions, using the technique known as linear systems identification, we solved for the other "input variable"; this latter variable is best described as a family of detection densities (t_d) that usually varies in accordance with the intensity and frequency of the RT signal. The reconvolution of each t_d density with the uniform t_r density provided very satisfactory approximations of the original RT distributions from which these two components were identified.

Our data indicate that the sensory-detection process at weak intensity levels is different from detection at relatively strong intensity levels. Specifically, the detection densities (t_d) at 20 and 40 phons (weak intensities) reveal a detection process that generates distributions ranging from gamma to normal, whereas the detection of RT signals at 60 and 80 phons (strong intensities) yields exponential distributions. Our theory is that as few as one "neural" impulse is required for the detection of a strong RT signal, whereas two or more impulses must accumulate before a relatively weak RT signal is detected. These data are consistent with Luce and Green's (1974) general view that the detection of weak signals (in the RT paradigm) may require a psychophysical process that is somewhat different from that required in the detection of strong signals.

As a final comment, we note, with some surprise, that previous evaluations of the power-function exponents for RT-intensity functions have focused exclusively on 1,000-Hz signals. In our present research, RT data are available across five frequencies, and one might be tempted to calculate the power exponents at the different frequencies to see how the values compare with the expected value of $-.3$ at 1,000 Hz (see, e.g., Figure 4 in Kohfeld et al., 1981). However, since the intensity levels which served as the RT signals are in phons (i.e., they are matched in loudness with 1,000 Hz), they do not form orderly RT-intensity functions with respect to the unique threshold (in dB, SL) for each frequency. Moreover,

there are only four data points in each intensity function (excluding the 110-dB value at 1,000 Hz). We think that at least 10 RT values, evenly spaced between 0 dB, SL, and the strongest intensity level (expressed in sound pressure units) at each frequency, should be obtained before detailed analyses of the RT-intensity functions are made. Accordingly, the next research article in our series on loudness and reaction time will deal with the following issues: (1) When mean detection times (Mt_d) are plotted as a function of stimulus intensity over a wide range of sound-pressure values, will the functions converge on $Mt_d = 0$ at the strongest intensity level (specified as T_R), as predicted by a successful deconvolution of t_r from the composite RT distributions at increasing intensities? (2) As we have already predicted, will the same densities of T_R as the one at 1,000 Hz also be generated at different frequencies? And (3) will these frequency-specific, t_r models, when deconvolved from overall RT distributions at increasing intensity levels within specific frequencies, generate families of t_d models which form power functions? Obviously, considerable parsimony would be achieved if similar power exponents could describe the general relation between stimulus intensity and reaction time, regardless of stimulus frequency. In any event, if the power law holds across stimulus frequencies, any similarities between the exponents for t_d scales (from reaction time) and the exponent for loudness would contribute to our knowledge of the power-transformation process.

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NOTES

1. We used a 3-point, moving-average smoothing technique to plot the continuous curve shown in Figure 2. This method of smoothing with weights of .25, .50, and .25 is called "hanning," after the Austrian meteorologist Julius Von Hann (see Chatfield, 1975, pp. 142-143). We have available a FORTRAN program for rapid computation of this smoothing function.

2. If we assume that trial-to-trial variability in the impulse rate for a given stimulus intensity can be described by a Poisson

probability distribution (Restle, 1961, pp. 165-170), it follows that the trial-to-trial variability in detection latency for a subject with a fixed criterion (set number of impulses) should form a gamma distribution with parameters λ (average impulse rate) and n (criterion impulse count). These parameters can be calculated from the detection densities, since it is known that $n/\lambda = \text{mean RT}$, and $n/\lambda^2 = \text{SD}^2$. By squaring both sides of the former equality, we get $n^2/\lambda^2 = \text{mean}^2$, and by substitution and reducing to lowest terms, we obtain $n = \text{mean}^2/\text{SD}^2$. In order to apply a gamma model to our detection densities, the time delay, t , must be subtracted from the mean detection latency (DL) for each density. Accordingly, the ratio $(t - \text{mean DL})^2/\text{SD}^2$ yields an estimate of the criterion count for a given detection density; these values are presented in the criterion column of Table 2.

3. A conservative criterion for selection of the "best" theoretical model for each detection density (t_d) would be to generate for example, a series of gamma distributions for each t_d at 20 and 40 phons, and subsequently to perform chi-square goodness-of-fit tests to evaluate the null hypothesis that the theoretical gamma distributions and the observed t_d densities have the same shapes. Another approach would be to examine the hazard functions of the t_d densities. As suggested by Luce (Note 1), hazard functions may reveal more about the forms of distributions than the actual density functions themselves (see also, Burbeck, 1979). Our plan is to present a detailed analysis of the forms of t_d , particularly at relatively weak signal intensities, in our next paper in this series on loudness and reaction time.

4. The ratio n/λ is the average interarrival time (IAT) between n pulses. Luce and Green (1972, 1974) have proposed a theory of the detection process in which the detector compares the average IAT between pulses with a criterion IAT value. This "timing" model assumes that the detector samples the impulse train to determine whether the average IAT within a fixed, brief interval is equal to or less than the criterion IAT; if so, the detector fires. We have offered a "counting" model of detection in this paper, but anticipate that other conceptualizations (e.g., a timing model) might also account for our detection densities. It is clear that both counting and timing models may be deduced from the same distributions, and thus are closely related.

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