Does perceived size depend on perceived distance? An argument from extended haptic perception

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Two experiments were directed at the comparison between two perspectives on the perception of size achieved by probing the gap between two occluded distal surfaces by means of a hand-held rod. One perspective was the classical size-distance invariance hypothesis developed for the problem of visual size perception with a central role for perceived distance; the other was the hypothesis that the extended haptic perception of gap size is specific to a physical invariant λ of the dynamics of probing. Experiment 1 examined the relation between haptically perceived gap size and haptically perceived gap distance. No causal connection between the two was found, and all the variance in perceived size was accounted for by λ . Experiment 2 manipulated the rotational inertia of the probe. Its effect was different for the two perceptions of size and distance, underscoring their independence. The indifference of perceived size to perceived distance was discussed in reference to identifying invariants for both the haptic and the visual perception of size at a distance.

In research on visual space perception, one of the most common assumptions is that the rules of Euclidean plane geometry that apply to physical objects in physical space also apply to perceived space. Under the most simplifying conditions,

$$S = \Omega D \quad (1)$$

expresses the relation between the physical size S of an object at a distance D from some reference point O , and the angle Ω subtended by S at the point O . When applied to the description of psychological events, this equation states that perceived size (S') and perceived distance (D') stand in a unique ratio determined by the visual angle (Θ) and is known as the *size-distance invariance hypothesis* (Kilpatrick & Ittelson, 1953):

$$S' = \Theta D'. \quad (2)$$

The equation summarizes a fact of common observation that if D' is constant, S' changes directly with the size of visual angle, or if S' is constant, D' changes inversely with Θ . The hypothesis has usually been tested by holding one of the variables constant and observing the degree of covariance of the remaining two. For instance, by manipulating the convergence distance of a focal target while keeping stimulus change minimal (thus, keeping Θ

constant), changes in D' are introduced that are accompanied by changes in S' congruent with Equation 2 (Gogel, Loomis, Newman, & Sharkey, 1985).

The adequacy of Equation 2 as a description of the observed relation between S' , D' , and Θ has been questioned since its formulation (see Sedgwick, 1986). For example, the empirically determined relation between S' and D' for a given Θ is sometimes opposite from that predicted by Equation 2—an effect referred to as the size-distance paradox (Gruber, 1954). Furthermore, the evidence in favor of the size-distance invariance ratio leaves open the possibility of independent determination of S' and D' within the perceptual system, and the link being a result of how the factors of size and distance are entangled in the environment (Hochberg, 1971). In spite of the contradictions, the assumption of a functional connection between S' and D' has endured. According to cognitive theories of perception (Gogel, 1990; Rock, 1977), stability and constancy in spatial perception are products of operations on coded representations of physical extents. The perceptual world is said to be fashioned by percept-percept couplings (e.g., Hochberg, 1974), or cue heuristics (e.g., Gilden, 1991), with the links among the percepts often assigned a causal interpretation: "Were we to extract the correlation between the conditioning percept and the focal percept, the partial correlation between proximal stimulus and focal percept would be zero" (Epstein, 1977, p. 169).

To enhance the explanatory use of Equation 2, modifications in the terms entering the equation have been introduced. The only nonperceptual term in the equation, Θ , has been replaced by Θ' , perceived angle (McCready, 1985), but no evidence of the changes in Θ' necessary to account for the size-distance paradox has been found.

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Registered distance (Rock & Kaufman, 1962), or effective distance (Baird & Wagner, 1991), has been proposed as a substitute for perceived distance. Registered or effective distance is defined as perceptual input resulting from distance cues that affect S' but do not necessarily affect D' . Other proposed modifications include known distance as a complementary factor to D' , weighted differentially according to perceptual circumstances that determine the observer's response (Gogel & DaSilva, 1987). Patently, the problem with these hypothesized entities is in empirically establishing their existence (Epstein, 1973).

Gibson's (1979/1986) account of size and distance perception stands in contrast to the kind of theory embodied by Equation 2. Epstein (1982) provides a succinct summary of the differences: For Gibson (1) there is no perceptual representation of size correlated with the object's retinal size, (2) S' and D' are independent direct functions of particular optical quantities, and (3) S' and D' are not causally linked nor is S' mediated by operations combining retinal angle and D' . In short, by Gibson's account, the appeal to Equation 2 and to perceptual entities in general is encouraged by incompleteness in the analysis of the conditions of stimulation. On the theory of perception as direct, a complete analysis is expected to reveal an invariant property of the structured energy distribution to which S' is related by a single-valued function. No "epistemic mediators" are needed (Turvey, 1977; Turvey, Shaw, Reed, & Mace, 1981).

Recent research has demonstrated the ability of human subjects to perceive the size of the separation or gap between two occluded distal surfaces by means of probing with a hand-held rod. In this case of extended haptic perception (Burton, 1993), the geometric relation expressed by Equation 1 can be readily defined under the conditions of a fixed axis of exploratory rotation (e.g., the wrist) that lies on the bisector of the angle that the gap of a given S at a given D forms with respect to the axis. It is a small step, therefore, to the hypothesis that Equation 2 may apply to the extended haptic perception of size at a distance—that is, that the angular excursion Θ of the hand-with-rod between the two gap edges determines an invariant ratio of the haptic S' and haptic D' (Hanley & Goff, 1974). Following Gibson's argument, however, one might ask whether there is an alternative hypothesis founded on an invariant of the mechanical energy distribution patterned by the activity of striking the gap. Mechanical analysis reveals a collective parameter λ that connects the muscular forces applied to the probe to the reactive forces impressed on the tissues of the arm (see Barac-Cikoja & Turvey, 1991, 1993, for the derivation of λ). The measurable quantity λ relates several variables:

$$\lambda = \sin\alpha [1 - (2a/b) + (ma^2/I_o)], \quad (3)$$

where α refers to the angular position of a limb-plus-probe system at the moment of contact and equals $\Theta/2$, a and b stand for distances to the center of mass of this

system and to the point of contact along the probe, respectively, m is the mass of the system and I_o is the rotational inertia of the system with respect to the fixed axis of rotation at O . For a given S and D and a given limb segment (e.g., hand, forearm) plus probe, λ is an invariant of the rigid body dynamics associated with striking back and forth between surfaces. Thus, at the moment of contact with a gap edge, there is a quantity available to the haptic system that is specific to a complex fact: a particular gap at a particular distance contacted by a particular probe. λ has been shown to accommodate the following experimental observations: (1) perception of a given S decreases with D , (2) for the same α , different S s are perceived as different, (3) perception of a given S changes with the inertial characteristics of limb segment(s) plus probe, and (4) discrimination among a given set of S s decreases at a specific rate as D increases (Barac-Cikoja & Turvey, 1991, 1993). In brief, the hypothesis that S' is fully constrained by and specific to λ can be expressed by rewriting Equation 3 as

$$S' \propto \sin\alpha [1 - (2a/b) + (ma^2/I_o)]. \quad (4)$$

The similarity between Equations 4 and 2 is evident. They both have an angular component and a quantity by which it is multiplied to yield S' . The natures of these two multipliers are profoundly different, however. Let the bracketed part of Equation 4 be symbolized by δ . Whereas D' is a hypothesized cognitive entity, δ is an observable, physically defined quantity (Barac-Cikoja & Turvey, 1991, 1993).

Two goals of the present research were to establish the relation between S' and D' in the case of haptic perception and to test the causal influence of D' on haptic S' . By extending the reach of the haptic system with a hand-held rod, conditions analogous to those of visual perception were created. Occluded gaps of various sizes were posted at various distances from the observer, who was asked to probe them with the hand-held rod. The task for the observer was to discern the size and the distance of the probed gap. The foregoing haptic analogs of visual size and distance perception were used in two experiments to distinguish between Equations 2 and 4 as explanations of S' . To date, comparisons have been made between Equation 4 and Equation 1 (Barac-Cikoja & Turvey, 1993) but not between Equation 4 and Equation 2, which is the psychological version of Equation 1.

EXPERIMENT 1

Experiment 1 addressed two issues. First, does S' enter into a constant ratio with D' , as Equation 2 predicts? Second, is the relation between S' and D' causal in accordance with percept-percept coupling? These two issues were examined in the context of the contrasting hypothesis that S' is indifferent to D' and dependent only on λ as defined by Equation 3. Previous demonstrations of the significance of λ have not excluded the potential

Figure 1. View of the experimental arrangement.

contribution of the subject's impression of the distal location of the probed gap.

Method

Subjects. Seven undergraduate students (4 men and 3 women) at the University of Connecticut participated in partial fulfillment of course requirements. They had no previous experience with the task. All subjects were right-handed.

Materials and Apparatus. The experimental arrangement is diagrammed in Figure 1. The subject was seated in front of a table (width $\times$ length = 150 $\times$ 200 cm) to the left of a curtain, which extended in the subject's sagittal plane throughout the length of the table and divided it in half. The curtain occluded the subject's right hand and the stimulus apparatus. The stimulus apparatus consisted of a pair of wooden blocks inserted in a base of support that allowed the between-block separation to be adjusted and provided stability during the exploratory striking. The separation between the blocks and their distance from the subject were adjusted by 1 of 2 experimenters. Two response devices, one used to reproduce gap size and one used to reproduce gap distance, were placed in front of the subject. The size-matching apparatus was similar to the stimulus apparatus except for the right block, which was fastened. The left block could be slid within a casing to produce a maximum 50-cm separation between it and the right block. Placed farther to the left was the distance-matching apparatus: a single block mounted on a rail that extended throughout the length of the table. The subject moved the block along the rail by pulling the rope attached to it through a system of pulleys. The range of responses permitted by the distance-matching apparatus was 0–200 cm. Both response devices were operated within a comfortable reach. Scale markings in centimeters were affixed to the back of each apparatus to be visible only to the experimenter who recorded the measurements. Response devices were viewed in a well-illuminated room, on a textured table top, with both eyes and with no restrictions on head movements, to ensure that the measurements of size and distance responses indeed represented haptically perceived magnitudes. Each of the five blocks had thin surfaces (width = 0.5 cm) attached to their frontal side in order to provide sharp edges. The size and distance of the stimulus gap could, therefore, be defined unambiguously, and the responses could be measured precisely. The edges of the explored aperture were padded with a thin foam to reduce substantially the sounds resulting from striking. The probe used in exploration was a 38-g wooden rod (60 cm long), with uniform mass distribution.

Procedure. The subject was required to contact the aperture edges with the rod held in the right hand. The rod was placed in the

hand by one of the experimenters at 5-cm distance from the wrist. The subject was instructed to hold the rod firmly and keep its position within a hand unchanged throughout the experiment. This was monitored by the experimenter. The right forearm was fastened to a horizontal support that provided a fixed position of the wrist throughout the experiment. At the beginning of each trial, the aperture was centered around the longitudinal axis of the forearm–hand–rod. The instruction stressed that exploratory movements should be performed in the horizontal plane. There were no limitations on the manner and the rate of exploratory movements nor on the time to produce a measurement. The subjects used the left hand to give responses. They were instructed to form an aperture of the same size as the one felt by adjusting the position of the left block in the size apparatus. In order to match perceived distance, the subjects were asked to place the response block in the same plane with the explored aperture. The subjects were informed that the wrist was aligned with the edge of the table and that it served as the common reference point for the measurement of stimulus and response distances. Initial position of the two manipulated response blocks could be at either extreme and was randomized across trials. On each trial, the subject was called to make either size or distance response first. Two trials (with size/distance conditions not used in the experiment) served for a demonstration of the task. There was no feedback information.

Design. Three aperture sizes (5, 10, or 15 cm) appeared at three distances (25, 45, or 65 cm measured from the wrist). On each trial, the subject's task was to match both the size and the distance of the aperture. Each size/distance combination was repeated four times, with a size response preceding a distance response on half of the trials. All 36 trials were individually randomized, as was the order in which size and distance reports were made. The experimental session lasted less than 1 h.

Results and Discussion

Perceived size and distance values averaged across trials and subjects for each of the size/distance conditions are presented in Table 1. As expected, the subjects were capable of discriminating apertures of different sizes. They also clearly distinguished three aperture distances. Further inspection of Table 1 reveals that perceived size steadily decreased with increased distance, while perceived distance appears not to have been affected by aperture size. Two repeated-measures analysis of variance (ANOVAs), for each task separately, were conducted on the individual subject data. The ANOVA on size responses showed significant main effects of size [$F(2,12) = 50.39, p < .0001$] and distance [$F(2,12) = 31.47, p < .0001$], as well as their interaction [$F(4,24) = 7.10, p < .001$]. The interaction reveals that apertures of

Table 1
S' and D' as a Function of S and D in Experiment 1

Actual Extent		Perceived Extent	
S	D	S'	D'
5	25	7.47	20.18
	45	5.84	29.41
	65	5.04	41.78
10	25	12.89	18.62
	45	10.22	34.97
	65	8.37	41.60
15	25	20.44	20.05
	45	13.10	32.90
	65	12.18	40.70

Note—All values are given in centimeters.

the same size felt increasingly smaller with an increase in distance. The ANOVA on distance responses revealed a significant effect of distance [$F(2,12) = 19.39, p < .001$]. Neither the effect of size [$F(2,12) < 1$] nor the size $\times$ distance interaction [$F(4,24) < 1$] was significant.

Three linear regressions of S' averaged across trials and subjects were conducted: on the product of mean D' and $2\sin\alpha$ (i.e., on S' as predicted by the size-distance invariance hypothesis), on the product of D and $2\sin\alpha$ (i.e., on S), and on the parameter λ . Less than 85% of the total variability in S' was accounted for by $D' \times 2\sin\alpha$ (Figure 2, top). Even less predictive was physical size or $D \times 2\sin\alpha$ ($r^2 = .69$, with regression coefficient = .91, and intercept = 1.49). These results suggest, as often found in visual perception, that given the angle, D' is a better predictor of S' than is D . Finally, when mean S' was regressed on λ , about 98% of the total variance was accounted for (Figure 2, bottom). To further test the contribution of D' to S' , the residuals from the simple regression of S' on λ were regressed on D' . It was found that D' was not related in any systematic way to the unaccounted 2% of the variance ($r^2 = .005, p = .8$). To the contrary, when the residual of the simple regression on

$\sin\alpha$ was regressed against D' , the contribution of D' proved significant ($r^2 = .44, p < .05$).

The preceding comparisons substantiate the claim that evoking perceptual entities in explanations of S' is necessitated by incompleteness of the stimulus analysis. A percept-percept coupling hypothesis rests on the assumption that the proximal stimulus in size perception is given by the angle that some linear extent in space determines relative to the nodal point occupied by receptors. This limited stimulus has to be supplemented with D' . However, according to Gibson's (1979/1986) direct perception hypothesis, concurrent percepts lose their causal role when a description of existing information is sufficiently rich. Analysis of partial correlations was conducted in order to further illuminate this distinction.

With one independent variable (IV) and two dependent variables (D' and S'), five causal relations, or models, are possible (Oyama, 1977): (1) changes in D' and S' result from the joint causal effect of IV (i.e., the two percepts are independent covariants of IV), (2) IV operates on S' through the change in D' , therefore S' is directly caused by D' , or (3) there is the same chain of causation, except that S' precedes D' , (4) IV influences S' directly, but also indirectly, via D' , thus S' is in direct causal relation with both variables, or (5) there is the same triangular causal arrangement, except that IV and S' determine D' . Analysis of partial correlations allows for separation of "true" from "spurious" correlation, and consequently, elimination of the inadequate causal models (Blalock, 1962). The relation described by Model 1 above demands that the correlation between S' and D' disappears when relational effects of IV , their common cause, are partialled out. The analysis with λ as IV fits this model: correlation between the two percepts when the relational effects of λ were partialled out was non-significant [$r_{(S'D':\lambda)} = .15, p > .05$; all correlations were calculated on 63 data points representing means over trials for nine conditions and 7 subjects]. To the contrary, the analysis with $\sin\alpha$ as IV allowed for causal connection between the two percepts: $r_{(S'D':\sin\alpha)} = .29$ ($p < .05$), and therefore for any of the remaining four models. For Models 2 or 3 to be accepted, partial correlation between $\sin\alpha$ and either S' or D' should go to zero when the other percept is held constant. This requirement was not met [$r_{(S'\sin\alpha:D')} = .83$, and $r_{(D'\sin\alpha:S')} = -.44$], leaving Model 4 or 5 as a possible model. However, the implied causal link between the two percepts dissolved when D or S was added as an independent determinant of D' or S' , respectively. Both second-order partial correlations between S' and D' with joint effect of $\sin\alpha$ and D or S held constant were insignificant [$r_{(S'D':\sin\alpha D)} = .17, p > .05$, and $r_{(S'D':\sin\alpha S)} = .18, p > .05$]. In sum, this finding undermines Models 4 and 5 and further demonstrates the need to attribute a causal role to a percept when available stimulation is not adequately captured.

The physical parameter λ , by definition, captures the relation among the geometric variables that describe the stimulus configuration (S , D , and $\sin\alpha$). Consequently, their effect on S' should be completely exhausted by λ —

Figure 2. S' as a function of the product of D' and $2\sin\alpha$ (top) and of λ (bottom) in Experiment 1.

that is, the requirements of Model 2, where λ is the mediating variable between I/V and S' , should apply. In compliance with the model, the correlation between S' and each of the geometric variables was nullified by partialing out the effect of λ [$r_{(S'S\lambda)} = -.003$; $r_{(S'D\lambda)} = .00$; $r_{(S'\sin\alpha\lambda)} = .07$]. None was affected, though, by partialing out the effect of D' . These findings imply that the relation between environmental layout, as represented by S , D , and $\sin\alpha$, and S' is not mediated by the other haptic percept, but is constrained by the invariant of the probing dynamics. Finally, when all manipulated variables that may affect the two percepts were partialled out, the correlation between S' and D' did not significantly dif-

fer from zero [$r_{(S'D'SD\sin\alpha\lambda)} = .167$, $p > .05$]. (A complete listing of simple correlations is given in the Appendix.)

A final comparison between the haptic size-distance invariance hypothesis and the λ hypothesis concerns changes in accuracy that occur with increases in D . A deviation of S' from S is used as an index of accuracy and plotted against D . There were two sets of predicted deviation values: one calculated by subtracting S' as given by Equation 2 from S ; the other calculated by subtracting S' as given by $\lambda \times 100$ (100 is a constant factor that brings λ values into the scale of S ; see Barac-Cikojia & Turvey, 1993) from the respective values of S . These expectations were plotted in Figure 3 (top and middle, respectively) for the comparison with deviations of measured S' from S (Figure 3, bottom). We can see that both Equation 2 and Equation 4 predict, and data confirm, an increase in error with an increase in S . Furthermore, Figure 3 (top) shows that the size-distance invariance hypothesis predicts a constant increase in undershoot in S' with increasing D . From the λ hypothesis, however, a change in the sign of error is expected—from overestimations at smaller D to underestimations at larger D (see Figure 3, middle). It is obvious from Figure 3 (bottom) that the obtained deviations follow the trend predicted by λ .

In sum, Experiment 1 replicated previous observations that S' is affected by both S and D (Barac-Cikojia & Turvey, 1991, 1993). In addition, it revealed that D' is unaffected by changes in S . Analyses suggest that the effect of aperture D does not translate into a causal effect of D' . Although D' predicts changes in S' better than does D , its contribution is manifest only when the conditions of mechanical stimulation are expressed in merely geometric terms. When a kinetically based observable (namely, λ) expressed what was invariant about the conditions of mechanical stimulation, the seeming effects of D' disappeared and better predictions of S' were obtained.

EXPERIMENT 2

In Experiment 2, two rods of different rotational inertia I_{rod} were used in aperture exploration. The objectives were (1) to establish the effect of this manipulation on S' and D' when both are reported for a given aperture on the same trial and (2) to probe further the role of D' on S' through the dependence of each on I_{rod} .

Method

Subjects. Seven undergraduate students (4 women and 3 men) at the University of Connecticut participated in partial fulfillment of course requirements. None of them had participated in similar experiments before. One man was left-handed.

Materials and Apparatus. The materials and apparatus were the same as in Experiment 1, with the exception of the rod that was used in probing the apertures. A wooden rod (60 cm in length and 1.2 cm in diameter) was weighted with a cylindrical piece of lead (9 cm in length, 1 cm in diameter) inserted into a hole drilled in its one end. The mass of this rod was 76 g, and its moment of inertia changed depending on the position of the weighted end with respect to the axis of rotation. When the end of the rod with the inserted lead piece was held in the hand (i.e., closer to the

Figure 3. Deviations of S' from S in Experiment 1 as predicted by the size-distance invariance hypothesis (top) and λ (middle), together with the observed deviations (bottom), plotted against D . S is the curve parameter.

Table 2
 S' and D' as a Function of I_{rod} , S , and D in Experiment 2

Actual Extent		Perceived Extent			
		$I_{rod} = 84.1 \text{ kgcm}^2$		$I_{rod} = 222.5 \text{ kgcm}^2$	
S	D	S'	D'	S'	D'
5	25	8.93	24.51	8.82	20.20
	45	6.45	34.60	6.45	33.66
	65	5.43	40.41	6.12	44.70
10	25	16.50	22.86	15.95	18.90
	45	11.98	35.87	11.32	37.59
	65	9.71	40.61	10.13	46.16
15	25	22.39	21.43	21.59	17.60
	45	16.48	34.41	15.32	34.69
	65	12.79	40.69	12.82	41.46

Note—All values are given in centimeters.

Table 3
 λ as a Function of D , S , and I_{rod} in Experiment 2

Actual Extent		λ	
		$I_{rod} = 84.1 \text{ kgcm}^2$	$I_{rod} = 222.5 \text{ kgcm}^2$
25	5	.066	.059
	10	.133	.117
	15	.199	.176
45	5	.049	.044
	10	.098	.089
	15	.146	.133
65	5	.037	.034
	10	.074	.068
	15	.111	.102

Note— D and S values are given in centimeters.

axis), moment of inertia of the rod about the axis at the wrist was 84.1 kg cm^2 . Placing the unweighted end of the rod in the hand yielded a moment of inertia about the axis at the wrist of 222.5 kg cm^2 .

Procedure. On each trial, the rod was placed in the subject's hand with its weighted end either proximal or distal to the wrist. In all other details, the procedure was the same as in Experiment 1.

Design. Apertures of three sizes (5, 10, or 15 cm) appeared at three distances (25, 45, or 65 cm, measured from the axis at the wrist). Nine aperture size/distance configurations were explored with both rods. Each subject was presented all size/distance/rod conditions four times in an individually randomized order. The experimental session lasted about 1 h.

Results and Discussion

The results are summarized in Table 2. Two repeated-measures ANOVAs were conducted on the individual subject data. The ANOVA on size responses revealed significant main effects of S and D [$F(2,12) = 52.74$, $p < .0001$, and $F(2,12) = 24.19$, $p < .0001$], respectively. Also significant was the $S \times D$ interaction [$F(4,24) = 6.39$, $p < .001$], revealing a decrease in S' with an increase in D . The main effect of I_{rod} was not significant, nor did any of its interactions reach significance. The ANOVA on distance responses revealed a significant main effect of D [$F(2,12) = 19.02$, $p < .001$]. The main effects of S and I_{rod} were not significant, but the effect of I_{rod} on D' was indicated by its significant interaction with D [$F(2,12) = 6.06$, $p < .05$]. No other interaction was significant. In sum, as in Experiment 1, S' was affected by both S and D , while D' did not vary with

changes in S . Manipulation of I_{rod} did not affect S' , but it did affect D' : D' was greater for the larger I_{rod} in agreement with previous research on probing to perceive distance (Carello, Fitzpatrick, & Turvey, 1992; Chan & Turvey, 1991). Although the effect of I_{rod} on S' was not statistically significant, the direction of the difference was consistent with λ and with previous research on probing to perceive S (Barac-Cikoja & Turvey, 1991, 1993): S' was numerically greater for the smaller I_{rod} .

Simple regression of S' on the product of $D' \times 2\sin\alpha$ resulted in $r^2 = .82$ (see Figure 4, top). The same analysis with λ as the independent variable resulted in $r^2 = .98$ (see Figure 4, bottom). Inspection of Figure 4 (bottom) indicates a subtle role of I_{rod} in affecting S' . It should be noted here that the effects of I_{rod} manipulation on λ are countered by the concomitant changes in a and m and are rather fine as shown in Table 3.

Finally, the partial correlation analysis confirmed the findings of Experiment 1. While the correlation between S' and D' was significant when the angle was held constant [$r_{(S'D', \sin\alpha)} = .32$, $p < .05$], suggesting a causal relation between the two variables, it turned nonsignificant when the relational effects of λ were controlled [$r_{(S'D', \lambda)} = .16$, $p > .05$]. When all the manipulated variables were

Figure 4. S' as a function of the product of $2\sin\alpha$ and D' (top) and λ (bottom) in Experiment 2.

Figure 5. D' (top) and δ (bottom) as a function of I_{rod} in Experiments 1 and 2. D is the curve parameter.

partialled out, the correlation between S' and D' was found nonsignificant [$r(S'D', SD \sin \alpha \text{rod} \lambda) = .247, p > .05$].

The upshot of the regression analyses together with the analyses of partial correlations is the implication that the effects of S , D , and I_{rod} on S' are all accounted for by a single observable λ , leaving no room for D' as a causal factor. The structure of the dynamically derived Equation 4 resembles the structure of the trigonometric Equation 1 and its perceptual derivative Equation 2: In all three equations, an angular component is multiplied by a quantity that is a distance or is related to distance. Clearly, the natures of these “distance quantities”— D , D' , and δ —are different. The relation between D and D' , under the conditions of Experiment 2, is apparent from Table 2. D' tended to increasingly underestimate D as D assumed larger values, an observation that characterized data from Experiment 1 as well. To highlight the contrast between D' and δ (the quantity that was shown to affect S'), D' and δ are presented in Figure 5 as a function of I_{rod} in Experiments 1 and 2. The three curves

in each plot represent the regression lines for three values of D . We see that at larger distances, D' showed a tendency to increase with increase in I_{rod} , whereas δ decreased. In sum, manipulation of I_{rod} broke the covariance of S' (dependent on δ) and D' , indicating different dynamical invariants underpinning these two perceptual tasks and attesting further to their independence.

GENERAL DISCUSSION

Experiments 1 and 2 were directed at the relation between the extended haptic perceptions of specific dimensions of environmental layout—namely, the S and D of apertures. Both experiments revealed a haptic capability to discriminate reliably among occluded gap sizes (in agreement with Barac-Cikoja & Turvey, 1991, 1993) and gap distances (in agreement with Carello et al., 1992; Chan & Turvey, 1991) when the gaps were contacted with a hand-held rod. In both experiments, it was shown that S' changed with changes in both S and D , whereas D' was unaffected by changes in S . Previous studies (Barac-Cikoja & Turvey, 1991, 1993) demonstrated a complex dependence of S' on inertial characteristics (I_{rod} , m , a) of the probe used in exploration of the surface layout, which was only hinted in the results of Experiment 2. Separate, significant effects of inertial characteristics of the probe on D' were demonstrated on the pooled data from the two experiments.

Traditionally, the relation between visually perceived size and distance has been expressed in terms of the size–distance invariance hypothesis, Equation 2. Consistent with Equation 2, the present research confirms that in extended haptic perception, D' predicts S' better than does D . But it also shows that the haptic equivalent of Equation 2 leaves a significant part of the variance in S' unaccounted for. When the analysis of the conditions of stimulation is conducted in terms of kinetic variables, as expressed in the alternative λ hypothesis, no room for a contribution of D' was left in either experiment. Thus, the advantage of D' over D in predicting S' seems to be rooted in the informational basis shared by the two percepts that allows D' to stand in for information more adequately expressed by λ .

Further analyses were provided to compare the causal interpretation of percept–percept coupling—an important corollary of the size–distance invariance hypothesis—and the λ hypothesis, as two contrasting interpretations of the basis for S' . A causal role of D' seems to be implied only when the haptic angle is assumed to be the entirety of the haptic input; in the context of the kinetic variables comprising λ , the causal role expected from percept–percept coupling was never confirmed.

By way of a final summarizing remark on the present data, we note that changes in visual S' that contradict what Equation 2 predicts on the basis of measured D' have sometimes been attributed to changes in “registered D .” The main problem with this strategy, as remarked in the introduction, is operationalizing “regis-

tered D ." As a quantity that is related to D that affects S' but not through the changes in D' , δ provides the appropriate operational definition in the present haptic case. It must be underscored, however, that δ is solely a description of haptic stimulation available in the situation of probing a gap and is completely independent of a perceiver's cognition and processing capabilities.

Research on haptic size-at-a-distance perception may have important implications for the more traditional problem of visual size-at-a-distance perception. Let us return to the bracketed part of Equation 3 symbolized by δ . Because $I = mK^2$, where K is the radius of gyration, δ can be rewritten as

$$\delta = 1 - (2a/b) + (a^2/K^2). \quad (5)$$

Furthermore, because K^2/a is the distance p of the center of percussion from the rotation point O , the expression for δ can be written even more simply as

$$\delta = 1 - (2a/b) + (a/p). \quad (6)$$

What Equation 6 brings out is the fact that δ is a configuration of different kinds of distances from O —namely, to center of mass (a), to point of contact (b), and to center of percussion (p) (Barac-Cikoja & Turvey, 1993). These "points" define the constant geometric structure of the probing. During rotational probing about a fixed point O , these distances remain invariant. Collectively, as the single quantity δ , they scale the angle through which the probe moves (specifically, the component of the contact force associated with a given angular displacement). It can be seen from Equation 6 and 4 that if p equals $b/2$, then perceived size would be completely determined by $\sin\alpha$. This would be analogous to the "law of retinal size," meaning a complete absence of size constancy under the perceptual conditions of dynamic touch.

In vision research, failures of size constancy have led to hypotheses of inferential biases established either through experience or through evolution (e.g., Gilinsky, 1955). In Experiments 1 and 2, failure of haptic size constancy was tied directly to observables within the experiments—namely, δ and b . In previous research, variations in the rate at which S' changes with S were predicted by the partial derivative of λ with respect to S —a quantity composed solely from the observables a , b , and p (Barac-Cikoja & Turvey, 1993). Given a hypothesis of information-perception specificity (e.g., Barac-Cikoja & Turvey, 1991, 1993; Gibson, 1979/1986), it is expected that failures of visual size constancy and variations in the actual size derivatives of perceived size are similarly explainable. In Equations 1 and 2, there is only D (corresponding to the quantity b in the haptic equations) and D' , respectively, to scale angle. These measures alone quantify "surface layout." Using Equation 4 in conjunction with Equation 6 as the haptic model, it is reasonable to assume that neither D nor D' are sufficient to capture the invariant geometry underlying the visual inspection (by mobile eyes in a mobile head) of an object of a given S at a fixed D from the observer. In short, Equations 1

and 2 are incomplete. The implication is that there are important aspects of the layout of surfaces and of the optic array lawfully engendered by that layout that are currently excluded from Equations 1 and 2. From the perspective of the haptic model given by Equation 6, Equations 1 and 2 are lacking the functional equivalents of a and p . In searching for these functional equivalents, guidance should be provided by the more abstract understanding of a and p as symmetries of the mass distribution—quantities that are invariant over well-defined classes of transformations. The present and previous research on the extended haptic perception of S pose, therefore, a new kind of question for the visual perception of S : Given a surface layout, and the optical structure engendered by it, what are the optical symmetries equivalent to a and p ?

REFERENCES

- BAIRD, J. C., & WAGNER, M. (1991). Transformation theory of size judgment. *Journal of Experimental Psychology: Human Perception & Performance*, **17**, 852-864.
- BARAC-CIKOJA, D., & TURVEY, M. T. (1991). Perceiving aperture size by striking. *Journal of Experimental Psychology: Human Perception & Performance*, **17**, 330-346.
- BARAC-CIKOJA, D., & TURVEY, M. T. (1993). Haptically perceiving size at a distance. *Journal of Experimental Psychology: General*, **122**, 347-370.
- BLALOCK, H. M. (1962). Four-variable causal models and partial correlations. *American Journal of Sociology*, **68**, 182-194.
- BURTON, G. (1993). Non-neural extensions of haptic sensitivity. *Ecological Psychology*, **5**, 105-124.
- CARELLO, C., FITZPATRICK, P., & TURVEY, M. T. (1992). Haptic probing: Perceiving the length of the probe and the distance of the surface probed. *Perception & Psychophysics*, **51**, 580-598.
- CHAN, T. C., & TURVEY, M. T. (1991). Perceiving the vertical distance of surfaces by means of a hand-held probe. *Journal of Experimental Psychology: Human Perception & Performance*, **17**, 347-358.
- EPSTEIN, W. (1973). The process of "taking-into-account" in visual perception. *Perception*, **2**, 267-285.
- EPSTEIN, W. (1977). What are the prospects for a higher-order stimulus theory of perception? *Scandinavian Journal of Psychology*, **18**, 164-171.
- EPSTEIN, W. (1982). Percept-percept couplings. *Perception*, **11**, 75-83.
- GIBSON, J. J. (1986). *The ecological approach to visual perception*. Hillsdale, NJ: Erlbaum. (Original work published 1979)
- GILDEN, D. L. (1991). On the origin of dynamical awareness. *Psychological Review*, **98**, 554-568.
- GILINSKY, A. S. (1955). The effect of attitude on the perception of size. *American Journal of Psychology*, **68**, 173-192.
- GOGEL, W. C. (1990). A theory of phenomenal geometry and its applications. *Perception & Psychophysics*, **48**, 105-123.
- GOGEL, W. C., & DASILVA, J. A. (1987). A two-process theory of the response to size and distance. *Perception & Psychophysics*, **41**, 220-238.
- GOGEL, W. C., LOOMIS, J. M., NEWMAN, N. J., & SHARKEY, T. J. (1985). Agreement between indirect measures of perceived distance. *Perception & Psychophysics*, **37**, 17-27.
- GRUBER, H. E. (1954). The relation of perceived size to perceived distance. *American Journal of Psychology*, **67**, 411-426.
- HANLEY, C., & GOFF, D. P. (1974). Size constancy in extended haptic space. *Perception & Psychophysics*, **15**, 97-100.
- HOCHBERG, J. (1971). Perception II: Shape and movement. In J. W. Kling & L. A. Riggs (Eds.), *Woodworth and Schlosberg's Experimental psychology* (3rd ed., pp. 475-550). New York: Holt, Rinehart & Winston.
- HOCHBERG, J. (1974). Higher-order stimuli and inter-response cou-

- pling in the perception of the visual world. In R. B. MacLeod and H. L. Pick (Eds.), *Perception: Essays in honor of J. J. Gibson* (pp. 17-39). Ithaca, NY: Cornell University Press.
- KILPATRICK, F. P., & ITTELSON, W. H. (1953). The size-distance invariance hypothesis. *Psychological Review*, **60**, 223-231.
- MCCREADY, D. (1985). On size, distance, and visual angle perception. *Perception & Psychophysics*, **37**, 323-334.
- OYAMA, T. (1977). Analysis of causal relations in the perceptual constancies. In W. Epstein (Ed.), *Stability and constancy in visual perception* (pp. 183-216). New York: Wiley.
- ROCK, I. (1977). In defense of unconscious inference. In W. Epstein (Ed.), *Stability and constancy in visual perception: Mechanism and processes* (pp. 321-373). New York: Wiley.
- ROCK, I., & KAUFMAN, L. (1962). The moon illusion II. *Science*, **136**, 1023-1031.
- SEDGWICK, H. A. (1986). Space perception. In K. R. Boff, L. Kaufman, & J. P. Thomas (Eds.), *Handbook of perception and human performance* (Vol. 1, pp. 1-57). New York: Wiley.
- TURVEY, M. T. (1977). Contrasting orientations to the theory of visual-information processing. *Psychological Review*, **84**, 67-88.
- TURVEY, M. T., SHAW, R. E., REED, E. S., & MACE, W. M. (1981). Ecological laws of perceiving and acting: In reply to Fodor and Pylyshyn (1981). *Cognition*, **9**, 237-304.

APPENDIX **Simple Correlations in Experiment 1**

	S'	D'	λ	$\sin\alpha$
S	.710	.019	.846	.686
D	-.395	.559	-.490	-.653
$\sin\alpha$	.822	-.362	.964	
λ	.841	-.260		
D'	-.141			

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