On the perception of a class of bilaterally symmetric forms*

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Two experiments were performed to determine the manner in which people perceive a class of bilaterally symmetric forms. In the first experiment, paired comparison dissimilarity judgments and unidimensional single stimulus ratings were collected from the Ss. The second experiment involved the collecting of semantic differential and paired comparison dissimilarity judgments from two groups of Ss. The judgments and ratings from both experiments were compared to each other and to a set of objective measures of the forms by a series of multivariate statistical techniques. It was found that three primary attributes, orientation, convexity-concavity, and curvature, accounted for the perceptual reports, but the manner in which the attributes were used differed for different response procedures and for the same response on different occasions.

Attneave and Arnoult (1956), while describing objective rules for generating form stimuli, indicated the need for a psychophysics of form perception. Since then, the perception of irregular polygons generated by their rules has been extensively analyzed. For example, Stenson (1968) found, with the aid of canonical correlation analysis, intersecting psychological and physical dimensions of forms generated by Attneave and Arnoult's fourth rule. In a parallel line of investigation, Brown and his associates (Aiken & Brown, 1969a; Behrman & Brown, 1968; Brown & Andrews, 1968) performed a sequence of studies to determine the dimensions of the psychological space for a class of forms generated by a modified version of Attneave and Arnoult's first rule, whose physical dimensions had been examined by Brown and Owen (1967). It is consistently found, as Richards (1972a) recently reconfirmed, that perimeter and area are important physical antecedents of perceptions of the irregular polygons.

The literature on form perception since Attneave and Arnoult's original contribution has been primarily concerned with irregular polygons. In order to develop a psychophysics of *form* perception, instead of irregular polygon perception, it will be necessary to apply sophisticated measurement algorithms to various other

domains of form stimuli. Dobson (1971) has recently investigated a domain of form stimuli, Minkowski patterns, which has a series of interesting properties. While the Attneave and Arnoult forms are usually presented as irregular polygons, the Minkowski patterns are regularly shaped and they are symmetric about three axes. The Minkowski patterns include rectilinear patterns, but most of the forms in the domain are curvilinear. Another interesting property of Minkowski patterns is the variety of well-known forms in the domain; these include a cross, diamond, circle, and square. Perhaps this feature may be of value in some applied investigation of form perception. Finally, the stimulus set may be parsimoniously described, since all forms in the domain may be obtained by adjusting a single parameter in the Minkowski unit distance equation. The unique properties of Minkowski patterns relative to irregular polygons suggest that the former may be useful in helping to extend our comprehension of form perception.

In the initial investigation of the Minkowski patterns (Dobson, 1971), magnitude estimation techniques were employed to scale size, convexity, and curvature. While the psychophysical function for size was consistent with prior experiments and produced reliable results between different groups of Ss and within the same group of Ss over successive testing intervals, the psychophysical functions for curvature and convexity, the two measures of shape, failed to show the same stability. The primary objective of the current investigation, therefore, is to extend the previous research with Minkowski forms by determining a set of psychophysical shape attributes which describe the perception of Minkowski forms. In order to assure the generality of the resulting psychophysical dimensions, two measurement techniques were employed. One is based on the semantic

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Fig. 1. Representations of the form stimuli for the experiment.

differential procedure and the other is predicated on multidimensional scaling of similarity judgments via Carroll and Chang's (1970) INDSCAL algorithm. A secondary objective of the investigation was to compare the two methods on the judgments collected.

EXPERIMENT I Method

Subjects

The Ss were 26 male and female undergraduate students enrolled in an introductory psychology course. Participation in the experiment met a course requirement.

Apparatus

A Digital Equipment Corporation PDP-8 computer was used to generate and display the stimuli and to collect the responses. The stimuli were presented to the Ss on a Sony television receiver. The system has been fully discussed by Jones, Johnson, & Young (1969).

Stimuli

The 16 stimuli used in this experiment are presented in Fig. 1. These stimuli are generated by varying the value of n in the equation

$$|x|^n + |y|^n = 1,$$

which is the equation for unit distance in a two-dimensional Minkowski space, where the parameter n is the Minkowski metric parameter. The values of n used in this experiment are as follows:

Stimulus:	1	2	3	4	5	6	7	8
n :	.01	.25	.30	.35	.40	.45	.55	.75
Stimulus:	9	10	11	12	13	14	15	16
n :	1.0	1.2	1.5	2.0	3.0	4.5	9.0	400.0

The patterns presented to the Ss were composed of 41 rows of dots, each row extending from the left to the right contour boundary. The maximum width of any form was 41 dots; the maximum height was also 41 dots. Note that the patterns as presented to the Ss were actually discrete versions of the continuous forms presented in Fig. 1, implying that the stimulus contour may be somewhat distorted from what would otherwise be a smooth curvilinear contour.

Procedure

The Ss were run in individual sound-attenuated cubicles, and were seated at a Teletype directly in front of the display unit. At the start of the experiment, the Ss were given a set of instructions which explained the experiment and the use of the Teletype. The Ss were run in groups of one to five (one S per cubicle).

The experiment was administered in two phases, with a break of approximately 5 min between phases. The first phase of the experiment was a paired comparison condition in which Ss were presented pairs of stimuli and were asked to judge their dissimilarity. All of the 120 unique pairs of the 16 stimuli were presented after an initial set of 16 practice pairs. The S was not informed that some of the pairs were for practice, and these pairs were in no way differentiated from the remainder. The 16 practice pairs were selected randomly from the full set of 120 pairs and were not included in the analysis. A different irregular order of pairs was given to each group of one to five Ss. The Ss were instructed to type a number on the Teletype from 1 to 9, depending on the judged dissimilarity of the stimuli. For a great degree of dissimilarity of the stimuli, they were instructed to type a 9, but for a small degree, they were to type a 1. Intermediate degrees of dissimilarity were to be assigned intermediate numbers.

The second phase of the experiment was a single stimulus condition in which Ss were presented stimuli one at a time and were asked to rate the stimuli on each of the following eight rating scales: pointed, tilted, irregular, flat, bulging, large, curved, and erect. The Ss were instructed to type a number from 1 to 9, depending on how well the scale described the stimulus. A 1 was to be used if the scale did not describe the stimulus at all, a 9 for a very accurate description. A new stimulus was not presented until it had been rated on all eight scales. The order in which the scales were presented was different for each stimulus, and the stimulus sequence was ordered randomly for each group of Ss.

Results

The dissimilarity judgments from the paired comparison phase of the experiment were submitted to Carroll and Chang's individual differences multidimensional scaling (INDSCAL) model. This model locates the stimuli in a nonrotatable Euclidean space, allowing individual differences in the salience of each dimension. The saliences are expressed as weights applied to the stimulus dimensions. For a complete description of this model, see Carroll and Chang (1970). Solutions were obtained in 1, 2, 3, and 4 dimensions. Since the four-dimensional solution accounted for more of the variance than did any of the other three solutions, and since it was interpretable, it is discussed below. The validity of a four-dimensional hypothesis is also treated below.

The coordinates of the Minkowski forms are listed in Table 1. The projections along Dimension 1 indicate that

it is primarily sensitive to the degree to which the sides of a stimulus are concave. Stimuli with sides which protrude out, or at least are not concave, project at approximately the same negative point on Dimension 1. As the stimuli become more concave, the projections become more positive, culminating in the cross, the stimulus with the most positive projection on Dimension 1. Dimension 2 reflects the direction of curvature. Forms which do not curve in or out have a zero or nearly zero projection on this dimension. As forms curve out, their projection of Dimension 2 becomes more negative, but as they curve in, the projection becomes more positive. The projections along Dimension 3 indicate that the dimension is sensitive to the orientation of a form. The diamond, which can be viewed as a square rotated 45 deg, has the most positive projection. A pattern which is neither upright nor rotated 45 deg is the circle, which has a projection on Dimension 3 close to zero. Dimension 4 is sensitive to the curvature of patterns. Both the rotated and upright square have extreme positive projections and the circle has the most negative projection.

Relation Between INDSCAL Solution and Rating Scale Judgments

The mean of the judgments on the eight rating scales for each of the 16 patterns was computed. Eight multiple correlation analyses were then performed between the INDSCAL solution and each scale (in turn) to determine the direction through the stimulus space in closest agreement with each rating scale. The magnitude of the multiple correlation coefficient measures the degree of overall linear agreement. The beta weights indicate the direction of closest agreement. The multiple correlation coefficient and beta weights are presented in Table 2. From the correlational analysis, it would appear

Table 2
Beta Weights and Multiple Correlation Coefficients Between INDSCAL Configuration and Eight Rating Scales

Scale	Dimensions				R
	1	2	3	4	
Pointed	.396	.437	.461	.235	.985
Tilted	-.148	.111	.867	.122	.879
Irregular	-.211	.202	.314	-.661	.776
Flat	-.159	-.016	-.559	.623	.918
Bulging	-.700	-.415	.122	-.344	.974
Large	-.880	-.106	-.132	.062	.987
Curved	-.502	-.001	.333	-.820	.959
Erect	.698	.349	-.072	.319	.930

that Dimension 1 should be labeled "size." This is a label which closely agrees with the "concavity" label given to the dimension in the interpretation above, since concavity and size are very closely related. Dimension 1 also is negatively related to the "bulging" scale, agreeing with the interpretation above. We will continue to call this dimension concavity, therefore. Dimension 2 does not seem to be simply related to any of the unidimensional scales, and we note that there are no unidimensional scales which clearly fit the interpretation of Dimension 2 as being "direction of curvature." The modest beta weights for "pointed" and "erect" seem to reflect inward curvature (one direction on Dimension 2), and the modestly negative beta weight for "bulging" reflects outward curvature. The unidimensional scales having the largest beta weights for Dimensions 3 and 4 are "tilted" and "curved," respectively, directly supporting the interpretation developed earlier. Finally, there is a strong relationship between the multidimensional solution and the several rating scales; all the correlations are significant beyond the .01 level and six are larger than .90.

The first experiment showed that a set of meaningful dimensions were perceptually relevant from two perspectives: first, the projection of the forms on the axes led to easy interpretations, and second, the coordinates of the forms in the perceptual space accounted for the ratings of the patterns in another judgment task.

EXPERIMENT II

The second experiment assesses the reliability of the first experiment's paired comparison analysis and compares the paired comparison analysis to an analysis of semantic differential judgments.

Method

Subjects

The Ss were 60 male and female students drawn from the same population as in Experiment I.

Table 1
INDSCAL Solution from Paired-Comparison Task: Experiment I

Stimulus	Dimension			
	1	2	3	4
1	.428	-.009	-.059	.135
2	.423	.013	-.113	.090
3	.342	-.028	.128	.121
4	.277	.231	.082	-.177
5	.170	.342	.149	-.193
6	.154	.348	.124	-.256
7	.035	.342	.185	-.150
8	-.106	.329	.258	-.057
9	-.157	-.052	.317	.403
10	-.195	-.157	.311	.303
11	-.184	-.395	.187	-.079
12	-.203	-.405	-.045	-.513
13	-.242	-.295	-.303	-.263
14	-.235	-.207	-.410	.010
15	-.269	-.067	-.409	.240
16	-.237	.011	-.402	.385

Table 3
Principal Components Solution from Semantic Differential
Judgments When Made First

Stimulus	Dimensions			
	1	2	3	4
1	0.379	1.707	-0.808	-0.198
2	1.328	0.579	0.470	-0.666
3	1.476	0.160	0.236	0.698
4	1.246	-0.804	0.178	0.520
5	0.690	-0.746	0.563	-0.082
6	0.592	-0.331	0.303	-2.464
7	0.284	-1.050	-0.132	0.088
8	0.864	-0.192	0.802	-0.053
9	-0.042	1.486	-2.446	0.458
10	-0.120	-1.376	-1.930	1.107
11	-1.184	-1.265	-0.319	-0.467
12	-1.605	0.506	-0.413	-2.108
13	-1.694	-0.402	0.806	0.650
14	-1.126	-0.137	1.044	0.819
15	-0.497	-0.160	0.154	0.693
16	-0.590	2.026	1.490	1.004

Apparatus and Stimuli

The stimuli, the visual display, and the response collection were identical to those of Experiment I.

Procedure

The students were divided into two groups of 30 Ss. Both groups of Ss participated in paired comparison and semantic differential conditions, but the order was counterbalanced for the two groups. There was a short rest period of approximately 5 min between experimental conditions. The paired comparison procedure was identical to that of Experiment I.

In the semantic differential condition, Ss were instructed to rate the stimulus being displayed on a 7-category bipolar scale typed on the Teletype. After responding with a number from 1 to 7, a new scale was presented. There were 10 scales. A new stimulus was not presented until it had been rated on all 10 scales, but the order in which the scales were typed was different for each pattern, and the stimulus sequence was ordered randomly for each group of Ss. The scales were: pointed-rounded, tilted-upright, regular-irregular, simple-complex, convex-flat, bulging-collapsed, symmetrical-asymmetrical, small-large, straight-curved, and erect-slanted.

The semantic differential judgments were analyzed by a principal components procedure with varimax rotation. All semantic differential dimensions are numbered in order corresponding to amount of variance accounted for, as is also true for the INDSCAL dimensions: Dimension 1 always accounts for the most variance.

The semantic differential procedure is different from traditional approaches in three respects: denotative instead of connotative scales are used, stimulus objects instead of verbal concepts are rated, and the focal point of the analysis is switched from the factor loading matrix to the factor score matrix, which contains the coordinates of the forms in the factor space.

Results

The results will be presented in three parts: the analysis of the first set of judgments, the analysis of the second set, and the interrelations between analyses. All

solutions for this experiment were extracted in four dimensions in order to remain comparable with the solution from Experiment I.

Solution for the First Set of Judgments

An interpretable solution was found in four dimensions for the semantic differential judgments. Dimension 1 in Table 3 is labeled "direction of curvature." Forms with convex sides, like a circle, have a negative projection on the dimension, but forms with concave sides have a positive projection. The diamond, a form which is neither concave nor convex, has a nearly zero projection on the dimension. Dimension 2 receives very positive projections from the square, diamond, and cross; other patterns have less positive projections or negative projections on the dimension. Since the forms with high-positive projections are standard geometric forms and since the high factor loadings for this dimension were on the "regular-irregular," "simple-complex," and "symmetrical-asymmetrical" scales, the dimension is labeled "simplicity." Dimension 3 measures the orientation of the forms, and the fourth dimension is primarily sensitive to curvature.

The four-dimensional solution for the INDSCAL analysis of the paired comparison judgments is shown in Table 4. Dimension 1 reflects the orientation of a pattern, Dimension 2 is sensitive to the curvature of patterns with convex sides, and Dimension 3 measures curvature of patterns with concave sides. Dimension 4 appears to assess primarily the degree to which the sides of a pattern protrude out: the cross, whose sides collapse in completely, has the most negative projection on Dimension 4, the convex patterns share the most positive projections on the axis.

Table 4
INDSCAL Solution from Paired Comparisons
Judgments When Made First

Stimulus	Dimensions			
	1	2	3	4
1	-.024	.090	-.188	-.370
2	-.137	.004	-.164	-.379
3	.136	-.096	-.081	-.338
4	.039	.049	.219	-.284
5	.054	.078	.339	-.219
6	.035	.107	.412	-.195
7	.071	.196	.421	-.079
8	.142	.255	.393	.040
9	.338	.311	-.212	.148
10	.392	.128	-.203	.191
11	.341	-.238	-.128	.220
12	.068	-.653	-.049	.231
13	-.275	-.407	-.081	.285
14	-.369	-.141	-.192	.281
15	-.421	.040	-.245	.246
16	-.391	.275	-.242	.222

Table 5
Principal Components Solution from Semantic Differential
Judgments When Made Second

Stimulus	Dimensions			
	1	2	3	4
1	1.436	0.024	-0.645	0.535
2	0.811	-1.359	-1.790	1.249
3	0.542	-1.014	-0.459	-0.239
4	-0.271	-1.335	0.859	0.462
5	-0.514	-1.046	0.283	-0.611
6	-0.679	-0.672	1.217	1.146
7	-0.664	-0.179	0.855	0.227
8	-0.457	-1.417	-0.209	-0.282
9	1.911	0.192	-0.080	-2.312
10	-0.189	0.190	1.494	-1.822
11	-1.452	0.841	-0.702	-0.296
12	-1.577	0.799	-2.144	-0.913
13	-0.555	1.331	-0.306	0.302
14	-0.353	1.169	0.476	1.128
15	0.373	0.996	1.208	0.479
16	1.639	1.480	-0.058	0.946

Table 6
INDSCAL Solution from Paired Comparisons
Judgments When Made Second

Stimulus	Dimensions			
	1	2	3	4
1	-.158	-.046	.319	-.299
2	-.157	-.124	.337	-.314
3	-.035	-.156	.272	-.305
4	.009	.177	.170	-.292
5	.103	.282	.902	-.252
6	.122	.328	.027	-.265
7	.181	.355	.024	-.170
8	.279	.358	.082	-.001
9	.208	.107	.250	.272
10	.298	-.080	.046	.274
11	.341	-.356	-.191	.216
12	.182	-.468	-.430	.138
13	-.155	-.320	-.423	.198
14	-.318	-.132	-.400	.222
15	-.463	.031	-.199	.270
16	-.438	.044	.026	.308

Solutions for the Second Set of Judgments

Table 5 shows the four-dimensional solution for the semantic differential judgments. Projections of the geometric stimuli on Dimension 1 reflect their curvature, and Dimension 2 is sensitive to convexity-concavity. Dimension 3 doesn't suggest an interpretation, but the projections along the fourth dimension are interpretable as orientation. While there are some projections on Dimension 4 which are not totally consistent with the interpretation, a dimension similar to this one has occurred in several previous solutions.

The four-dimension INDSCAL solution for the paired comparison judgments is shown in Table 6. Dimension 1 reflects orientation, and Dimension 2 assesses convexity-concavity. The projections along Dimensions 3 and 4 are both determined by conjoint stimulus attributes. For Dimension 3, the attributes are direction of curvature and curvature. Curved patterns, like the circle, have more negative projections than rectilinear ones, but concave patterns have more positive projections than convex ones. The conjoint attributes determining projections along the fourth dimension are curvature and enclosed area. Curved forms tend to have a projection on Dimension 4 which approaches zero, but patterns enclosing a larger area have more positive projections than ones which contain a smaller area.

Interrelations Between the Solutions

There are four scaling solutions for this experiment. Two solutions were obtained by using the INDSCAL model with paired comparison data, and two by using the principle components model on semantic differential data. These four solutions may be compared to each other and to the INDSCAL solution in the first experiment. Each of these five analyses describes the manner in which people see a set of forms generated by the two-dimensional Minkowski unit distance equation. Each of the perceptual spaces were obtained in four dimensions, and the interpretations provided for the dimensions are summarized in Table 7. The attribute of

Table 7
The Interpretations for the Dimensions of the Various Solutions

Solution	Dimension 1	Dimension 2	Dimension 3	Dimension 4
INDSCAL solution from Experiment I	Concavity	Convexity-Concavity	Orientation	Curvature
First INDSCAL solution from Experiment II	Orientation	Convex Curvature	Concave Curvature	Convexity
Second INDSCAL solution from Experiment II	Orientation	Convexity-Concavity	Convexity-Concavity & Curvature	Curvature & Area
First SD solution from Experiment II	Convexity-Concavity	Simplicity	Orientation	Curvature
Second SD solution from Experiment II	Curvature	Convexity-Concavity	—	Orientation

Table 8
Canonical Correlation for Pairs of Multidimensional Solutions

Multidimensional Solution Pair		Dimensions			
INDSCAL Experiment I also with INDSCAL Experiment II	First INDSCAL Experiment II	1.00†	1.00†	.99†	.98†
First INDSCAL Experiment II	Second INDSCAL Experiment II	1.00†	.99†	.98†	.95†
First INDSCAL Experiment II	Second SD Experiment II	.98†	.90†	.82†	.61*
Second INDSCAL Experiment II	First SD Experiment II	.97†	.87†	.75	.25
First SD Experiment II	Second SD Experiment II	.98†	.98†	.83†	.47

* $p < .05$ † $p < .01$

orientation was represented by a dimension in every solution. Convexity-concavity, curvature, or a combination of these two dimensions was also present in every solution. The diversity of response procedures, analysis procedures, and experimental orderings did not obfuscate this set of common perceptual attributes.

The relations between the perceptual spaces are quantitatively summarized in Table 8, which lists the canonical correlations between pairs of solutions. Canonical analysis may be used to compare two multidimensional solutions by transforming them so that the transformed variables, called canonical variates, have the largest possible correlations between corresponding dimensions and zero correlations between noncorresponding dimensions (Cooley & Lohnes, 1971). The magnitude of the correlations reveals the degree of similarity between solutions, and the number of canonical dimensions with statistically significant correlations indicates the number of common dimensions in the intercorrelation matrix for the two solutions. This multivariate method has been used at least three times in the form perception literature: twice to determine the similarity of one or more pairs of multidimensional solutions (Aiken & Brown, 1969b; Richards, 1972b) and once to determine the similarity of a multidimensional solution, which was based on judgments, and a battery of physical measures on the same set of forms (Stenson, 1968). The canonical transformation may alter the relation of the distances to the judgments, as it may have in the previous three applications, but this consideration is of no concern, since the focus of attention has switched from the relation of the judgments to the distances to the relation of two alternative multidimensional solutions.

The canonical correlations in Table 8 show that the INDSCAL solutions both within Experiment II and between Experiments I and II may be transformed to become highly congruent. For both comparisons, all four canonical correlations are statistically significant at beyond the .01 level. This outcome supports the four-dimensional hypothesis of Experiment I. The two

semantic differential solutions of Experiment II shared only three common dimensions, as revealed by the three statistically significant canonical correlations. The degree of similarity between the INDSCAL and semantic differential solutions depended on the order in which the procedures were administered. When the similarity judgments for the INDSCAL solution were administered first, four canonical correlations were significant between the INDSCAL and semantic differential solutions, but when the order of procedures was reversed, there were only two significant canonical correlations.

Psychophysical Analysis

As may be noted by examining Table 7, the primary psychological attributes of Minkowski patterns are orientation, convexity-concavity, and curvature. In an earlier psychophysical analysis of patterns from the Minkowski shape continuum, Dobson (1971) proposed objective indexes of both convexity and curvature. The convexity definition will be extended to a convexity-concavity index by allowing it to assume negative values for patterns with sides which protrude out and positive values for patterns with sides which collapse in. Orientation will be defined by the function

$$\left| \frac{\arctan |y'|}{\arctan 1} - 1 \right|$$

where y' is the first derivative of the Minkowski two-dimensional unit distance function. The first derivative was computed for the x-coordinate corresponding to that point along the perimeter in the first quadrant which is 1/10 of the first quadrant perimeter from the right-hand extreme of the form at (1,0). The index of orientation varies from 0 for the diamond or rotated square through 1 for the upright square and the cross.

The psychophysical analysis was implemented by regressing the INDSCAL solution from Experiment I and

the first INDSCAL solution from Experiment 2 against each of the objective indexes or orientations, convexity-concavity, and curvature, in turn. These INDSCAL solutions were chosen since the canonical analysis showed that these configurations had strong internal consistency, and since both these solutions are obtained from Ss who had no previous experience with Minkowski patterns.

Table 9 lists the beta weights for each dimension and the multiple correlations for each index for solutions from Experiments I and II. All the multiple correlations are significant at beyond the .01 level of significance. The beta weights for the orientation index have the greatest absolute magnitude for the dimensions interpreted as orientation dimensions in both experiments. There were dimensions interpreted as sensitive to convexity-concavity and curvature in Experiment I, and these dimensions produce the largest beta weights for the convexity-concavity and curvature indexes, respectively. Dimension 4 from Experiment II, which was interpreted as being sensitive to convexity, has the largest beta weight for the convexity-concavity index, while Dimension 2, which was interpreted as being sensitive to convex curvature, has the largest weight for the curvature index.

GENERAL DISCUSSION

The primary objective of the investigation reported here is to determine the psychophysical attributes underlying the perception of Minkowski forms. These attributes, as verified by a couple of analyses, are orientation, curvature, and convexity-concavity. The two different scaling models, which are based on unique data collection procedures, repeatedly uncovered these attributes, and it was further possible to successfully relate the perceptual dimensions from the various solutions to objective indexes of the Minkowski patterns which also measured orientation, curvature, and convexity-concavity. While these attributes seem to underlie the perception of the forms, their use does not remain totally invariant between judgment procedures or even for different occasions with the same judgment procedure. The variance in the manner in which the three attributes underlie the judgments supports the validity of a perceptual space of more than three dimensions, like the four-dimensional spaces reported above.

A secondary objective for this report is to compare Carroll and Chang's (1970) INDSCAL model to a semantic differential procedure (Osgood, Suci, & Tannenbaum, 1957) for uncovering the perceptual dimensions. The most important difference between the two approaches was shown by the canonical analysis. The INDSCAL solutions consistently shared four statistically significant canonical correlations, but the semantic differential solutions shared only three

Table 9
Beta Weights and Multiple Correlation Coefficients (R) Between Experiments I and II Solutions and Objective Indices

		Indices		
		Orien- tation	Convexity- Concavity	Curva- ture
Experiment I Dimensions	1	.494	.307	.239
	2	.190	.613	.092
	3	-.873	.243	-.348
	4	-.343	-.013	-.685
	R	.954	.920	.782
Experiment II Dimensions	1	-.822	.104	-.391
	2	-.362	.286	-.646
	3	.161	.407	.507
	4	-.541	-.505	-.302
	R	.954	.916	.844

statistically significant canonical correlations. On the other hand, the semantic differential solutions of Experiment II accounted for 89% and 91% of the variance of their respective judgments, while the INDSCAL solutions from Experiment II accounted for 70% and 76% of the variance of their respective judgments. The INDSCAL model, though, attempts to predict individual judgments, while the semantic differential technique, as used here, attempts only to predict the average of the judgments across Ss. The INDSCAL model also generates saliences of the dimensions for individual Ss, but, since the experiments were not concerned with individual differences, they are not discussed.

The significance of the experiments reported here may be judged by the new knowledge which they contribute to our comprehension of form perception and the number of subsequent experiments which are generated from their outcome. One major outcome is, of course, the three attributes which were uncovered. Stenson (1966), however, using curved and rectilinear Attneave and Arnoult (1956) forms, found 3 similar attributes (i.e., curvature, orientation, curved concavity) among the 10 that accounted for 94% of the physical measures on 20 patterns. Brown and Owen's (1967) investigation of five sets of 200 Attneave and Arnoult irregular polygons found 3 factors (i.e., Factors 3, 4, and 5), which generally refer to orientation. Because they used only polygons, it was impossible for them to find curvature and convexity-concavity indexes. The tentative conclusion which may be drawn is that orientation, curvature, and convexity-concavity describe regular, symmetric forms, like Minkowski patterns, as well as irregular patterns, like the Attneave and Arnoult patterns. Future investigations may explore the impact of one or more of these attributes on the dimensionality of the perceptual space for patterns to uncover the ways in which they influence the perception of forms. One approach to the last problem is completed, and its report will be forthcoming.

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