Visual form discrimination: Multidimensional analyses¹

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Nonmetric multidimensional scaling techniques were used to evaluate discrimination latency as a similarity measure. A three dimensional Euclidian solution was found for a sample of random polygons. The dimensions were labeled compactness, jaggedness, and elongation. The spatial configuration was highly similar to results obtained from similarity estimates and the dimensions were systematically related to physical shape measures.

This study is the second in a series designed to determine some of the dimensions of visual form and to evaluate the use of multidimensional scaling as a psychophysical technique. The appropriateness of discrimination latency as a distance measure was also investigated.

The first two goals are interdependent and may be viewed within the context of attempts to develop a psychophysics of form perception (Attneave & Arnoult, 1956; Brown & Owen, 1967; Michels & Zusne, 1965). Specifically, Brown and Owen (1967) have suggested using multidimensional scaling techniques to define perceptual spaces which may, in turn, be used to refine physical spaces for selected stimulus domains. One study (Behrman & Brown, 1968) presented encouraging results based upon similarity estimates. This study is an attempt to extend these results to discrimination latency as a similarity measure.

The question of using discrimination latency as a distance measure is based upon the necessity of defining similarity in terms which are independent of the response it is being used to explain (Attneave, 1950). Assuming that a perceptual space exists for a stimulus domain, distances in this space may be assumed to represent perceived similarities. Shepard (1960) lists five methods of similarity measurement: amplitude of a generalized conditioned response, percentage of errors in a paired associates task, discrimination latency, sorting time, and judged similarity. The choice of discrimination latency as a similarity measure is based upon the assumption that the more similar two stimuli are, the longer it should take to discriminate them. There is evidence that this is true even when stimuli are quite different (Woodworth & Schlosberg, 1954). In addition, choice time has been found to be a linear function of information transmitted (Bricker, 1955; Hyman, 1953). If shapes are viewed as multidimensional displays, information is related to both the dimensionality of the similarity space and the ordering of shapes on those

dimensions. A spatial analysis of discrimination latencies should, therefore, reflect both of these aspects of similarity.

METHOD

Subjects

Twenty male and 20 female undergraduates participated in the experiment as part of the introductory psychology course requirements.

Patterns

Sixteen four-sided random polygons were selected from a larger sample of 200 which were generated in a 100 by 100 unit matrix by a modified version of Method 1 of Attneave and Arnoult (1956). These same 16 shapes were used in an earlier study (Behrman & Brown, 1968), and their quantification has been described in detail by Brown and Owen (1967). These particular shapes were selected to represent the range of five factor dimensions (compactness, jag-

FACTOR									
	1		2		3		4		5
▶	1.11	◀	1.01	▲	1.76	▶	2.25	▲	2.43
▲	.81	▶	.85	◀	1.26	▶	1.40	◀	1.41
◀	.78	◀	.85	▲	.79	▶	1.20	▶	.66
▲	.78	◀	.61	▲	.70	◀	1.08	◀	.64
◀	.77	▲	.44	◀	.58	▶	.84	▶	.42
▶	.76	◀	.41	◀	.48	▶	.47	▶	.32
▶	.74	◀	.29	◀	-.04	▲	.31	◀	.15
◀	.54	▲	.10	▶	-.07	◀	-.22	▶	.06
▶	.37	▶	.05	◀	-.16	◀	-.24	◀	.05
◀	.18	◀	.02	▶	-.18	◀	-.48	◀	.04
◀	-.05	◀	-.36	◀	-.23	▲	-.58	▲	-.16
◀	-.42	▲	-.40	▶	-.53	◀	-.73	▶	-.35
▲	-.61	▶	-1.68	◀	-.86	◀	-.81	◀	-.60
▶	-.84	▶	-1.81	▶	-.94	▲	-1.09	▲	-.79
◀	-1.77	▶	-2.11	◀	-1.07	◀	-1.10	◀	-1.19
◀	-2.91	◀	-3.73	▶	-1.82	◀	-4.53	◀	-1.66

Fig. 1. The ordering of shapes and their scores on the first five factors from an analysis of 80 shape measures.

gedness, y-axis areal asymmetry, x-axis areal asymmetry, and rotation) and to minimize the linear correlation among measures. The orthogonal factor score dimensions were based upon the linear independence among 80 initial physical measures (Brown & Owen, 1967). Linear correlations among factor scores in this sample ranged between .00 and .49. Their ordering on the factor dimensions is shown in Fig. 1.

Apparatus and Procedure

The apparatus consisted essentially of a rear-projection viewing booth, two tachistoscopically-shuttered, random-access slide projectors, a response panel, and circuitry to record response latency.

One hundred twenty problems were constructed from the paired comparisons of the 16 patterns and photographed as 35 mm slides. Each problem consisted of a standard and two test patterns, one like, and one unlike, the standard. The unlike pattern was designated "correct." Each problem could be presented in one of eight different ways, depending on which of the two patterns of each pair served as the standard, whether the correct pattern appeared on the right or left, and which sequence was used. The two sequences were defined by using the Ross pairings (Ross, 1934) in their forward and backward order. The 40 Ss were randomly assigned to these eight conditions with three of one sex and two of the other assigned to each.

The S was instructed to place his preferred hand on the 4 x 4 in. response plate, activating a signal light which remained on until a problem was ready for presentation. When the signal light was extinguished, S could remove his hand from the plate and present the standard at the top of the screen with the two test patterns immediately below it, and activate a Hunter timer. To select which of the test patterns matched the standard, S pressed one of two buttons on the response panel. If the response were correct, the problem was extinguished, the timer was stopped, and the signal light was activated for the next trial. For incorrect responses, the problem remained on the screen until the correct response was made.

The Ss were tested individually in 50 min sessions. They were instructed to respond as rapidly as possible without making errors. Ten practice trials preceded the experimental testing.

RESULTS

Latencies were treated as similarity estimates and analyzed by nonmetric (Kruskal, 1964a, b) multidimensional scaling methods. Prior to the major analyses, the matrix of between-subject correlations was factored to determine the degree of homogeneity of Ss. The principal component results were rotated orthogonally with the first two factors accounting for 20 and 10% of the variance, respectively, and with small, equal

eigenvalues for subsequent factors. The first two factors were clearly related to the two orders of the Ross pairings used. When mean latencies for the two groups were plotted over trials (shape pairs) for the two groups, a linear decrease in latency occurred over the first 40 trials. The mean difference in latency between the two orders on a given problem was computed on each of the first 40 trials and plotted against trial number. The least squares linear regression equation was taken as an estimate of the practice effect and was subtracted from both problems occurring on each trial. The mean of these two sets of latencies, corrected for practice effects, was used as the 16 (shapes) by 16 (shapes) matrix ($S = [s_{ij}]$) whose entries may be viewed as psychological distances among all patterns.

The nonmetric solution was run with Minkowski r -metrics of $r=1$ (city block model), $r=1.5$, $r=2$ (Euclidean model), and $r=3.0$. The stress curves are shown in Fig. 2. Using stress as a criterion, it is clear that the city block solution is not parsimonious, that there is little basis for selecting among the other metrics, and that a two, three, or four dimensional solution may be appropriate. Kruskal (1964a) describes 10% stress as a "fair" fit, 5% as "good," and 2.5% as "excellent." By those criteria, at least four dimensions would be required when $r=1.5$, 2.0, or 3.0 for a "fair" fit of a monotone relationship between dissimilarities and distances. In view of the fact that previous results with similarity judgments

Fig. 2. Stress with different Minkowski r metrics with varying dimensionality of solutions.

Table 1. Correlations Between the Dimensions of the Different Scaling Solutions

	r = 1.5					r = 2.0					r = 3.0				
	Two D		Three D			Two D		Three D			Two D		Three D		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	—	-.19	.95	-.16	.21	.46	-.35	.68	-.42	.28	.55	-.04	.68	-.22	-.29
2		—	-.16	.91	-.59	-.84	.53	-.48	-.06	-.91	-.90	-.01	-.83	-.18	-.63
3			—	-.18	.10	.49	-.26	.69	-.30	.26	.55	.06	.67	-.12	-.31
4				—	-.21	-.94	.19	-.62	-.37	-.62	-.84	-.40	-.78	-.53	-.40
5					—	.19	-.90	-.03	-.61	.77	.52	-.74	.48	-.62	.68
6						—	-.22	.78	.26	.70	.91	.40	.90	.46	.23
7							—	.03	.73	-.74	-.59	.80	-.51	.67	-.57
8								—	.09	.28	.64	.51	.78	.34	-.30
9									—	-.14	-.10	.82	-.14	.95	.00
10										—	.87	-.29	.79	-.09	.78
11											—	.00	.96	.09	.41
12												—	.06	.91	-.43
13													—	.06	.26
14														—	-.13
15															—

of these same shapes provided a meaningful solution in three dimensions (Behrman & Brown, 1968), and the fact that Kruskal's criteria were based upon limited experience with experimental data, these analyses were carried out with two and three dimensional solutions for all but the city block model.

Table 1 presents product-moment correlations among the projections of shapes on the dimensions of all solutions. Table 1 shows that there is a high degree of similarity among the solutions for the different metrics except for the first dimension of the two and three dimensional, $r=1.5$ solutions. They are highly correlated with each other, but not with dimensions from any other solutions. Since Euclidean solutions are convenient to present, the outcome is generally characteristic of the other solutions, and comparisons are to be made with metric Euclidean results, subsequent analyses were limited to the two and three dimensional Euclidean solutions.

Fig. 3. The two dimensional Euclidian configuration.

When attention is restricted to only the Euclidean results, it is clear that the dimensionality of the solution is in question; both dimensions of the two dimensional solution are highly correlated with the third dimension of the three dimensional solution. This also occurred with the $r=3$ results and raises a question concerning the need for the third dimension. The two dimensional solution, shown graphically in Fig. 3, is intuitively appealing and indicates that shapes were grouped on the basis of angle characteristics (jaggedness) for Dimension 1 and ordered on an elongation basis for the second dimension. Further interpretation was referred to a psychophysical analysis in which the projections of shapes on dimensions of the scaling solutions were correlated (Pearson, product-moment) with 80 shape measures (Brown & Owen, 1967). Since this analysis involves many correlations, and since many measures are highly intercorrelated, six measures were selected on an a priori basis and represent hypotheses concerning the nature of the dimensions. The first five measures were based upon the factor structure used to select the shapes. The sixth measure, the maximum ratio of vertical to horizontal extent (Max/VE/HE), measures the elongation of shapes without regard to any particular spatial orientation and was included because it was uniquely predictive of previous results (Behrman & Brown, 1968). Correlations among these measures and between measures and dimensions are shown in Table 2. It is clear that the first dimension of the two dimensional solution is highly correlated with the physical measure of compactness (P) and jaggedness (IA2). Inspection of Fig. 3 leads to the conclusion that this dimension is one in which Ss grouped shapes on the basis of the size of the largest interior angle (jaggedness) with a clear separation of those shapes with an interior obtuse angle from those with all interior acute angles. The high correlation with compactness (P) appears to be due to a

Table 2. Correlations among Shape Measures and Between Shape Measures and Dimensions of Scaling Solutions (decimals omitted)

Measures								Metrics and Dimensions (D)														
P	IA ₂	M _{3x}	M _{3y}	VE/HE	Max VE/HE	M _{2y}	r = 1.5					r = 2.0					r = 3.0					
							Two D		Three D			Two D		Three D			Two D		Three D			
							1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
P	—	94	54	04	-20	45	41	47	-88	42	-83	49	87	-52	71	-13	74	92	06	90	10	26
IA ₂	—	—	-42	-13	-13	18	34	35	-91	30	-93	36	93	-31	72	16	73	88	28	87	34	33
M _{3x}			—	-25	11	38	-19	-29	24	-23	27	-09	-35	11	-45	10	-05	-32	-16	-30	-11	33
M _{3y}				—	27	03	33	44	14	28	27	31	-13	40	-06	-56	02	07	-44	08	-51	-16
VE/HE					—	-14	69	19	16	20	27	19	-13	-05	-16	-09	06	-07	-15	-03	-22	09
Max VE/HE						—	39	39	-34	35	-10	64	17	-85	04	-73	50	50	-67	43	-63	28

P: The perimeter of the shape.

IA₂: The variance of the interior angles.

M_{3y}: The third moment of the distribution of area units collapsed to the y axis.

M_{3x}: The third moment of the distribution of area units collapsed to the x axis.

VE/HE: The ratio of the difference between the largest and smallest y coordinate (Vertical Extent) and the largest and smallest x coordinate (Horizontal Extent).

Max

VE/HE: Same as VE/HE but computed in 5° steps through 360° rotation and the maximum value selected.

sampling problem which led to a high correlation between P and IA₂ ($r = .94$). It is worth noting that this same dimension appeared in all solutions. The second dimension is most clearly related to elongation (Max^{VE/HE}). This dimension is characteristic of all solutions accept the two dimensional, $r = 1.5$ solution.

It is impossible to evaluate the three dimensional solution based solely upon the correlations presented in Tables 1 and 2 due to the high correlation between P and IA₂. In an earlier study (Behrman & Brown, 1968), it was concluded that a three dimensional Euclidean solution was appropriate, the dimensions were labeled dispersion-compactness, jaggedness, and

elongation, and were related monotonically to P, IA₂, and Max^{VE/HE}, respectively. These dimensions resulted from both a metric and a nonmetric analysis. The three dimensional solution was evaluated by comparison with the physical measures and by comparison with the previously obtained metric solution. These comparisons are summarized in Fig. 4a-c, and in Fig. 5a-5c.

From Table 2 it is clear that the first and third dimensions may be predicted equally as well from either P or IA₂ and that the second dimension was most closely related to Max^{VE/HE}. In Fig. 4a-c, each dimension is plotted against the physical measures

Fig. 4a. The relation between Dimension 1 (jaggedness) and IA₂ (rescaled 1-10).

Fig. 4b. The relation between Dimension 2 (elongation) and Max^{VE/HE} (rescaled 1-10).

Fig. 4c. The relation between Dimension 3 (dispersion) and P (rescaled 1-10).

with which it is most highly correlated. The first dimension (Fig. 4a) was plotted against IA_2 ($r = .72$), and it seems clear that the high correlation with P ($r = .71$) is due to the correlation between P and IA_2 ($r = .94$). This is considered to be a jaggedness dimension since the highly dispersed, but not jagged, shape (coordinates, 2.91, 0.44) was ordered below the jagged shapes and the only shape with an obtuse interior angle which was low on this dimension (coordinates, 3.91, -1.90) was one for which it is doubtful that the angle was perceived.

The second dimension was plotted against $Max^{VE/HE}$ (Fig. 4b) and is interpreted as an elongation dimension. The third dimension was interpreted as a dispersion dimension and is shown plotted against P (Fig. 4c).

Figure 5a-c shows the plots of the dimensions from the present solution which were most highly correlated with the jaggedness (Fig. 5a, $r = .78$), elongation (Fig. 5b, $r = -.80$), and dispersion (Fig. 5c, $r = .86$) dimensions from the previous metric analysis of similarity estimates (Behrman & Brown, 1968).

DISCUSSION

The use of latency as an indicant of the nature of perceptual and judgmental processes has a long history and has been recently summarized within both the context of choice reaction time experiments and perceptual judgment tasks (Smith, 1968). In the present study, the logic may be stated simply. It is assumed that latency is monotonically related to distance in a spatial configuration with the task of the multidimensional analysis being to determine the dimen-

Fig. 5a. The relation between the jaggedness dimensions from the nonmetric analysis of discrimination latency and the metric analysis of similarity judgments.

sionality and metric of the space. This does not necessitate assuming that latency in the perceptual judgment task in which it is measured is related to similarity judgments. The high degree of congruence between the analysis of similarity estimates and the analysis of discrimination latencies was gratifying, therefore, from two points of view. First, even with relatively complex and unfamiliar stimuli, it leads to the conclusion that the specific pattern attributes which underlie similarity estimates are also those attributes which are useful in other perceptual judgment tasks such as the present discrimination problems. The similar findings suggest that similar perceptual processes underlie superficially different perceptual tasks. Secondly, if supported by future studies, powerful methods of analysis may be applied to new subject populations. The usual procedures which are used to collect similarity estimates are highly demanding of Ss and typically require relatively complex verbal instructions. These constraints preclude the use of routine procedures of data collection with certain Ss (e.g., infants, nonverbal animals, brain damaged and severely disturbed adults, etc.). To the extent that multidimensional scaling analyses of latencies and other dependent variables are possible, important new similarities and differences in perceptual processes may be detected for these subject populations. It does not seem unreasonable to assume, for instance, that an important difference underlying perceptual performance in both ontogenetic and phylogenetic comparisons is the dimensionality of the perceptual space.

Accumulating psychophysical (e.g., Sternberg, 1967)

Fig. 5b. The relation between the elongation dimensions from the nonmetric analysis of discrimination latency and the metric analysis of similarity judgments.

and neurophysiological data (Hubel, 1963) make it almost trite to say that perceptual performance involves abstracting specific pattern characteristics from complex stimulus arrays. Certainly the present data support such a conclusion. From the point of view of developing a psychophysics of pattern perception, however, the results are both encouraging and discouraging. On the positive side, it seems clear that patterns varying in P , IA_2 , and Max^{VE}/HE (or correlated measures of the same pattern characteristics) would be differentially discriminable. Moreover, the functions relating dimensions of the "discrimination space" to the physical measures are monotonic if it is assumed that variation from a perfect monotonic function simply represents experimental error. However, when Fig. 4a-c is inspected in detail, this assumption appears untenable. Considerable bipolar clustering of shapes appeared for the jaggedness dimension (Fig. 4a) and the distribution of shape projections on the other two axes were highly skewed. Similar results were obtained from the similarity data. This suggests, once again, the need for detailed work concerning subjective spacing on individual dimensions of the spatial configuration and the study of the relation of such subjective units to the selected physics of patterns.

References

- Attneave, F. Dimensions of similarity. *Amer. J. Psychol.*, 1950, 63, 516-556.
- Attneave, F., & Amoult, M. D. The quantitative study of shape and pattern perception. *Psychol. Bull.*, 1956, 53, 452-471.
- Behrman, B. W., & Brown, D. R. Multidimensional scaling of form: A psychophysical analysis. Submitted to *Percept. & Psychophys.*
- Bricker, P. D. Information measurement and reaction time: A review. In H. Quastler (Ed.), *Information theory in psychology: Problems and methods*. Glencoe, Illinois: Free Press, 1955.
- Brown, D. R., & Owen, O. H. The metrics of visual form: Methodological dyspepsia. *Psychol. Bull.*, 1967, 68, 243-259.
- Hubel, D. H. Integrative processes in central visual pathways of the cat. *J. Opt. Soc. Amer.*, 1963, 53, 58-66.
- Hyman, R. Stimulus information as a determinant of reaction time. *J. exp. Psychol.*, 1953, 45, 188-196.
- Kruskal, J. B. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika*, 1964a, 29, 1-27.
- Kruskal, J. B. Nonmetric multidimensional scaling: A numerical method. *Psychometrika*, 1964b, 29, 28-42.
- Michels, K. M., & Zusne, L. Metrics of visual form. *Psychol. Bull.*, 1965, 63, 74-86.
- Ross, R. T. Optimum order for the presentation of pairs in the method of paired comparisons. *J. educ. Psychol.*, 1934, 25, 375-382.
- Shepard, R. N. Similarity of stimuli and metric properties of behavioral data. In H. Gulliksen & S. Messick (Eds.), *Psychological scaling: theory and applications*. New York: Wiley, 1960.
- Smith, E. E. Choice reaction time: An evaluation of the major theoretical positions. *Psychol. Bull.*, in press.
- Sternberg, S. Two operations in character recognition: Some evidence from reaction-time measurements. *Percept. & Psychophys.*, 1967, 2, 45-53.
- Woodworth, R. S., & Schlosberg, H. *Experimental psychology*. New York: Holt, Rinehart & Winston, 1954.

Note

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Fig. 5c. The relation between the dispersion dimension from the nonmetric analysis of discrimination latency and the metric analysis of similarity judgments.