DORF2R.BAS: Analyzing signal-detection theory rating data in the BASIC programming language

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A standard method of estimating signal-detection theory parameters and confidence intervals for a rating ROC curve of an individual observer has been the application of a maximum likelihood computer algorithm (e.g., Dorfman, 1982; Dorfman & Alf, 1969). The present program fills the need for a maximum likelihood solution written in a very simple form of Microsoft BASIC.

The accompanying program was written in Microsoft BASIC on the Epson Geneva, a portable 8-bit machine. It was written to be consistent with MBASIC, GWBASIC, BASICA, and most other forms of BASIC. Simple variable names were chosen so the program would be easily transportable from machine to machine.

The program is very similar to the original program by Dorfman and Alf (1969), with the following changes:

1. The normal curve integral and inverse integral routines were obtained by expanding the normal curve equation in a Taylor series, and iterating until the integral converges to the desired decimal accuracy. This is a needless addition for the present program, since the Geneva has only 6 significant figures in single precision. However, an almost identical version has been written for the Cyber, where convergence is carried to 14 significant figures, primarily for research purposes.

2. When the cumulative probabilities are zero or one, the expected cell value is estimated using the Bayesian rule of succession. This method gives the expected cell probability, assuming arbitrarily that all probabilities are equally likely prior to gathering of the data.

3. Rather than use of ordinary least squares to obtain the initial estimates, a weighted least squares procedure is used that weights each data point according to its stability. Furthermore, error is assumed on both axes, rather than on only the y-axis. As a result, the initial estimates are very close to the final ones, and frequently the initial chi-square is smaller than the chi-square value after iterations are complete. Thus, the number of iterations is generally reduced, and, more importantly, the procedure rarely fails to converge.

4. The program estimates the positive $z(k)$ values, rather than the negative of these values. This is simply a minor difference in sign, but it also makes a difference in the signs in the variance-covariance matrix.

5. The additional indices and their variances provided by Dorfman (1982) are given, together with a few others that may be useful for theoretical purposes. Specifically, the indices DELTA M, D(E), A(Z), and D(A) are as defined by Dorfman, and as presented in Appendix D of Swets and Pickett (1982). In addition, the estimates and variances are given for

SD—The standard deviation of the signal-plus-noise distribution, assuming the noise distribution to have a standard deviation of 1.00;

DELTA SD—equal to $SD - 1$;

(DELTA M)/(DELTA SD)—The ratio of the difference in means to the difference in variances for the noise and signal-plus-noise distributions. It is sometimes expected that this ratio will be about 4.00;

Z(A)—The normal z-score below which the area A(Z) lies.

Appendix A is the program listing. Appendix B is an example using Subject Number 3 from Appendix D of Swets and Pickett (1982, p. 218). This reference also provides guidance that should be helpful in using the program, and provides comparisons of the difference in output.

The program is slow, particularly on a slow computer such as the Epson Geneva. However, even on the Geneva, printing begins about 52 sec after the data have been entered. A compiled version of the program is available on disk for the IBM personal computer and compatibles. The compiled version is considerably faster, completing the iterations in just a few seconds.

Availability. A compiled copy of the program for the IBM PC and compatibles can be obtained by writing either author. To cover the costs of disks, duplication, and postage, please include a check made out to SDSU PSYCHOPHYSICS in the amount of \$6.

REFERENCES

- DORFMAN, D. D. (1982). RSCORE-II. In J. A. Swets & R. M. Pickett, *Evaluation of diagnostic systems: Methods from signal detection theory* (pp. 212-232). New York: Academic Press.
- DORFMAN, D. D., & ALF, E., JR. (1969). Maximum likelihood estimation of parameters of signal detection theory and determination of confidence intervals—Rating method data. *Journal of Mathematical Psychology*, 6, 487-496.
- SWETS, J. A., & PICKETT, R. M. (1982). *Evaluation of diagnostic systems: Methods from signal detection theory*. New York: Academic Press.

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APPENDIX A
Listing of DORF2R.BAS

```

10 REM DORF 2R
20 INPUT "HOW MANY JUDGEMENT CATEGORIES ARE THERE";K9
30 K = K9
40 J = K-1
50 L = K+1
60 DIM O(K,2),N(K,2),P(K,2),F(J,2),G(J,2),V(J,2)
70 DIM Z(K,2),X(K,2),W(J,2),Y(K,2),Q(K,2),E(K,2)
80 DIM S(L),R(L),C(L),A(L,L),B(L,L),H(L,L),T1(L,2*L),D(9,1)
90 J = 0
100 K = 0
110 REM
120 REM
130   FOR J = 1 TO K9
140       PRINT "HOW MANY NOISE STIMULI ARE IN RESPONSE CATEGORY";J;
150       INPUT O(J,1)
160       PRINT "HOW MANY SIGNAL STIMULI ARE IN RESPONSE CATEGORY";J;
170       INPUT O(J,2)
180       PRINT " "
190 REM
200       N(J,1) = N(J-1,1) + O(J,1)
210       N(J,2) = N(J-1,2) + O(J,2)
220   NEXT J
230 REM
240 REM
250   FOR J = 1 TO K9-1
260       FOR K = 1 TO 2
270           P(J,K) = N(J,K)/N(K9,K)
280           IF N(J,K) = 0 THEN 320
290           IF N(J,K) = N(K9,K) THEN 320
300           F(J,K) = P(J,K)
310           GOTO 330
320           F(J,K) = (N(J,K)+1)/(N(K9,K)+2)
330       NEXT K
340   NEXT J
350 REM
360 P(K9,1) = 1
370 P(K9,2) = 1
380 REM
390   FOR J = 1 TO K9-1
400       FOR K = 1 TO 2
410           P = F(J,K)
420           GOSUB 3580
430           G(J,K) = Z
440           V(J,K) = P*(1-P)/N(K9,K)/Y/Y
450       NEXT K
460   NEXT J
470 REM
480 D0 = 0
490 M0 = 0
500 D3 = 0
510 M3 = 0
520 V0 = 0
530 V1 = 0
540 D4 = 0
550 REM
560   FOR J = 1 TO K9-1
570       D0 = D0 + G(J,1)*G(J,1)/(V(J,1)+V(J,2))
580       M0 = M0 + G(J,1)/(V(J,1)+V(J,2))
590       D3 = D3 + G(J,2)*G(J,2)/(V(J,1)+V(J,2))
600       M3 = M3 + G(J,2)/(V(J,1)+V(J,2))
610       V0 = V0 + 1/(V(J,1)+V(J,2))
620   NEXT J
630 REM
640 D1 = SQR((V0*D0-M0*M0)/(V0*V0))
650 D2 = SQR((V0*D3-M3*M3)/(V0*V0))
660 B0 = D2/D1

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APPENDIX A (Continued)

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670 REM
680   FOR J = 1 TO K9-1
690     D4 = D4 + (G(J,2) - B0*G(J,1))/(V(J,2) + B0*B0*V(J,1))
700     V1 = V1 + 1/(V(J,2) + B0*B0*V(J,1))
710   NEXT J
720 REM
730 A0 = D4/V1
740 REM
750   FOR J = 1 TO K9-1
760     Z(J,1) = G(J,1)/V(J,1) + B0*(G(J,2)-A0)/V(J,2)
770     Z(J,1) = Z(J,1)/(1/V(J,1) + B0*B0/V(J,2))
780     Z(J,2) = A0 + B0*Z(J,1)
790   NEXT J
800 REM
810 S(1) = -A0
820 S(2) = B0
830 REM
840   FOR J = 1 TO K9-1
850     S(J+2) = Z(J,1)
860   NEXT J
870 REM
880 C = 0
890 REM
900   FOR J = 1 TO K9-1
910     FOR K = 1 TO 2
920       Z = Z(J,K)
930       GOSUB 3500
940       Q(J,K) = P
950       Y(J,K) = Y
960     NEXT K
970   NEXT J
980 REM
990 Q(K9,1) = 1
1000 Q(K9,2) = 1
1010 REM
1020 X2 = 0
1030 L1 = 0
1040 REM
1050   FOR J = 1 TO K9
1060     FOR K = 1 TO 2
1070       X(J,K) = Q(J,K) - Q(J-1,K)
1080       E(J,K) = X(J,K)*N(K9,K)
1090       X2 = X2 + (O(J,K) - E(J,K))^2/E(J,K)
1100       L1 = L1 + O(J,K)*LOG(X(J,K))
1110     NEXT K
1120   NEXT J
1130 REM
1140   FOR J = 1 TO K9-1
1150     FOR K = 1 TO 2
1160       W(J,K) = (P(J,K)-P(J-1,K))/X(J,K) - (P(J+1,K)-P(J,K))/X(J+1,K)
1170     NEXT K
1180   NEXT J
1190 REM
1200 R(1) = 0
1210 R(2) = 0
1220 REM
1230   FOR J = 1 TO K9-1
1240     R(1) = R(1) - N(K9,2)*Y(J,2)*W(J,2)
1250     R(2) = R(2) + N(K9,2)*Y(J,2)*Z(J,1)*W(J,2)
1260     R(J+2) = N(K9,2)*Y(J,2)*B0*W(J,2) + N(K9,1)*Y(J,1)*W(J,1)
1270   NEXT J
1280 REM
1290   FOR J = 1 TO K9+1
1300     FOR K = 1 TO K9+1
1310       A(J,K) = 0
1320     NEXT K
1330   NEXT J
1340 REM
1350   FOR J = 1 TO K9-1

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APPENDIX A (Continued)

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1360  A(1,1)=A(1,1)+N(K9,2)*Y(J,2)*(Y(J,2)-Y(J-1,2))/X(J,2)
1370  A(1,1)=A(1,1)-N(K9,2)*Y(J,2)*(Y(J+1,2)-Y(J,2))/X(J+1,2)
1380  A(1,2)=A(1,2)-N(K9,2)*Y(J,2)*Z(J,1)*(Y(J,2)-Y(J-1,2))/X(J,2)
1390  A(1,2)=A(1,2)+N(K9,2)*Y(J,2)*Z(J,1)*(Y(J+1,2)-Y(J,2))/X(J+1,2)
1400  T1 = N(K9,2)*Y(J,2)*Z(J,1)
1410  A(2,2)=A(2,2)+T1*(Y(J,2)*Z(J,1)-Y(J-1,2)*Z(J-1,1))/X(J,2)
1420  A(2,2)=A(2,2)-T1*(Y(J+1,2)*Z(J+1,1)-Y(J,2)*Z(J,1))/X(J+1,2)
1430  T1 = N(K9,2)*Y(J,2)*B0
1440  A(J+2,1) = -T1*(Y(J,2)-Y(J-1,2))/X(J,2)
1450  A(J+2,1) = A(J+2,1) + T1*(Y(J+1,2)-Y(J,2))/X(J+1,2)
1460  A(1,J+2) = A(J+2,1)
1470  A(J+2,2) = T1*(Y(J,2)*Z(J,1)-Y(J-1,2)*Z(J-1,1))/X(J,2)
1480  A(J+2,2) = A(J+2,2) - T1*(Y(J+1,2)*Z(J+1,1)-Y(J,2)*Z(J,1))/X(J+1,2)
1490  A(2,J+2) = A(J+2,2)
1500  A(J+2,J+2)=N(K9,2)*Y(J,2)*Y(J,2)*B0*B0*(1/X(J,2)+1/X(J+1,2))
1510  A(J+2,J+2)=A(J+2,J+2)+N(K9,1)*Y(J,1)*Y(J,1)*(1/X(J,1)+1/X(J+1,1))
1520  NEXT J
1530 REM
1540 A(2,1) = A(1,2)
1550 REM
1560  FOR J = 1 TO K9-2
1570    A(J+2,J+3) = -N(K9,2)*Y(J,2)*Y(J+1,2)*B0*B0/X(J+1,2)
1580    A(J+2,J+3) = A(J+2,J+3) - N(K9,1)*Y(J,1)*Y(J+1,1)/X(J+1,1)
1590    A(J+3,J+2) = A(J+2,J+3)
1600  NEXT J
1610 REM
1620 M = K9 + 1
1630 R = M
1640 D5 = 1
1650 REM
1660  FOR I = 1 TO M
1670    FOR J = 1 TO M
1680      T1(I,J) = A(I,J)
1690    NEXT J
1700    T1(I,I+M) = 1
1710  NEXT I
1720 REM
1730 TL = 1/1000/M
1740 REM
1750  FOR P = 1 TO M
1760    IF ABS(T1(P,P))>=TL THEN D5 = D5*T1(P,P):GOTO 1820
1770    R = R - 1
1780    FOR U = 1 TO 2*M
1790      T1(P,U) = 0
1800    NEXT U
1810    GOTO 1950
1820    PIV = T1(P,P)
1830    FOR J = 1 TO 2*M
1840      T1(P,J) = T1(P,J)/PIV
1850    NEXT J
1860 REM
1870  FOR I = 1 TO M
1880    IF I = P THEN 1940
1890    PIV = T1(I,P)
1900 REM
1910  FOR J = 1 TO 2*M
1920    T1(I,J) = T1(I,J) - PIV*T1(P,J)
1930  NEXT J
1940  NEXT I
1950  NEXT P
1960 REM
1970  FOR I = 1 TO M
1980    FOR J = 1 TO M
1990      H(I,J) = T1(I,J)
2000      B(I,J) = T1(I,J+M)
2010    NEXT J
2020  NEXT I
2030 REM
2040  FOR I = 1 TO M

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APPENDIX A (Continued)

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2050   C(I) = 0
2060   NEXT I
2070 REM
2080   FOR I = 1 TO M
2090     FOR J = 1 TO M
2100       C(I) = C(I) + B(I,J)*R(J)
2110     NEXT J
2120   NEXT I
2130 REM
2140 IF C>0 THEN 2190
2150 REM
2160 GOSUB 2800
2170 GOSUB 2990
2180 REM
2190 S(0) = 0
2200 REM
2210   FOR I = 1 TO M
2220     S(I) = S(I) + C(I)
2230     S(0) = S(0) + ABS(C(I))
2240   NEXT I
2250 REM
2260 A0 = S(1)
2270 B0 = S(2)
2280 REM
2290   FOR J = 1 TO K9 - 1
2300     Z(J,1) = S(J+2)
2310     Z(J,2) = B0*Z(J,1) - A0
2320   NEXT J
2330 REM
2340 C = C + 1
2350 PRINT "*";
2360 REM
2370 IF S(0)>.001 THEN 900
2380 D(1,0) = A0/B0
2390 D(1,1) = B(1,1)/B0/B0 + B(2,2)*A0^2/B0^4
2400 D(1,1) = D(1,1) - 2*B(1,2)*A0/B0^3
2410 REM
2420 D(2,0) = 2*A0/(1+B0)
2430 D(2,1) = 4*B(1,1)/(1+B0)^2 + 4*B(2,2)*A0^2/(1+B0)^4
2440 D(2,1) = D(2,1) - 8*B(1,2)*A0/(1+B0)^3
2450 REM
2460 D(3,0) = A0/SQR(1+B0^2)
2470 D(3,1) = B(1,1)/(1+B0^2) + B(2,2)*A0^2*B0^2/(1+B0^2)^3
2480 D(3,1) = D(3,1) - 2*B(1,2)*A0*B0/(1+B0^2)^2
2490 REM
2500 D(4,0) = SQR(2)*D(3,0)
2510 D(4,1) = 2*D(3,1)
2520 REM
2530 D(5,0) = 1/B0
2540 D(5,1) = B(2,2)/B0^4
2550 REM
2560 D(6,0) = D(5,0) - 1
2570 D(6,1) = D(5,1)
2580 REM
2590 D(7,0) = A0/(1-B0)
2600 D(7,1) = B(1,1)/(1-B0)^2 + B(2,2)*A0^2/(1-B0)^4
2610 D(7,1) = D(7,1) + 2*B(1,2)*A0/(1-B0)^3
2620 REM
2630 Z = D(3,0)
2640 GOSUB 3520
2650 D(8,0) = P
2660 D(8,1) = D(3,1)*Y*Y
2670 REM
2680 PRINT " "
2690 INPUT "READY PRINTER, THEN HIT RETURN:";B$
2700 GOSUB 2990
2710 GOSUB 3180
2720 INPUT "READY PRINTER, THEN HIT RETURN:";B$
2730 GOSUB 3420

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APPENDIX A (Continued)

```

2740 REM
2750 REM
2760 END
2770 REM
2780 REM
2790 REM
2800 LPRINT " "
2810 LPRINT "RATING","OBSERVED FREQUENCIES:"
2820 LPRINT "CATEGORY:","NOISE:","SIGNAL:"
2830 FOR J = K9 TO 1 STEP -1
2840 LPRINT J, O(J,1),O(J,2)
2850 NEXT J
2860 LPRINT "-----","-----","-----"
2870 LPRINT "SUM:",N(K9,1),N(K9,2)
2880 REM
2890 LPRINT " "
2900 LPRINT "RATING", "OBSERVED PROPORTIONS:"
2910 LPRINT "CATEGORY:","NOISE:","SIGNAL:"
2920 FOR J = K9 TO 1 STEP -1
2930 LPRINT J, (O(J,1)/N(K9,1)),(O(J,2)/N(K9,2))
2940 NEXT J
2950 LPRINT " "
2960 RETURN
2970 REM
2980 REM
2990 LPRINT " "
3000 LPRINT , , "ITERATION";C
3010 LPRINT , "PARAMETER","ESTIMATE","FIRST"
3020 LPRINT "PARAMETER:","ESTIMATE:","VARIANCE:","PARTIAL:"
3030 LPRINT " "
3040 LPRINT "A",S(1),B(1,1),R(1)
3050 LPRINT " "
3060 LPRINT "B",S(2),B(2,2),R(2)
3070 FOR J = 1 TO K9-1
3080 LPRINT " "
3090 LPRINT "Z";J,S(J+2),B(J+2,J+2),R(J+2)
3100 NEXT J
3110 LPRINT " "
3120 LPRINT "CHI SQUARE=";X2
3130 LPRINT " "
3140 LPRINT "LOG LIKELIHOOD=";L1
3150 RETURN
3160 REM
3170 REM
3180 LPRINT " "
3190 LPRINT , , "OTHER INDICES:"
3200 LPRINT , "INDEX:","ESTIMATE:","VARIANCE:"
3220 LPRINT " "
3230 LPRINT , "DELTA M",D(1,0),D(1,1)
3240 LPRINT " "
3250 LPRINT , "D(E)",D(2,0),D(2,1)
3260 LPRINT " "
3270 LPRINT , "A(Z)",D(8,0),D(8,1)
3280 LPRINT " "
3290 LPRINT , "D(A)",D(4,0),D(4,1)
3300 LPRINT " "
3310 LPRINT , "SD",D(5,0),D(5,1)
3320 LPRINT " "
3330 LPRINT , "DELTA SD",D(6,0),D(6,1)
3340 LPRINT " "
3350 LPRINT "(DELTA M)/(DELTA SD)",D(7,0),D(7,1)
3360 LPRINT " "
3370 LPRINT , "Z(A)",D(3,0),D(3,1)
3380 LPRINT " "
3390 RETURN
3400 REM
3410 REM
3420 LPRINT , "VARIANCE-COVARIANCE MATRIX:"

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APPENDIX A (Continued)

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3430   FOR I = 1 TO L
3440     FOR J= 1 TO L
3450       LPRINT USING "##.#### ";B(I,J);
3460     NEXT J
3470   LPRINT " "
3480 NEXT I
3490 RETURN
3500 REM
3510 REM
3520 Y = EXP(-Z*Z/2)/SQR(2*3.14159)
3530 W = Z:N = 1:D = Z:U = Z*Z
3540 N = N + 2:D = D*U/N:W = W + D
3550 IF ABS(D)>.000001 THEN 3540
3560 W = W*Y:P = W + .5
3570 RETURN
3580 REM
3590 REM
3600 Q = P
3610 Z = (Q - .5)/.4
3620 GOSUB 3520
3630 Z = Z + (Q - P)/Y
3640 IF ABS(Q-P)>.000001 THEN 3620
3650 RETURN

```

APPENDIX B

Sample Output for Subject 3 in the User's Manual

RATING	OBSERVED FREQUENCIES:	
CATEGORY:	NOISE:	SIGNAL:
6	47	348
5	43	66
4	42	32
3	26	26
2	99	44
1	334	81
----	----	----
SUM:	591	597

RATING	OBSERVED PROPORTIONS:	
CATEGORY:	NOISE:	SIGNAL:
6	7.952622E-02	.5829146
5	7.275803E-02	.1105528
4	7.106599E-02	5.360134E-02
3	4.399323E-02	4.355109E-02
2	.1675127	7.370184E-02
1	.5651438	.1356784

PARAMETER:	PARAMETER ESTIMATE:	ITERATION 0	
		ESTIMATE VARIANCE:	FIRST PARTIAL:
A	1.22866	6.391967E-03	-1.241581
B	.716576	4.27494E-03	1.736123
Z 1	.1679222	2.672896E-03	.9062052
Z 2	.6098079	2.748772E-03	-10.40587
Z 3	.7698475	2.929841E-03	14.39708
Z 4	1.019365	3.472101E-03	-8.004211

APPENDIX B (Continued)

Z 5 1.415658 5.35187E-03 3.254386

CHI SQUARE= 2.951889

LOG LIKELIHOOD=-1577.705

PARAMETER:	PARAMETER ESTIMATE:	ITERATION 2 ESTIMATE VARIANCE:	FIRST PARTIAL:
A	1.224568	6.382348E-03	4.032135E-03
B	.7156026	4.268837E-03	8.068085E-04
Z 1	.1659488	2.688653E-03	7.288933E-03
Z 2	.6062839	2.769077E-03	-1.345062E-02
Z 3	.770981	2.958003E-03	1.079559E-03
Z 4	1.016232	3.492769E-03	4.867554E-03
Z 5	1.41673	5.38901E-03	-6.160736E-04

CHI SQUARE= 2.828581

LOG LIKELIHOOD=-1577.664

INDEX:	OTHER INDICES: ESTIMATE:	VARIANCE:
DELTA M	1.711241	1.418252E-02
D(E)	1.427566	5.042343E-03
A(Z)	.8403391	1.515155E-04
D(A)	1.408347	5.132929E-03
SD	1.397424	1.627878E-02
DELTA SD	.3974236	1.627878E-02
(DELTA M)/(DELTA SD)	4.305836	1.418941
Z(A)	.9958516	2.566465E-03

VARIANCE-COVARIANCE MATRIX:

0.0064	0.0034	0.0020	0.0017	0.0015	0.0011	0.0003
0.0034	0.0043	0.0008	0.0000	-0.0004	-0.0010	-0.0025
0.0020	0.0008	0.0027	0.0021	0.0019	0.0016	0.0013
0.0016	0.0000	0.0021	0.0028	0.0026	0.0024	0.0022
0.0015	-0.0004	0.0019	0.0026	0.0030	0.0028	0.0027
0.0011	-0.0010	0.0016	0.0024	0.0028	0.0035	0.0035
0.0003	-0.0025	0.0013	0.0022	0.0027	0.0035	0.0054