PROGRAM ABSTRACTS/ALGORITHMS

A simple algorithm to obtain nonparametric response bias estimates using Hodos's method

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Since its origination in 1954 (Tanner & Swets, 1954), the theory of signal detection (TSD) has gained wide acceptance in the study of perception and memory as an alternative to classical psychophysical methodologies. The biggest advantage of the TSD over classical psychophysical methods is that TSD provides independent parameters for estimating sensory discrimination and response biases in the tasks requiring assessment of a subject's ability to discriminate the occurrence of discrete binary events (Gardner & Boice, 1986).

Normally, application of the TSD-based parameters requires specific assumptions about the underlying distributions. However, some nonparametric indices of sensitivity and bias are also available for detection/recognition experiments in which specific underlying distributions are not assumed (Grier, 1971).

According to Treisman and Williams (1984) and Gardner and Boice (1986), researchers in psychophysics in general have concentrated primarily on the indices of sensory sensitivity and neglected the response bias indices. This neglect is reflected in the fact that although several nonparametric measures of sensitivity have been developed, there is still only one index of response bias: B'_H (Hodos, 1970).

B'_H is a rich index of response bias in that it preserves the notion of "yea" sayers, "nay" sayers, and unbiased respondents. The index is very fruitful in those research situations in which one does not wish to make distributional assumptions (see, e.g., Singh & Churchill, 1986). However, a major drawback to the utilization of the index B'_H has been its cumbersome calculation. The purpose of this paper is to provide a simple algorithm for computing B'_H quickly and efficiently. B'_H is a measure of response bias based upon the location of a point, determined by the respondent's hit rate and false-alarm rate, in the unit square (see Hodos, 1970). In Figure 1, the positive diagonal line represents chance performance. Points below this diagonal are due to a subject's malingering,

Figure 1. Unit square.

that is, the subject is deliberately saying "yes" when he/she thinks he/she should have said "no," and vice versa. Points falling to the right and left of the negative diagonal represent a tendency to say "yes" and "no," respectively. The points falling on the negative diagonal indicate unbiased performance.

Originally, as proposed by Hodos (1970), B'_H was "calculated" by graphical estimation. Fortunately, Grier (1971) provided the following functional expressions for computing B'_H : for points to the left of the negative diagonal,

$$B'_H = 1 - \frac{x(1-y)}{y(1-x)}, \quad (1)$$

and for points to the right of the negative diagonal,

$$B'_H = \frac{y(1-y)}{x(1-x)} - 1, \quad (2)$$

where x and y are the false-alarm and hit rates, respectively.

Unfortunately, even in the Grier (1971) formulation given above, the false-alarm rate and the hit rate must be plotted on the unit square prior to the calculation of B'_H . If one is dealing with a number of subjects, plotting their responses in the unit square may become an exacting task. In the following section, we propose an algorithm to calculate B'_H , which completely eliminates the need for plotting the point to determine the respondent type.

This project was supported in part by the University of Kansas General Research Fund allocation 3401-20-0038. Support was also provided by the University of Kansas, School of Business Research Fund. The ideas and opinions expressed herein are solely those of the authors. Address correspondence to Surendra N. Singh, Faculty Suite-Summerfield Hall, School of Business, University of Kansas, Lawrence, KS 66045-2003.

Algorithm

Step 1. Determine if $y > x$. Otherwise, the respondent is either malingering ($y < x$) or operating by chance ($y = x$).

Next, determine whether the subject is a "yea" sayers, "nay" sayers, or an unbiased respondent:

Step 2. Let $F = 1 - \frac{x(1-x)}{y(1-y)}$.

Step 3. If $F = 0$, then the respondent is unbiased.

This follows from the fact that $F = 0$ if and only if $y = x$ or $y = -x + 1$. Since $y > x$ and (x, y) is in the unit square, $F = 0$ if and only if (x, y) is on the negative diagonal of the unit square.

Step 4. If $F > 0$, then the point (x, y) will fall to the left of the negative diagonal and the respondent is a "nay" sayers. Moreover, in this case $B'_H = F$ as represented in Equation 1.

Step 5. If $F < 0$, then the point (x, y) will fall to the right of the negative diagonal and the respondent is a "yea" sayers. In other words,

$$B'_H = \frac{y(1-y)}{x(1-x)} - 1.$$

The validity of the last two steps follows from the fact that for any given point (x, y) in the unit square, both formulas for B'_H (as shown in Equations 1 and 2, respectively) have the same sign. Moreover, that common sign is positive when (x, y) is to the left of the negative diagonal and negative when (x, y) is to the right.

Proof of Claim. Let Equations 1 and 2 be represented by NB'_H and YB'_H , respectively. That is, let

$$NB'_H = 1 - \frac{x(1-x)}{y(1-y)} = \frac{y(1-y) - x(1-x)}{y(1-y)},$$

and

$$YB'_H = \frac{y(1-y)}{x(1-x)} - 1 = \frac{y(1-y) - x(1-x)}{x(1-x)}.$$

Notice that the denominators of NB'_H and YB'_H are both positive since $0 < y < 1$ and $0 < x < 1$, respectively. Thus, the signs of both NB'_H and YB'_H are determined by the signs of the respective numerators, which in each case is $y - y^2 - x + x^2$. In other words, the signs of NB'_H

and YB'_H are always identical in the unit square and determined by the sign of $y - y^2 - x + x^2$. Since a level curve for the function $y - y^2 - x + x^2$ is a hyperbola with asymptotes $y = x$ and $y = -x + 1$, the common sign for NB'_H and YB'_H is positive when the respondent is a "nay" sayers and negative when the respondent is a "yea" sayers.

Summary

The proposed algorithm greatly simplifies the computation of B'_H compared with the existing methods and is easily programmed on a personal computer. This algorithm should be helpful to those researchers who want to compute a TSD-based nonparametric index of response bias that preserves the notion of "yea" saying, "nay" saying, and unbiased response. In this sense, the proposed algorithm should be potentially useful to researchers not only in psychology, but also in disciplines such as educational psychology (Cronbach, 1946, 1950) and advertising (Singh & Churchill, 1986; Wells, 1961, 1963).

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(Revision accepted for publication July 16, 1987.)