SESSION IX SYMPOSIUM SIMULATING NEURAL NETWORKS

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Statistical analysis of fuzzy-set data from neuronal networks

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In this study, we describe the process of transforming neuron polarization values that meet the assumptions of ratio data into ordinal data. This is a crisp-to-fuzzy set transformation. A computerized simulation of the application of this process is discussed and nonparametric methods of analyzing fuzzy-set data are presented.

The applications of fuzzy-set theory to neural networks have been demonstrated by Kosko (1988) and Togai (1988). Their research was among the first to apply the theory developed by Zadeh (1972) to the emerging science of neural networks. Although little research has been done in the application of statistics to fuzzy sets, it is generally believed that fuzzy-set theory is "nonstatistical" with regard to the analysis-of-variance model (Kandel, 1986). The purpose of this paper is to demonstrate the application of nonparametric statistics to fuzzy sets developed by neuronal networks.

The main characteristic of fuzzy-set theory that is of interest in this paper is that class membership in a fuzzy set is a function that can vary, rather than a sharp dichotomy like class membership in a traditional set (crisp set). An example of a crisp set in the behavioral sciences might consist of whether or not an adult Yanomamo Indian male has killed an enemy and thus belongs to a class called *unokais*. The adult male Yanomamo population could be dichotomized into two crisp sets: those who are *unokais* and those who are not. It turns out that 44% of adult males estimated to be over the age of 25 are *unokais*

(Chagnon, 1988). However, the definition of *adult male* constitutes a fuzzy set, since it implies a range of ages whose minimum is 25 years and whose maximum is age at death. While it is operationally simple to deal with different ages at death, the definition of the lower limit of the set creates problems. This limit is particularly fuzzy in the present example, because the Yanomamo have no precise way of determining age (Chagnon, 1988). One assumes that if Chagnon had reported the percentage of males who were *unokais* starting at approximately age 18, it would have been smaller.

The definition of variables such as adulthood is a problem more typical of the behavioral sciences, including neuropsychology. Therefore, the assignment of class membership by the rules of fuzzy-set theory is of particular interest to these disciplines. It turns out that fuzzy-set membership involves basically an ordinal scale. Therefore, the fuzzy sets have *irreflexive*, *asymmetrical*, and *transitive* properties. Presented symbolically, any $x > x$, if $x > y$, then $y \not> x$; and if $x > y$ and $y > z$, then $x > z$.

Actually, fuzzy sets are not single sets; they are, instead, matched pairs of sets. In the case of the age of the Indians, one of the sets contains the actual ages of the Indians in years. This, then, would be a vector containing the ages of the male Indians in the tribe. Obviously, the data in this vector meet the assumptions of ratio data. The

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second set is a dichotomous vector, and there is a one-to-one correspondence between the elements in the two vectors. The elements (1,0) of the second vector are dichotomous and are driven or defined by the elements of the first vector. For those male Indians who have reached the age of 25 (have a value of 25 or more in the first vector), a 1 will be in the second vector. Otherwise, the value will be 0. The second criterion of whether or not an adult male Indian has killed an enemy is imposed on the second vector to obtain the *end* fuzzy set.

While the ages of the male Indians are, because of their origins, ratio data, the final vector of scores for their ages comprises interval data. By modifying, or changing, the rigor with which the criterion for being an adult is applied, membership in the final set can be changed or manipulated. Hence the difference between fuzzy and crisp sets.

Building on this analogy, let us consider the neuron. The state of neuronal polarization is most often viewed as dichotomous, but it is well known that there are a number of factors affecting it. The crisp sets to which neuron data might belong are polarized or depolarized, and often the polarized state is assigned a value of 70 mV. In actuality, the polarized state is a range of values with a minimum of 60 mV and a maximum of 80 mV (Patton, 1961). This is a difference of electrical potential that can be measured, and such data meet the assumption of interval or, possibly, even ratio data. The value of 70 mV is the mean of these values. A vector of the level of polarization of neurons can be used to develop a matching, dichotomous vector, denoting whether or not a neuron has fired. For a neuron to be a member of the depolarized set, its electrical state must be less than its polarized state. If 70 mV is taken as the median for the polarized set, then 60 mV can serve as the maximum for the depolarized set. We can define the polarized fuzzy set of neurons as having a range of voltages from 60 mV to 80 mV, and class membership in the depolarized fuzzy set can be defined as a difference of potential less than 60 mV. From this, it can be seen that the condition of a neuron across time can easily be expressed as a fuzzy set: depolarization ≤ 60 mV.

In real experiments with small populations of neurons, the prediction of membership in a set as a result of its level of depolarization sometimes appears chaotic. In a Pavlovian conditioning study involving a population of related neurons in the sea slug, Mpitso (1987) was unable to predict the classification or depolarization of motoneurons. In fact, Mpitso calls this response *deterministic chaos*. Mpitso reported that, although the organism responded on every trial by opening its mouth, the nine motoneurons responsible for that response were not predictable. Nevertheless, the gross evoked potential from all nine neurons was correlated with the conditioned response. This poses a challenging neuronal network. Stated formally, the task was to design a population of nine motoneurons whose output is a conditioned response

(CR) on every trial, but whose nine individual motoneurons approach random firing (chaos).

METHOD

The apparatus used in this study was the Micro SAINT (1985) simulation program, which was disk-resident in an IBM PC. The program is specifically designed to simulate serial and parallel processes (Bremner & Eddy, 1987) and neuron networks (Bremner & Yost, 1988). The program, which is menu driven, uses spreadsheet-like functions to set the contingencies and logical operands that model the behavior of neurons. These functions include "if" statements and "greater than/less than" relationships, as well as probability-distribution functions. Therefore, a neural network of nine motoneurons was built that collectively gave a CR, but the polarization of each motoneuron was normally distributed with a mean of 70 mV, a standard deviation of 4 mV, a maximum of 80 mV, and a minimum of 60 mV. A diagram of the resulting network is presented in Figure 1.

In Figure 1, *sgen* represents the apparatus that drives the experiment, including stimulus generation; *cs* represents transduction of the conditional stimulus; and *mouth* represents the muscles that make the CR. All of the other components of the networks are neurons (i.e., $x, p, qq, r \dots y$). Neurons x and $x1$ represent triggering neurons, which, when fired, attempt to depolarize the nine motoneurons (p through y) by 5 mV. Each of the nine motoneurons has a polarization equation that controls its threshold. This equation takes into account the fluctuating nature of neuron polarization. The fluctuation is the outcome of local depolarizations due to the many synapses impinging upon the motoneurons, in addition to those from x and $x1$ (Mpitso & Cohan, 1986a, 1986b). These fluctuations were normally distributed with a mean of 70 mV, a minimum of 60 mV, a maximum of 80 mV, and a standard deviation of 4 mV. For convenience, we made the polarization vary in integer steps from 60 mV to 80 mV. The polarization equation was of the following form: polarization = normal (70 mV, 4 mV), max (80 mV), min (60 mV).

The threshold for depolarization was set at 60 mV, so that when polarization fell below 60 mV for any motoneuron, it triggered the CR. The polarization equation was loaded in each neuron so that their thresholds were not synchronized with or dependent upon each other. The x and $x1$ neurons decreased polarization by 6 mV. Therefore, when the state of polarization due to random fluctuation was 66 mV or less, the motoneuron would be triggered by x or $x1$, resulting in the CR. The form of the triggering equation was as follows: if polarization minus 6 mV ≤ 60 mV, then CR.

With nine motoneurons, the probability that one of the nine would be depolarized and produce a CR was high, but the probability that an electrode in any one of nine motoneurons would correlate with the CR was low.

Figure 1. Neuron network.

RESULTS AND DISCUSSION

The results from one electrode for running the neural network for 100 trials are given in Table 1. The recording electrode is in motoneuron *p*. Note that the firing pattern is irregular and that it would be difficult to predict when it would trigger the CR, even though motoneuron *p* fires approximately 15% of the time. The fact that there is always a CR is due to the firing of at least one of the other motoneurons. In this simulation, we have assumed that one motoneuron is sufficient to trigger a CR. The mean and median polarization of motoneuron *p* is 70 mV. The standard deviation is 4 mV, strongly suggesting a normal distribution of these values. The range is 20 mV, the maximum is 80 mV, and the minimum is 60 mV.

The level of polarization across time is the interval or ratio-level data that describe the condition of the neuron. Cells having a polarization level less than 66 mV at the time that neuron *x* fires are considered to cause the CR. When a neuron fires (has a polarization less than 66 mV), it is given a value of 1 in the fuzzy set, which represents the CR.

Two methods in this simulation can be used to influence membership in the fuzzy set. First, if the criterion (66 mV) for depolarization is decreased, then the probability of the CR's occurring will increase. Since there is

precedence for the range of polarizations and the threshold for polarization, we chose not to deviate from them. Second, if we had changed the dispersion or shape of the distribution, we could have modified the probability of a CR's occurring. For example, by increasing the standard deviation, the area under the ends of the curve would

Table 1
Data and Summary Statistics

Trial	Data	Trial	Data
1	70	11	65
2	72	12	80
3	67	13	76
4	70	14	75
5	64	15	68
6	63	16	74
7	73	.	.
8	71	.	.
9	67	.	.
10	71	100	63

Neuron					
	<i>p</i>	<i>qq</i>	<i>r</i>	<i>s</i>	<i>t</i> ... <i>y</i>
Mean	70.1	70.8	69.5	70.4	69.8
Standard deviation	4.4	3.8	4.1	3.8	4.2 ...
Maximum	80	80	79	78	80
Minimum	60	61	60	60	61

Note—All values are for millivolts.

increase, and, correspondingly, the probability of obtaining a CR would also increase. Similarly, by skewing the distribution to either the left or the right, the probability of a CR's occurring would increase or decrease, respectively.

There are many differences in the statistics that can be used with the data in the two matching sets. Since the polarization data, which are measured in mV, are interval or ratio in nature, most of the statistical analyses in the literature are parametric. Given the difficulty in gathering this type of data, the reliability of the data, and, frequently, the limited size of the data sets, it is difficult to believe that all of the assumptions implicit in the use of parametric statistics are met.

The data in the fuzzy sets are dichotomized and simple, and there are a number of nonparametric statistics that can be used with them. Since there are fewer underlying assumptions implicit in the use of nonparametric statistics with this type of data, the statistics can be more valid. Also, by adjusting the criteria for inclusion or exclusion in the fuzzy sets, an investigator can run analyses that evaluate the "best-" and "worst-case" scenarios within a study.

Statistics such as chi Square, the McNemar test for the significance of changes, the Cochran Q test, contingency coefficients, and the binomial test are appropriate for use with fuzzy data sets.

In the neuronal network simulation, we found that the probability of one of the p through y neurons' depolarizing when x depolarized was 0.15 ($p = .15$). If we had been interested in the probability of the CR's having been associated with the depolarization of two or three or four or more of the nine neurons, the binomial distribution could have been employed. There are a number of nonparametric statistics that are appropriate for use with a number of different experimental designs.

In this study, we view the data from this neuron as a fuzzy set that takes on a range of values representing the fluctuating nature of the polarization of the neuron. The neuron conceptualized here is very different from the McCullough-Pitts dichotomous neuron so often used in neuronetworks (Braitenberg, 1986). As Mpitsos and Cohan (1986a and 1986b) point out, the multiple connections among neurons lead to high variability in the be-

havior of individual neurons. Moreover, this variability is so high that Mpitsos (1987) considers it chaotic. The fuzzy-set nature of polarization alone can be used to simulate the behavior of neurons in a conditioning experiment. This does not mean that we believe that the very complex and elegant work of Mpitsos is explained by our simple equations, but that we believe it can be done as we have shown. In short, we have presented a possible, but not necessarily sufficient, explanation of the behavior of some motoneurons.

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