Notes and Comment

Are there any psychophysical applications of single-observation confidence intervals?

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After setting up the equipment for a forced-choice psychophysical experiment, an experimenter might vary the stimulus upward and downward continuously, using his or her own eye or ear, to get an idea of the vicinity of the threshold. This is an informal application of the method of adjustment. The information that it yields can be useful in selecting stimulus values to be used in a non-sequential forced-choice method, or the start-up parameters in a sequential one.

If just one adjustment trial were allowed before the forced-choice experiment was begun, it seems that there would be information useful only for the choice of the center of the stimulus range, but not the size of the range, since no variability estimate would be available.

A remarkable theorem (Abbott & Rosenblatt, 1963; Machol & Rosenblatt, 1966) suggests that only a single adjustment trial might yield information for determining both the center and the size of the range of stimuli for the following run of forced-choice trials. According to the theorem, a single observation from a normally distributed population with unknown mean and variance can determine a finite confidence interval (CI) for the mean. It has seemed to us that the most likely application, if any, might be in psychophysics, and perhaps in a problem somewhat like the one described above. It seems unlikely that a single-observation CI would be reported as the final outcome of any experiment. It would more likely be used in sequential decision making in some multistage experiment.

Actually, there are several theorems that provide CIs for the mean and standard deviation of a normal distribution with unknown mean and variance, and one that provides a CI for the median of any continuous symmetric unimodal distribution.

In each of these three cases, the user starts by specifying an *a priori* guess, A , of the center of the distribution. Then the observation, X , is obtained, and the further steps make use of the absolute difference, $|X - A|$. The prior value of A may be chosen in any arbitrary way, but it is advantageous if A is near the true center of the distribution. The widths of all three CIs are proportional to $|X - A|$, but the proportionality factors are different. These widths have been calculated for several confidence levels, for $|X - A| = 1$. The values are given in Table 1. The expressions for the CIs, and the proofs, are compli-

cated enough so that they are not presented here. The remaining discussion concerns only the CI for the mean. The questions that have occurred to us seem to cluster around three main points: (1) apparent anomalies, (2) relation to more familiar CIs, and (3) possible applications.

Apparent Anomalies

One apparent anomaly is the role of A . This seems to introduce some element of subjectiveness into any possible application. The theorem itself, however, merely makes the assumption that A is any real number and that it is chosen independently of X . Subject to that condition, the value may be chosen subjectively, objectively, or entirely at random. The theorem still holds.

Another apparent anomaly is the coincidental possibility that $X = A$. In that case, the width of the CI would shrink to zero. This is no more of a problem, however, than that which arises when $X_1 = X_2$, in a two-observation sample, in the usual formulation of a CI based on Student's t . It can happen due to overly coarse intervals with a discrete measurement scale. In either case, an obvious solution is to increase the precision of measurements.

Comparison with a More Familiar CI

For the single-observation CI for the mean, the limits are symmetrical about the observed mean (the single observation), at a distance proportional to $|X - A|$. For a two-observation CI, of the usual t -distribution kind, the limits are likewise symmetrical about the observed mean, at a distance proportional to $|X_1 - X_2|$. With the proportionality factors for the case of $\alpha = .05$, the widths are $19.36 |X - A|$ and $17.97 |X_1 - X_2|$, respectively. The first of these widths is from Table 1. The second is from the critical t value of 12.7 and the simple relation between $|X_1 - X_2|$ and the standard deviation, S , of two observations, that is, $S = |X_1 - X_2|/\sqrt{2}$. In the latter case, the width is $2St$.

These widths lead to some interesting comparisons. First, consider taking, for the value of A , a prior independent observation from the same distribution. In that case, the t interval is about 92% as wide as the "single-observation" interval. The ratio of widths obtained is nearly the same with $\alpha = .01$ as with $\alpha = .05$. So, it is slightly advantageous to use the familiar t interval when two independent observations from the same distribution are available. Under what conditions might the advantage swing the other way? A necessary condition is that A be based on more information than is available in a single observation. The ideal extreme case would be the one in which A is equal to the true mean. Then $19.36 |X - A|$ is distributed as chi (the square root of chi-square) with 1 df and a scale factor of 19.36. This is comparable to

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Table 1
Widths of Confidence Intervals for the Mean, Standard Deviation, and Median, for
Several Confidence Levels, with $|X - A| = 1$

Parameter	Confidence Level						
	.50	.60	.70	.80	.90	.95	.99
Mean	2.0000	2.4392	3.2328	4.8412	9.6790	19.359	96.780
SD	1.4826	1.9069	2.5953	3.9472	7.9582	15.949	79.820
Median	2.0000	3.0000	4.6667	8.0000	18.000	38.000	198.00

the two-observation case in that $17.96|X_1 - X_2|$ is distributed as chi with 1 df and a scale factor of $17.96\sqrt{2} = 25.4$. Thus, in the ideal case, the single-observation CI shows about a 25% advantage over the two-observation CI, on the average. If A is at a fixed distance from the true mean, on the other hand, $|X - A|$ is distributed as noncentral chi and would tend to be quite large.

Another issue of comparison is robustness under violations of assumptions. Other things being equal, the mean of two observations is likely to be more nearly normally distributed than a single observation. Other than the CI for the median, the theorems (Abbott & Rosenblatt, 1963; Machol & Rosenblatt, 1966) do not address effects of violating the assumption of normality, so it would be desirable to obtain Monte-Carlo evidence. The assumption of equal means and variances of the populations from which X_1 and X_2 are sampled for the two-observation CI has no counterpart with the single-observation CI.

These comparisons serve to elaborate the concept of the single-observation CI, and relate it to something more familiar. It is not proposed that a single observation is ever preferable to two, when two are available.

Example of a Possible Application

Under most imaginable circumstances, a finding based on a single observation would not be reportable as a scientific finding. A case for the use of a single-observation CI might be one in which the conditions are repeatable but the observation must be used in a design decision before any repetition is feasible. Consider a possible example.

A dark-adaptation experiment is to be performed in which the two eyes are presented with different conditions. Initially, both eyes are light adapted. Then the light-adaptation conditions are maintained constant for the left (standard) eye, while the right (comparison) eye is allowed to adapt to the dark. During the dark adaptation, the subject is to make a series of fairly rapid adjustments of the brightness of an incremental stimulus presented to the right eye to match that of a standard incremental stimulus to the left eye. It is likely that the experimenter, or a computer program, could make a fairly good prediction of the adjustment for a given trial by extrapolation from the adjustments made on preceding trials.

It is well known that sensitivity can change over several orders of magnitude during dark adaptation. Continuously variable stimulus equipment often does not have the neces-

sary dynamic range, so it is common to use a discrete series of neutral-density filters, to shift the center of the continuously variable range several times during a run. In such an experiment, a single-observation CI might be used to decide when to change to a new neutral-density filter. The trialwise CIs would be formed with A as the extrapolated prediction and X as the actual observed adjustment. Change of filters would be prescribed whenever one limit of the CI extends outside the currently available continuously variable stimulus range. With proper automation, the filter changes could be made rapidly enough to allow the trials to proceed without interruption, at points closely spaced in time.

Against the procedure described above, it might be objected that the experimenter, or computer program, could just as well calculate a running estimate of variability of the observed adjustments about the respective extrapolations. Thence, confidence intervals could be calculated, in the sequence, by the familiar t method. That might be a better technique, but it assumes time invariance of the variability that is being estimated and zero bias of the extrapolated prediction at each time point, whereas the single-observation method does not.

Also against this procedure it could be argued that vision researchers have been plotting dark-adaptation curves satisfactorily for a century or so, with little explicit use of sequential statistical methodology.

Conclusion

Because our best example is the hypothetical dark-adaptation experiment, and because it is not a very convincing application of the single-observation CI, perhaps it should be concluded that there are no real practical applications. That is not a deductive argument, however, and it is possible that some psychophysicist colleague knows of a problem for which this curious CI is the ideal solution.

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