The effect of fundamental frequency on the discriminability between pure and tempered fifths and major thirds

JOOS VOS and BEN G. van VIANEN

Institute for Perception TNO, Soesterberg, The Netherlands

Thresholds were determined for discrimination between pure and tempered musical intervals (DTs) consisting of simultaneous complex tones. Tempered intervals are characterized by small frequency differences between those harmonics that coincide in pure intervals. Interference of these nearly coinciding harmonics gives rise to the perception of beats. Two parameters of beats were varied independently: (1) beat frequency and (2) depth of level variation (temporal envelope) as a measure of DT. DTs were determined for musical fifths and major thirds at a tone duration of 0.5 sec. The geometric mean of the fundamental frequency of the tones was varied in octave steps from 92.5 to 740 Hz. The beat frequency of the intervals was varied within a range of 2 to 64 Hz. The major result of our experiment was that, for a frequency range comprising 3 to 4 octaves, DTs do not depend on the frequency of the fundamentals, provided that the DTs are ordered according to beat frequency. When tempering (T) is expressed in cents and only those conditions are considered in which the degree of T corresponds to that found in the relevant tuning systems, it can be concluded that perceptual differences between tempered and pure fifths are increasingly irrelevant for tones lower than about middle C (262 Hz). Tempered major thirds, even those in a compromise tuning system such as that proposed by Silbermann (T=6 cents), can be discriminated from pure major thirds for all fundamental frequencies that are relevant in musical composition.

Discrimination between pure and tempered musical intervals consisting of simultaneous complex tones has been investigated in previous studies. Tempered intervals are characterized by small frequency differences between harmonics that coincide in pure intervals. Interference of these nearly coinciding harmonics gives rise to the perception of beats. Thresholds for discrimination (DTs) are expressed in terms of the depth of the level variation (temporal envelope) of these beating harmonics.

The main results obtained in one of the first studies (Vos, 1982) were that the DTs are higher (lower sensitivity) for major thirds than for fifths, are highest for low degrees of tempering, and decrease with increasing tone duration. In a second paper, Vos (1984) showed that for moderately tempered fifths, the value of DT depends mainly on the degree of interference of the first pair of nearly coinciding harmonics, whereas for major thirds, interference of other harmonics may play a role as well. For a large set of intervals varying in size between and including unison and twelfth, Vos and van Vianen (1985) showed that DTs gradually increase with the sum of the integers in the frequency ratio of the pure interval. In ad-

dition, they concluded that for all intervals investigated, DT is determined by the interference between nearly coinciding harmonics. DT, providing an objective measure for sensitivity to tempering, was related to tonal consonance and tuning procedures (Vos & van Vianen, 1985).

In these experiments, DTs were determined for only a restricted frequency range: for fifths and major thirds, the fundamental frequencies were between 261.6 and 523.2 Hz, that is, using staveless notation (Young, 1939), between C4 and C5. It was the aim of the present research to determine to what extent previous findings might be generalized to a much wider frequency range. Since most music comprises several octaves, knowledge of these results may be significant for students in music perception.

Physical and Perceptual Aspects

For a pure interval in which, by definition, the ratio of the fundamental frequency, f_1 , of the lower tone (tone 1) and the fundamental frequency, f_2 , of the higher tone (tone 2) is equal to $p:q$, with p and q ($p < q$) being small integers, the harmonics with frequencies of npf_1 and nqf_1 Hz ($n=1, 2, \dots$) coincide. For two simultaneous complex tones, a moderately tempered interval is characterized by small frequency differences between the harmonics that coincide in pure intervals. Perceptually, the most relevant aspect of the interference of these nearly coinciding harmonics is variation in amplitude. The depth (in decibels) of the level variation (temporal envelope) of two interfering harmonics with amplitudes a_1 and a_2 is given by

This research was supported by Grant 15-29-05 from the Netherlands Organization for the Advancement of Pure Research (ZWO). The authors are indebted to Reinier Plomp, to Ino Flores d'Arcais, and to Rudolf Rasch for their comments on an earlier draft of this paper.

Requests for reprints may be sent to Joos Vos, Institute for Perception TNO, Kampweg 5, 3769 DE Soesterberg, The Netherlands.

$$D = 20 \log \left| \frac{a_1 + a_2}{a_1 - a_2} \right| = 20 \log \left(\frac{10^{\Delta L/20} + 1}{10^{\Delta L/20} - 1} \right), \quad (1)$$

in which ΔL is the absolute level difference between a_1 and a_2 in decibels [$\Delta L = |20 \log(a_1/a_2)|$]. D approaches ∞ for $a_2 \rightarrow a_1$. Because discriminability between pure and tempered intervals is determined by the interference of nearly coinciding harmonics (Vos & van Vianen, 1985), DTs will be expressed in terms of D .

For the n th pair of interfering harmonics, the frequency of the level variation is equal to

$$f_{bn} = |npf_2 - nqf_1|. \quad (2)$$

For tones with many relatively strong harmonics, Vos (1984) showed that the dominantly perceived beat frequency was equal to that of the first pair of nearly coinciding harmonics (f_b). In our experiment, the amount of tempering will therefore be expressed in terms of f_b .

Tuning systems, however, are described differently (see, e.g., Rasch, 1983): tempering (T) of the various intervals is expressed as the difference between the logarithmically defined interval size of the tempered interval and that of the pure interval. The relationship between f_b and T (in cents) is given by

$$f_b = |qf_1(2^{T/1200} - 1)|,$$

and, since $(2^{T/1200} - 1) \sim \ln 2^{T/1200} = T/1731$, it follows that

$$T = (1731 \cdot f_b)/(qf_1). \quad (3)$$

In Figure 1, T is plotted for the fifth and the major third, separately, as a function of f_1 with f_b as parameter. For the octave range between C2 and C3, even relatively slow beats correspond to large temperings, especially for the fifth. For the equally tempered major third ($T = 13.7$ cents), high values of f_b are obtained for the octave range between C5 and C6.

Critical Bandwidth

In order to constitute a relatively smooth-sounding musical interval, the frequencies of two simultaneous tones have to be separated by a certain minimum value. This minimal frequency distance is required because the resolving power of the ear is limited. This resolving power may be expressed in the critical bandwidth of the ear (see Scharf, 1970, for a survey). Thus, when the frequencies of two simultaneous tones are within a single critical band, the result is an overall roughness sensation. This sensation becomes more and more manifest with decreasing frequency distance of the tones, up to a maximum at about 25% to 40% of the bandwidth; for smaller frequency dif-

Figure 1. Absolute tempering according to Equation 3 as a function of the fundamental frequency of the lower tone, with beat frequency $|pf_2 - qf_1|$ as a parameter, given for the fifth and the major third in panels a and b, respectively.

ferences, the beat sensation decreases to zero when the tones coincide. Comparison of estimates of critical bandwidth that are based on various experimentally investigated phenomena, such as loudness summation, masking, and frequency analysis, indicates that the values may differ by a factor of 2 (see Plomp, 1976).

With respect to discrimination between pure and tempered intervals, estimation of the critical bandwidth from experiments in which presence versus absence of beats or roughness was investigated seems to be most relevant here. Such experiments have been carried out by Cross and Goodwin (1893), by Mayer (1894), and by Plomp and Steeneken (1968). The results of these three studies are very similar; with the help of curvilinear regression (least squares solution), we found that critical bandwidth (CBW), in terms of the frequency distance between its lower and higher bounds—being dependent on the mean

frequency ($\bar{f}$) of these two bounds—can be described adequately ($r^2 = 0.98$) by

$$\text{CBW} = 1.22 (\bar{f})^{0.69}. \quad (4)$$

Comparison of the width of the critical band with that of musical intervals is facilitated by the expression of CBW in cents (CBW'). CBW', derived from the data reported by Cross and Goodwin (1893), Mayer (1894), and Plomp and Steeneken (1968) is plotted in Figure 2 as a function of $\bar{f}$ for the three studies separately. Based on all data points in Figure 2, CBW' can be represented excellently ($r^2 = 0.92$) by

$$\text{CBW}' = 2371 [\log_{10}(\bar{f})]^{-2.06}. \quad (5)$$

For the octave range between C1 and C2, CBW' may be expected to be larger than a musical fifth. Extending the frequency range investigated in our experiment to frequencies lower than C2 would not be interesting for the perception of music. For the octave between C2 and C3, CBW' is about 270 to 150 cents larger than a pure major third. Because of this, discrimination thresholds will not be determined for major thirds with fundamental frequencies below 130 Hz. Interval distributions computed from the chords of a number of musical compositions reveal that major thirds and fifths are avoided for lower octaves and therefore strongly suggest that critical bandwidth plays an important role in music (Plomp & Levelt, 1965).

Sensitivity to Beats as a Function of Frequency

It has been shown (Vos, 1982) that, for moderately tempered intervals ($T < 20$ cents), thresholds for discrimination are similar to thresholds for the detection of beats. This same study demonstrated that when the tempered intervals deviated from purity by more than 20 to 30 cents, perception of the difference between the size of the tempered interval and that of the pure interval could also become a relevant cue for discrimination. Since, in the present experiment, absolute tempering was smaller than 20 cents in more than 75% of the conditions, investigation of whether DT depends on fundamental frequency or, say, mean frequency of the nearly coinciding harmonics seems to be of primary concern here.

To our knowledge, no experimental results on the relation between fundamental frequency and threshold for the perception of beats that can be heard in tempered intervals consisting of two complex tones have been reported. Only the level-variation thresholds for beats caused by the addition of two sinusoids of nearly the same frequency (Riesz, 1928) and for beats resulting from amplitude modulation (AM) of one sinusoid (Zwicker, 1952a) have been studied systematically for various (fundamental) frequencies.

For a beat frequency of 3 Hz, Riesz (1928) varied the mean frequency of the interfering sinusoids from 35 Hz up to 10 kHz at various sensation levels of the tones. An effect of mean frequency on the threshold for level variation was found at very low sensation levels. For the fre-

Figure 2. Estimates of critical bandwidth as a function of mean frequency of two simultaneous tones that are just separated enough to be free of beats and roughness, adopted from three studies. Computation of the inserted regression function was based on all data points ($r^2 = 0.92$).

quency range relevant to the present investigation and at sensation levels exceeding 20 dB, however, no effect could be found.

Zwicker (1952a) varied carrier frequency of AM and FM sinusoids in octave steps from 65 to 8000 Hz. Since he was primarily interested in relative differences between thresholds for the detection of amplitude and frequency modulation, Zwicker never reported an effect of carrier frequency on these absolute thresholds. Our detailed analysis of his data (Zwicker, 1952b), however, revealed that there is a moderate, but consistent, effect of carrier frequency on the threshold for the detection of amplitude modulation. Averaged across modulation frequencies of 2, 4, and 8 Hz, the thresholds are plotted in Figure 3 for three loudness levels of the tones, separately. As can be seen in Figure 3, the threshold decrease with increasing frequency of the carrier is most prominent at a low loudness level. The threshold increase for carrier frequencies higher than 4 kHz was confirmed in an additional experiment (Zwicker, 1952b) in which frequencies higher than 8 kHz were also included.

Carrier frequency of AM sinusoids was also varied by Terhardt (1968). At a modulation frequency of 5 Hz, he found that the perceived strengths of beats at carrier frequencies of 125, 500, and 2000 Hz were about equal. For higher modulation frequencies, Terhardt's data are not conclusive on the effect of carrier frequency on perceived strength of the beats because, in his pertinent experiment, carrier frequency and modulation frequency were confounded.

In conclusion, the results discussed above are mutually inconsistent with respect to the effect that mean frequency of the interfering sinusoids may have on sensitivity to beats.

Figure 3. Reanalysis of data from Zwicker (1952b), who determined thresholds for the detection of amplitude modulation. These thresholds were converted into the depth of the level variation (Equation 1). Averaged across modulation frequencies of 2, 4, and 8 Hz, they are plotted separately as a function of carrier frequency for three loudness levels of the tones.

Masking and Overall Roughness in Complex Tones

For complex tones, perception of level variations may be obscured by the masking effect of adjacent components (see Vos & van Vianen, 1985). Zwicker (1954) has shown that, for noise bands masked by two sinusoidal tones at frequencies below and above the noise band, the masked threshold remains constant up to a certain frequency separation, but decreases progressively beyond that value. The interval size in cents between the two sinusoids up to which the masked threshold remains at maximum decreases as a function of the mean frequency of the maskers. On the assumption that the masking pattern for the detection of level variations resembles that for the detection of a band of noise, fundamental frequency, which will be varied in our experiment, may have an effect on discrimination between pure and tempered intervals.

A second factor that may be relevant is the overall roughness of the pure intervals: harmonics that fall within one single critical band are responsible for a roughness sensation. It may be expected that relatively rough pure intervals are less well differentiated from, again relatively rough, tempered intervals than are smooth intervals. For example, the intervals between the harmonics of the lower and the higher tones of the fifth are, up to the fourth harmonic of the lower tone, larger than or equal to about 500 cents, whereas for the major third this value is about 100 cents. Because estimates of the critical bandwidth, based on the presence versus absence of beats or roughness (see Figure 2) strongly depend on the frequency of the sinusoids, an additional effect of frequency on the discrimination thresholds is not precluded a priori, especially for the major thirds.

THE EXPERIMENT

The aim of the present experiment was to determine to what extent previous findings on discriminability between pure and tempered intervals might be applied to a much wider range of fundamental frequencies. The frequency range in the experiment corresponds to that given in Figure 1. It has been shown above that, due to the limited resolving power of the ear, extending the range to frequencies lower than C3 (for the major thirds) or C2 (for the fifths) would not be relevant for the perception of music.

Method

Stimuli and Apparatus. The stimuli were intervals consisting of two simultaneous complex tones, 1 and 2. The frequency ratio $f_1:f_2$ was equal to or slightly different from $p:q$. We presented fifths ($p=2, q=3$) and major thirds ($p=4, q=5$). The overall sound level of tones 1 and 2 together was about 85 dB SPL. The duration of the tone bursts was 0.5 sec. The complex tones consisted of 20 harmonics with amplitude a_n proportional to $1/n$. A detailed description of the waveforms and the temporal envelopes of the tones, as well as the way in which the level-variation depth of the beating harmonics was manipulated (Equation 1), was given in a previous

paper (Vos, 1982, Experiment 1). Also, details on the apparatus used are given in full in that report.

Subjects. Six subjects, students of either the Institute of Musicology or the Conservatory at Utrecht, were tested over four or five half-day sessions. Their hearing thresholds were determined with pure tones between 125 and 8000 Hz. None of them had a hearing loss larger than 15-dB hearing level (ISO 38 g) in any part of the audiogram. The subjects were paid for their participation.

Experimental design. The independent variables were: (1) interval (fifth and major third); (2) fundamental frequency [mean central frequency, f_c (= the geometrical mean $(f_1 f_2)^{1/2}$), of the pure intervals could be 92.5, 185.0, 370.0, and 740.0 Hz]; (3) attenuation of tone 1 versus attenuation of tone 2; and (4) degree of tempering (beat frequency, f_b , of the first pair of nearly coinciding harmonics could be 2, 4, 8, 16, 32, and 64 Hz). For the lowest frequency range ($f_c = 92.5$ Hz), the fundamental frequencies of the major third were clearly within the same critical band; therefore, no thresholds were determined for these conditions. Beat frequencies lower than 2 Hz were not presented because, for a tone duration of 0.5 sec, these beat frequencies would result in stimuli that contained less than one beating period. In addition, temperings larger than ± 35 cents were not investigated. This means that, with respect to both fundamental frequency and beat frequency, we have an incomplete factorial design. For the fifth, thresholds were determined for the following 14 combinations of f_c and f_b : $f_c = 92.5$ Hz, $f_b = 2$ and 4 Hz; $f_c = 185$ Hz, $f_b = 2, 4$, and 8 Hz; $f_c = 370$ Hz, $f_b = 2, 4, 8$, and 16 Hz; and $f_c = 740$ Hz, $f_b = 2, 4, 8, 16$, and 32 Hz. For the major third, there were 15 combinations: $f_c = 185$ Hz, $f_b = 2, 4, 8$, and 16 Hz; $f_c = 370$ Hz, $f_b = 2, 4, 8, 16$, and 32 Hz; and $f_c = 740$ Hz, $f_b = 2, 4, 8, 16, 32$, and 64 Hz. For each of these 29 conditions, the level of tone 1 (L_1) could be decreased relative to that of tone 2 (L_2), and vice versa. For the resulting 58 conditions, all of which were presented to the six subjects, two thresholds were determined.

Procedure. The task for the subject was to discriminate between pure and tempered musical intervals in a 2-AFC paradigm. The tempered interval was presented at random in the first or the second interval of a trial. Between the intervals, there was a silent period of 1.5 sec. By means of two response buttons, the subject indicated whether the first or the second interval was tempered. It was clear to the subjects that, in addition to beats, differences between the sizes of the tempered and the pure intervals (stretched or compressed) could also provide a valid cue for discrimination. Feedback was given by a light above the correct button. The next trial was presented 1.7 sec after the response to the preceding trial. Discrimination thresholds (DTs) were determined by means of an adaptive procedure, in which the absolute difference between L_1 and L_2 , ΔL , was increased by 2 dB after three correct responses and decreased by the same amount after a single incorrect response. Each reversal of the direction of change in ΔL over successive trials was counted as a turnaround. There were eight turnarounds. In the first run (a number of successive trials), however, ΔL , which was 0 dB at the very first trial, was increased by 3 dB after each correct response. The DT was defined as the mean value of ΔL on the trials that resulted in the last six turnarounds, corresponding with 79.4% correct responses in a nonadaptive 2-AFC procedure (Levitt, 1971). This value was converted into a D value (Equation 1). The standard deviation of the values of ΔL for the last six turnarounds was considered as a measure of the reliability of DT. Standard deviations greater than 5 dB were exceptional. If they occurred, the series was repeated until results with a standard deviation less than 5 dB were obtained. Three subjects were presented first with the block that comprised the 28 series of the fifth; the remaining three subjects were presented first with the block that comprised the 30 series of the major third. When thresholds had been determined for all 58 series, the blocks were presented for the second time. Presentation order of the series was randomized within each block. To avoid the use of absolute pitch as a cue, fundamental frequencies were varied from trial to trial in such a way that f_c of the pure

interval could be equal to 92.5, 185, 370, or 740 Hz ± 25 , ± 75 , ± 125 , ± 175 , or ± 225 cents, with equal probability of occurrence. Details about the computation of f_1 and f_2 of the pure and tempered intervals are given in Vos (1982, 1984). For each trial, the probability that either f_1 or f_2 had been altered and that the tempered interval had been stretched ($pf_2 - qf_1 > 0$) or compressed ($pf_2 - qf_1 < 0$) was 50%. The data were combined for stretched and compressed intervals. A session comprised about 30 series. The subjects received a few training series before a new experimental block (fifth or major third) started. In the $\Delta L < 10$ dB conditions, L_1 and L_2 were attenuated to a small extent to satisfy our criterion that the overall level of the interval should remain at 85 dB SPL.

Results

The DTs, averaged across subjects, replications, and the $L_1 < L_2$ and $L_1 > L_2$ conditions, are presented in Figure 4 as a function of beat frequency. In this figure, DTs are given separately for the fifth and major third in panels a and b, respectively, with mean central frequency as a parameter. DTs determined the second time were, on the average, 1 dB lower than those determined the first time, the difference being statistically significant. However, because replication did not interact significantly with the other variables, it will be treated as a within-cell variance.

Differences between the DTs which were due to whether D was manipulated in the $L_1 < L_2$ or the $L_1 > L_2$ conditions did not depend on mean central frequency. Because large differences between L_1 and L_2 are only rarely found in performed music, and differences between the DTs for the $L_1 < L_2$ and $L_1 > L_2$ conditions have been described at length in Vos (1982) and in Vos and van Vianen (1985), this variable will not be considered any further here.

As can be seen in Figure 4a, for the fifth, the DTs from the various frequency ranges, when plotted as a function of f_b , are very similar. Although there is more scatter in the data and the DTs for the mean central frequency of 740 Hz tend to be higher than those for the other frequencies, the DTs for the major thirds were also not significantly affected by the frequency of the fundamentals. For both intervals, the DTs decreased when beat frequencies increased from 2 to 8 Hz. It can also be seen in Figure 4 that, in general, the DTs for the fifths were considerably lower than those for the major thirds. The significance of these effects was tested by subjecting various sets of data to ANOVAs. For example, two sets of threshold data consisted of a 6 (subjects) $\times$ n (mean central frequency) $\times$ m (beat frequency) $\times$ 2 (interval) $\times$ 2 ($L_1 < L_2$ vs. $L_1 > L_2$) factorial design, all repeated measures and with replication as within-cell variance. Set I comprised three different mean central frequencies and the three lowest beat frequencies, whereas Set II comprised mean central frequencies of 370 and 740 Hz and beat frequencies of 2, 4, 8, and 16 Hz. The two ANOVAs showed (see Table 1 for F values and significance levels) that DTs did not depend on mean central frequency, and that the effects of beat frequency and musical interval were significant. Furthermore, neither first-order nor second-order interaction effects with mean central frequency were sta-

Figure 4. Discrimination thresholds as a function of beat frequency, with mean central frequency as a parameter, given for the fifth and the major third in panels a and b, respectively.

tistically significant. The highly significant differences between subjects tested against within-cell variance are due to the DTs for one subject, whose thresholds were considerably higher than those of the other five subjects. The results, summarized in Table 1, were confirmed by two ANOVAs that were performed on additional sets of DTs for either the fifth or the major third.

DISCUSSION

Measures of the Amount of Tempering

The major result of our experiment was that, over a frequency range of 4 octaves, discriminability between pure and tempered intervals did not depend on the frequency of the fundamentals. It should be emphasized, however, that the DTs shown in Figure 4 were ordered

according to beat frequency. As discussed above, both in musical and perceptual contexts, it may be more appropriate to present the data as a function of tempering, T , in cents (Equation 3). Therefore, the DTs from Figure 4 were replotted as a function of T in Figure 5.

Fifth

Figure 5a, in which the DTs for the fifths are given, suggests that tempered fifths in the meantone tuning system ($T = -5.4$ cents) and in equal temperament ($T = -2.0$ cents) can be discriminated from pure fifths, as long as the tones are not lower than about middle C. Thus, within the range of relevant tuning systems, and for intervals with a tone duration that has a high probability of occurrence in many musical compositions, perceptual differences between tempered and pure intervals are expected to be increasingly irrelevant for tones lower than about C4.

The DTs in Figure 5a suggest that the inferred effect of frequency for small temperings is maintained for temperings larger than 15 cents. This was tested by subjecting all DTs with mean absolute temperings of 15.5 and 31 cents to an ANOVA. The effect of fundamental frequency was, indeed, significant [$F(3,15) = 8.1$, $p < .002$]. A Newman-Keuls paired-comparison test (Winer, 1970) showed that only the DTs for the fifths between C2 and C3 were significantly higher than those with higher fundamental frequencies ($\alpha \geq .05$).

Major Third

Comparison of the DTs for the fifth with those for the major third reveals, in accordance with Equation 3, that, for corresponding mean central frequencies, the smallest temperings presented for the major third are nearly twice

Table 1
Summary of the F-Values and Corresponding Probabilities
of Two ANOVAs, One for Each Data Set (see Text)

	Data Set	
	I	II
A	$F(2,10) = 2.90$ n.s.	$F(1,5) = 2.89$ n.s.
B	$F(2,10) = 7.24$ $p < .01$	$F(3,15) = 4.39$ $p < .02$
C	$F(1,5) = 18.2$ $p < .01$	$F(1,5) = 19.7$ $p < .007$
A $\times$ B	n.s.	n.s.
A $\times$ C	n.s.	n.s.
B $\times$ C	n.s.	n.s.
A $\times$ B $\times$ C	n.s.	n.s.
S	$F(5,216) = 26.8$ $p < .00001$	$F(5,192) = 21.5$ $p < .00001$

Note—A = mean central frequency, B = beat frequency, C = musical interval, and S = subjects.

Figure 5. Thresholds for discrimination between pure and tempered intervals at the different beat frequencies, as given in Figure 4, replotted as a function of the corresponding mean absolute values of T in cents. Thresholds are given for the fifth and the major third in panels a and b, respectively.

as small as those for the fifths. In addition, Figure 5b indicates that tempered major thirds in the Pythagorean tuning system ($T = 21.5$ cents) and even those in compromise tuning systems such as, for example, that proposed by Silbermann ($T = 5.9$ cents) and equal temperament ($T = 13.7$ cents), can be discriminated from pure major thirds for all fundamental frequencies that are relevant in musical composition.

It can be seen in Figure 5b, however, that discriminability is very low for small temperings. For temperings of between 3 and about 13 cents, the data suggest that DTs at a mean central frequency of 185 Hz are higher than those at the other frequencies, whereas the reverse seems to be the case for temperings higher than 13 cents. This interaction effect, however, was not significant ($p > .07$).

Implications for Intervals Played on Keyboard Instruments

If we assume that the amount of tempering does not depend on the central frequency of the tones constituting

the interval, our experimental results may be related to the perception of intervals played on fixed-pitch keyboard instruments. For musical instruments that produce tones with strictly harmonic partials, such as the pipe organ, the assumption of equality of tempering for the whole keyboard seems to be justified.

For a restricted frequency range of 2 octaves between C4 and C6, fundamental frequencies for two organ stops were reported by Grützmacher and Lottermoser (1935). From their data it can be concluded that although the temperaments set did not correspond exactly to equal temperament, the standard deviations (SDs) of the differences were, in general, not greater than about 2 cents. More importantly, a trend towards either stretched or compressed tuning could not be found.

From more extended frequency measurements, performed on four different pipe organs for the octaves between C2 and C6 (Vos, 1981), it was also concluded that convincing evidence for stretched or compressed tuning could be found for neither 4-ft nor 8- or 16-ft stops. For a set of 15 stops, the slopes of the linear functions that fitted the deviation of the successive tones from equal temperament were very flat ($M = -0.3$ cents/octave; $SD = 1.1$ cents/octave) and the variance (r^2) explained by the linear functions was low ($M = 0.10$; $SD = 0.11$).

Similar frequency measurements performed on pianos and grand pianos indicate that our experimental results can be applied only to a limited part of the keyboards of pianos. For a number of grand pianos, Grützmacher and Lottermoser (1935) found that for the keys between C4 and C6, a trend towards stretched or compressed tuning could not be found. By investigating the whole frequency range of pianos and grand pianos, Railsback (1938; see Schuck & Young, 1943, for more details) and Meinel (1954, 1957) found that, indeed, for tones between about C4 and C6, the frequencies hardly deviated from equal temperament in a systematic way. For tones higher than about C6 and lower than C3, however, there was in most cases a tendency towards stretched tuning, extending to about 4-10 cents/octave. This phenomenon has been related to the inharmonicity of piano strings (Schuck & Young, 1943; Young, 1954; see also Rasch & Heetvelt, in press).

In conclusion, the experimental results described above can be applied to intervals played on the pipe organ. The same probably holds for the intervals played on the harpsichord, because the inharmonicity of its long and thin strings is very low. For the frequency range between C3 and about C6, the results can also be applied to the intervals played on the piano. Stretched tuning, found for the extreme upper and lower frequency ranges of the piano, may result in fifths and major thirds that are, relative to a tuning system with pure octaves, less tempered. For octaves that are stretched by 3.3 cents, for example (see Kolinski, 1959), pure fifths in a regular tuning system result in major thirds that are only 1.2 cents larger than those in equal temperament. Similarly, fifths that are tempered by 2 cents result in major thirds that deviate 6.8

cents from the pure major third. For intervals consisting of tones with inharmonic partials, the relation between tempering and beat frequency is different from that given for tones with strictly harmonic partials (Equation 3). It has recently been suggested (Rasch & Heetvelt, in press), however, that at least for the middle and descant registers of the piano, both the increase of inharmonicity with increasing frequency and the stretched tuning of the octaves lead to beat frequencies that are not much different from those expected for intervals with harmonic tones.

GENERAL CONCLUSIONS

Over a frequency range of 4 octaves between C2 and C6, thresholds for discrimination between pure and tempered fifths do not depend on the frequency of the fundamentals, provided that the thresholds are ordered according to beat frequency.

For a frequency range between C3 and C6, the same holds true for major thirds, although there is more scatter in the data.

Because frequency failed to show an effect on discrimination, it seems justified to generalize previous findings with respect to, for example, the effect of tone duration (Vos, 1982), and the effect of spectral content of the tones (Vos, 1984; Vos & van Vianen, 1985), to a wider frequency range.

When tempering is expressed in cents, and only those conditions are considered in which the degree of tempering (T) corresponds to that found in the relevant tuning systems ($-5.4 \leq T < 0$), it can be concluded that perceptual differences between tempered and pure fifths are expected to be increasingly irrelevant for tones lower than about C4.

Within the same musical constraints, tempered major thirds ($0 < T \leq 21.5$), even those in a compromise tuning system such as that proposed by Silbermann ($T = 5.9$ cents), can be discriminated from pure major thirds for most fundamental frequencies that are relevant in musical composition.

Differences in discriminability between fifths and major thirds, as well as the effect of beat frequency, reported in previous studies (Vos, 1982, 1984), could be confirmed.

Analysis of temperaments, determined for various organ stops and pianos, reveals that our experimental results may be related to the perception of intervals played on the pipe organ, and, for the frequency range between C3 and about C6, they may also be related to the perception of intervals that are played on the piano.

REFERENCES

- CROSS, C. R., & GOODWIN, H. M. (1893). Some considerations regarding Helmholtz's theory of consonance. *Proceedings of the American Academy of Arts and Sciences*, 27, 1-12.
- GRÜTZMACHER, M., & LOTTERMOSER, W. (1935). Über die Stimmung von Flügeln. *Physikalische Zeitschrift*, 36, 903-911.
- KOLINSKI, M. (1959). A new equidistant 12-tone temperament. *Journal of the American Musicological Society*, 12, 210-214.
- LEVITT, H. (1971). Transformed up-down methods in psychoacoustics. *Journal of the Acoustical Society of America*, 49, 467-477.
- MAYER, A. M. (1894). Researches in acoustics, No. IX. *Philosophical Magazine Journal of Science, Series 5*, 37, 259-288.
- MEINEL, H. (1954). Zur Stimmung der Musikinstrumente. *Acustica*, 4, 233-236.
- MEINEL, H. (1957). Musikinstrumentenstimmungen und Tonsysteme. *Acustica*, 7, 185-190.
- PLOMP, R. (1976). *Aspects of tone sensation*. London: Academic Press.
- PLOMP, R., & LEVELT, W. J. M. (1965). Tonal consonance and critical bandwidth. *Journal of the Acoustical Society of America*, 38, 548-560.
- PLOMP, R., & STEENEKEN, H. J. M. (1968). Interference between two simple tones. *Journal of the Acoustical Society of America*, 43, 883-884.
- RAILSBACK, O. L. (1938). A study of the tuning of pianos. *Journal of the Acoustical Society of America*, 10, 86.
- RASCH, R. A. (1983). Description of regular twelve-tone musical tunings. *Journal of the Acoustical Society of America*, 73, 1023-1035.
- RASCH, R. A., & HEETVELT, V. (in press). String inharmonicity and piano tuning. *Music Perception*.
- RIESZ, R. R. (1928). Differential intensity sensitivity of the ear for pure tones. *Physical Review*, 31, 867-875.
- SCHARF, B. (1970). Critical bands. In J. V. Tobias (Ed.), *Foundations of modern auditory theory* (Vol. 1, Chap. 5, pp. 157-202). New York: Academic Press.
- SCHUCK, O. H., & YOUNG, R. W. (1943). Observations on the vibrations of piano strings. *Journal of the Acoustical Society of America*, 15, 1-11.
- TERHARDT, E. (1968). Über akustische Rauigkeit und Schwankungsstärke. *Acustica*, 20, 215-224.
- VOS, J. (1981). [Tuning of pipe organs, as found in practice.] Unpublished manuscript (in Dutch), Institute for Perception TNO, Soesterberg, The Netherlands.
- VOS, J. (1982). The perception of pure and mistuned musical fifths and major thirds: Thresholds for discrimination, beats, and identification. *Perception & Psychophysics*, 32, 297-313.
- VOS, J. (1984). Spectral effects in the perception of pure and tempered intervals: Discrimination and beats. *Perception & Psychophysics*, 35, 173-185.
- VOS, J., & VAN VIANEN, B. G. (1985). Thresholds for discrimination between pure and tempered intervals: The relevance of nearly coinciding harmonics. *Journal of the Acoustical Society of America*, 77, 176-187.
- WINER, B. J. (1970). *Statistical principles in experimental design* (International Student ed.). London: McGraw-Hill.
- YOUNG, R. W. (1939). Terminology for logarithmic frequency units. *Journal of the Acoustical Society of America*, 11, 134-139.
- YOUNG, R. W. (1954). Inharmonicity of piano strings. *Acustica*, 4, 259-262.
- ZWICKER, E. (1952a). Die Grenzen der Hörbarkeit der Amplitudenmodulation und der Frequenzmodulation eines Tones. *Acustica*, 2, 125-133.
- ZWICKER, E. (1952b). *Die Grenzen der Hörbarkeit der Amplituden- und Frequenzmodulation von Tönen und ihre Berücksichtigung in der Übertragungstechnik und der Hörphysiologie*. Doctoral thesis, Technical University Stuttgart.
- ZWICKER, E. (1954). Die Verdeckung von Schmalbandgeräuschen durch Sinustöne. *Acustica*, 4, 415-420.

(Manuscript received November 26, 1984;
revision accepted for publication May 23, 1985.)