A theory of bidirectional judgments

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This paper is concerned with a scaling theory for "bidirectional" judgments, for which the order of judgment is reversible, as in fractional and multiple ratio estimation judgments. With the assumption that judgments are mediated by perceived relations of pairs of stimuli, the theory is developed for judgments of comparison stimuli in relation to standards, taking explicit account of the location of the comparison stimulus relative to the standard. The theory of bidirectional judgments, based on a theory of relative judgment by Fagot (1978, 1979), entails a partial nesting of models characterized by a progressive weakening of the constraints placed on the structure of the data. The weakest model, the relative bias/directional standard (RBDS) model, allows each standard to have two biasing effects, depending on the location of the standard above or below the comparison stimulus. Tests of the theory were carried out on the ratio estimation of brightness and weight data of Engen and Levy (1955) and the part-sum estimation data of Goude (1962). Only the RBDS model was found acceptable for all three data sets.

This paper is concerned with the development of a scaling theory for bidirectional judgments. Judgments are defined as bidirectional if the order of judgment is reversible. For example, denote, by R_{ab} , the estimation of the ratio of the magnitude of stimulus a to the magnitude of stimulus b. Then the order of judgment may be reversed, with R_{ba} denoting the estimated ratio of b to a.

Then, for each presentation pair (a, b) and some judgment, J, bidirectional theory is concerned with empirically distinguishable judgments, J_{ab} and J_{ba} . Judgments made in one direction only will be called *unidirectional*. A precise definition of bidirectional judgments will be given after presentation of some theory.

The approach taken in this paper is that judgments are mediated by perceived relations of pairs of stimuli. We shall be concerned with judgments of a *comparison* stimulus in relation to a *standard* stimulus. Surprisingly, research in psychophysics on the effect of standards has been limited for the most part to studies of the effects on the power function (Engen & Levy, 1955; Engen & Ross, 1966; Jones & Woskow, 1966; Macmillan, Moschetto, Bialostozky, & Engel, 1974; Poulton, 1968; Ross & DiLollo, 1970). But surely this is bypassing the more basic measurement question of the effect of standards on *judgments*? For example, if standards bias judgments of comparison stimuli, are there differential biasing effects depending on the location of the standard above or below the comparison stimulus? If there are, and given a model that attempts to characterize the effects of

standards on bidirectional judgments of comparison stimuli, what conditions on observed bidirectional judgments are entailed? That is, what are the testable consequences of the theory that must be satisfied prior to scale construction?

These and other questions are addressed by the theory of bidirectional judgments to be presented, based on a theory of relative judgment by Fagot (1978, 1979). First the theory will be outlined briefly, followed by further theoretical development that takes explicit account of bidirectionality. Finally, the theory will be tested with ratio estimation (based on brightness and weight data from Engen & Levy, 1955) and part-sum estimation of weight data (Goude, 1962).

THEORY

The theory (Fagot, 1978, 1979) deals with any set of judgments J_{ab} for which there exists a response transformation J_{ab}^* that is a function of sensory ratios ψ_a/ψ_b and possible bias parameters. Three specific model equations were analyzed:

$$J_{ab}^* = \psi_a/\psi_b, \quad (1)$$

$$J_{ab}^* = \alpha\psi_a/\psi_b, \quad (2)$$

$$J_{ab}^* = \alpha_b\psi_a/\psi_b, \quad (3)$$

where α and α_b are bias parameters and ψ is the scaling function (a ratio scale). Equations 1, 2, and 3 are referred to as the *classical* (C), *constant bias* (CB), and *relative bias* (RB) models, respectively.

Three structural conditions, each imposing constraints on the observed judgments, are entailed for all a, b, x, y:

This project was supported in whole or in part by BRSG Grant S07RR07080 awarded by the Biomedical Research Support Grant Program, Division of Research Resources, National Institutes of Health.

Ratio Consistency (C1):

$$J_{ab}^* = J_{ax}^* J_{xb}^* = J_{ay}^* J_{yb}^*, \quad (4)$$

Product Constancy (C2):

$$J_{ax}^* J_{xb}^* = J_{ay}^* J_{yb}^*, \quad (5)$$

Ratio Constancy (C3):

$$J_{ax}^* / J_{bx}^* = J_{ay}^* / J_{by}^*. \quad (6)$$

It can be shown that Equation 1—the C model—entails C1, C2, and C3 (but C2 and C3 are entailed by C1). C1 is written in x and y form to make explicit that C2 is nested in C1. Equation 2—the CB model—entails C2 and C3; and Equation 3—the RB model—entails C3 (Fagot, 1978).

To apply the theory to a given scaling model, it is only necessary to derive a *response transformation* J^* , satisfying Equation 1, 2, or 3 (Fagot, 1979).

The theory is illustrated via two examples, ratio estimation and unidimensional similarity judgments. For ratio estimation, the basic assumption generally made is that $R_{ab} = \psi_a / \psi_b$. This equation is of the same form as Equation 1, and hence the response transformation is simply $J_{ab}^* = R_{ab}$ and the theory of J_{ab}^* applies directly to the R_{ab} . For example, C1 is $R_{ab} = R_{ax} R_{xb}$. Explication of the theory applied to ratio estimation, along with empirical tests, is presented by Fagot (1978).

Eisler and Ekman (1959) proposed a simple model of similarity judgments for unidimensional attributes:

$$S_{ab} = \psi_a / \frac{1}{2}(\psi_a + \psi_b) \quad (\psi_a < \psi_b), \quad (7)$$

where S_{ab} is the judged similarity between the two percepts and $0 \leq S_{ab} \leq 1$. In order to find the response transformation J_{ab}^* , we must find S_{ab}^* such that $S_{ab}^* = \psi_a / \psi_b$, conforming to Equation 1. This is done by solving Equation 7 for ψ_a / ψ_b , which is $\psi_a / \psi_b = S_{ab} / (2 - S_{ab})$. Hence, the response transformation is $J_{ab}^* = S_{ab}^* = S_{ab} / (2 - S_{ab})$ and the theory on J_{ab}^* applies to S_{ab}^* (not S_{ab}). Application of the theory to this model is presented in Fagot (1979) along with empirical tests.

These examples show how to find the response transformation and demonstrate the wide applicability of the theory to any judgment model for which a response transformation exists satisfying Equation 1, 2, or 3.

The theory of relative judgment represented by Equations 1-6 applies to both the bidirectional case in which both J_{ab} and J_{ba} are defined as well as the unidirectional case in which only one J applies. The extensive data base for ratio estimation and similarity judgments examined by Fagot (1978, 1979) consisted

of unidirectional judgments. In the case of ratio estimation, this was because the standard was always the larger stimulus of the presentation pair. The Eisler and Ekman (1959) similarity model assumed that judgments were symmetric; that is, $S_{ab} = S_{ba}$. Hence, in neither case were the complications of bidirectional theory needed.

Equations 1-6 are not restricted to the case of unidirectional judgments, as applied by Fagot (1978) to ratio estimation. If judgments are bidirectional, however, the number of test cases for the structural conditions is substantially increased, as will be shown below.

No special problems arise in the application of the C or CB model to the bidirectional case. However, one extension is required relating the bidirectional judgments. For the C model, Equation 1 entails $J_{ba}^* = \psi_b / \psi_a$, and hence a *reciprocal relation*, $J_{ab}^* = 1 / J_{ba}^*$, which we state in product form¹:

$$J_{ab}^* J_{ba}^* = 1. \quad (8)$$

We are now in a position to give a more precise definition of bidirectional judgments. Let J_{ab} be any judgment of stimulus a relative to stimulus b. Then J_{ab}^* and J_{ba}^* are bidirectional judgments if (1) J_{ab} and J_{ba} are empirically distinguishable and (2) J_{ab}^* is a reciprocal function of J_{ba}^* ; in particular, if responses are unbiased, then Equation 8 holds.

The requirement of a reciprocal relation distinguishes bidirectionality from other order effects, such as a *position* effect. For example, suppose R_{ab} denotes the ratio estimation of a to b when a is on the left, and R_{ba} when b is on the left. Then, in the absence of bias, both R_{ab} and R_{ba} estimate the sensory ratio ψ_a / ψ_b —that is, $R_{ab} = \psi_a / \psi_b$ and $R_{ba} = \psi_a / \psi_b$, therefore $R_{ab} = R_{ba}$ —and hence the reciprocal relation does not hold. It would be possible to model both bidirectionality and a position effect, but that line will not be pursued here.

If judgments are biased, the reciprocal relation will include bias parameters, the precise form of which will depend on the model. For the CB model, Equation 2 entails that the product form of the reciprocal rule is

$$J_{ab}^* J_{ba}^* = \alpha^2. \quad (9)$$

In applying the RB model to bidirectional judgments, we need to consider certain special cases, corresponding to different possible sources of bias. Equation 3 can be used to model the effect of standards, associating a bias α_b with each standard (with the bias operating on the comparison stimulus). For bidirectional judgments, Equation 3 will be referred to as the *relative bias/standard* (RBS) model. Note that for the RBS model we also have $J_{ba}^* = \alpha_a \psi_b / \psi_a$,

and hence the product form of the reciprocal rule is

$$J_{ab}^* J_{ba}^* = \alpha_a \alpha_b. \quad (10)$$

Another possible source of bias in bidirectional judgments is the *direction* of the standard relative to the comparison stimulus. Let α_H denote the bias parameter if the standard is higher (larger in magnitude) than the comparison stimulus, and α_L denote the bias parameter if the standard is lower. Then the representation taking into account *solely* the directional effect is given by the following special case of Equation 3:

$$\begin{aligned} J_{ab}^* &= \alpha_H \Psi_a / \Psi_b \\ J_{ba}^* &= \alpha_L \Psi_b / \Psi_a \end{aligned} \quad (a < b) \quad (11)$$

and the product form of the reciprocal relation is

$$J_{ab}^* J_{ba}^* = \alpha_H \alpha_L. \quad (12)$$

Equations 11 will be referred to as the *relative bias/directional* (RBD) model.

Finally, the representation taking account of the combined effects of standard and direction is

$$\begin{aligned} J_{ab}^* &= \alpha_{bH} \Psi_a / \Psi_b \\ J_{ba}^* &= \alpha_{aL} \Psi_b / \Psi_a \end{aligned} \quad (a < b) \quad (13)$$

and

$$J_{ab}^* J_{ba}^* = \alpha_{aL} \alpha_{bH}. \quad (14)$$

Equations 13 will be referred to as the *relative bias/directional standard* (RBDS) model. The RBDS model allows each standard to have two biasing effects, depending on the location of the standard relative to the comparison stimulus.

To place the C and CB models in perspective, we see that the C model assumes that none of the biasing effects exist, whereas the CB model assumes that all biases are equal, that is, that the bias does not depend on the location of the standard.

The bias parameters for all models are derived from one or more forms of the function $J' = J_{ab}^* J_{bc}^* / J_{ac}^*$. For each triple, there are six forms of J' for bidirectional judgments since there are three pairs (a, c), and each pair has two directions. In Table 1, the first column lists the six forms of J' and the remaining columns indicate, for each model, which bias parameter corresponds to each form of J' .

The second column of Table 1 states that, for each form, $J' = 1$, which is a statement of C1. Note that, for each triple, there are six forms of C1 for bidirectional judgments (and hence six test cases), but only

Table 1
Formulas for Bias Parameters (for all $a < b < c$)

Form of J'	Model (Equation)				
	C (1)	CB (2)	RBD (11)	RBS (3)	RBDS (13)
(1) $J_{ab}^* J_{bc}^* / J_{ac}^* =$	1	α	α_H	α_b	α_{bH}
(2) $J_{cb}^* J_{ba}^* / J_{ca}^* =$	1	α	α_L	α_b	α_{bL}
(3) $J_{ba}^* J_{ac}^* / J_{bc}^* =$	1	α	α_L	α_a	α_{aL}
(4) $J_{ca}^* J_{ab}^* / J_{cb}^* =$	1	α	α_H	α_a	$\alpha_{aL} \alpha_{bH} / \alpha_{bL}$
(5) $J_{ac}^* J_{cb}^* / J_{ab}^* =$	1	α	α_L	α_c	$\alpha_{cH} \alpha_{bL} / \alpha_{bH}$
(6) $J_{bc}^* J_{ca}^* / J_{ba}^* =$	1	α	α_H	α_c	α_{cH}

one for unidirectional judgments (form 1 or form 2, depending on the direction of the judgments). C1 places the strongest constraints on the structure of the data and is entailed solely by the C model.

The bias parameters in Table 1 may be derived in the following manner. Consider form 2 and write the algebraic identity $(\Psi_c / \Psi_b)(\Psi_b / \Psi_a) = (\Psi_c / \Psi_a)$. To find the bias parameter for the RBDS model, we note, from Equation 13, that $(\Psi_c / \Psi_b) = (J_{cb}^* / \alpha_{bL})$, $(\Psi_b / \Psi_a) = (J_{ba}^* / \alpha_{aL})$, and $(\Psi_c / \Psi_a) = J_{ca}^* / \alpha_{aL}$. Therefore, $(J_{cb}^* / \alpha_{bL})(J_{ba}^* / \alpha_{aL}) = J_{ca}^* / \alpha_{aL}$ and $\alpha_{bL} = J_{cb}^* J_{ba}^* / J_{ca}^*$, as shown in Table 1.

Since the RBS model ignores direction, the subscript "L" is deleted and, for form 2, $J_{cb}^* J_{ba}^* / J_{ca}^* = \alpha_b$, as shown in column 5. And, since the RBD model assumes that *only* direction need be considered, the subscript "b" is deleted, as indicated in column 4. Finally, since the CB model assumes that bias is constant over standard and direction, all subscripts are deleted, as indicated in column 3. In a similar manner, the other bias parameters may be derived for the other forms in Table 1.

Note that forms 4 and 5 in Table 1 do not define a unique parameter for the RBDS model. However, deletion of appropriate subscripts leads to unique parameters for the other models based on forms 4 and 5.

Parametric Forms of C2 and C3

It can be inferred from Equation 5 that, for the bidirectional case, C2 is a system of 12 equations for each tetrad of stimuli, and all 12 equations are entailed by both the C and CB models. The RBD model provides a means of classifying this system into five forms of C2, as shown in Table 2.

The (H) after form 1 and the (L) after form 2 in Table 2 indicate that all judgments entering into forms 1 and 2 are made with respect to a high or low standard, respectively. For form 3, a high and low standard enters on each side of the equation.

In forms 4 and 5, each side is operated on by the reciprocal of a different RBD bias parameter, and,

Table 2
Parametric Forms of C2 Entailed by RBD Model

Form	C2 ($a < b < c < d$)	N
1 (H)	$J_{ab}^* J_{bd}^* = J_{ac}^* J_{cd}^*$	1
2 (L)	$J_{db}^* J_{ba}^* = J_{dc}^* J_{ca}^*$	1
3 (LH)	$J_{ba}^* J_{ac}^* = J_{bd}^* J_{dc}^*$	6
4	$(1/\alpha_L) J_{cb}^* J_{ba}^* = (1/\alpha_H) J_{cd}^* J_{da}^*$	2
5	$(1/\alpha_H) J_{ab}^* J_{bc}^* = (1/\alpha_L) J_{ad}^* J_{dc}^*$	2

Note—Deleting α_L and α_H transforms Table 2 to five forms of C2. N = number in each form per tetrad.

hence, only forms 1-2-3 are entailed by the RBD model. Forms 4 and 5 are, of course, converted to C2 forms by setting $\alpha_H = \alpha_L = \alpha$, with α canceling out.

Table 2 makes explicit five forms of C2 when α_L and α_H are deleted from forms 4 and 5. Hence, when we speak of “forms of C2,” we will be referring to the forms listed in Table 2 with bias parameters deleted in forms 4 and 5. The value of this classification lies in the operation of different effective biases in each form, together with its use in choosing between the CB and RBD models.

To see that the RBD model entails forms 1-3 of C2, it is necessary only to extend the definitions of α_L and α_H in Table 1 to four stimuli. For example, $\alpha_H = (J_{ab}^* J_{bd}^* / J_{ad}^*) = (J_{ac}^* J_{cd}^* / J_{ad}^*)$, from which form 1 of Table 2 follows. Similar reasoning shows that forms 2 and 3 of Table 2 are entailed by the RBD model.

To see that form 4 of C2 is not entailed by the RBD model (i.e., $J_{cb}^* J_{ba}^* \neq J_{cd}^* J_{da}^*$), note that $\alpha_L = J_{cb}^* J_{ba}^* / J_{ca}^*$ and $\alpha_H = J_{cd}^* J_{da}^* / J_{ca}^*$. Solving both equations for J_{ca}^* and equating resultants, we get the parametric form of form 4 indicated in Table 2, and hence in general $J_{cb}^* J_{ba}^* \neq J_{cd}^* J_{da}^*$. Similar reasoning gives the parametric form 5 of C2, showing that $J_{ab}^* J_{bc}^* \neq J_{ad}^* J_{dc}^*$.

Setting $\alpha_L = \alpha_H = 1$ (C model) and $\alpha_L = \alpha_H = \alpha$ (CB model) in forms 4 and 5 shows that all forms of C2 are entailed by the C and CB models. It is easy to show that no forms of C2 are entailed by the RBS or RBDS models.

The last column of Table 2 gives the number of C2 equations for each form per tetrad of stimuli. For the unidirectional case, only form 1 or form 2 applies, resulting in only one test case per tetrad.

Table 3 presents the parametric forms of C3 entailed by the RBDS model. The (H), (L), and (LH) after forms 1-2-3 have the same interpretation as in Table 2. Deleting all α from the last three forms transforms Table 3 into a table of six C3 forms.

The Table 3 classification is useful in exhibiting the C3 entailments of the various models: The presence of bias parameters in forms 4, 5, and 6 indicates

that only forms 1, 2, and 3 of C3 are entailed by the RBDS model. Deleting b, c from the bias parameters (only direction counts) indicates that forms 1, 2, 3, and 4 are entailed by the RBD model, but *not* forms 5 and 6. Deleting L and H (direction does not count) indicates that *all* forms of C3 are entailed by the RBS model, since all bias parameters cancel. Setting all parameters equal to one or α shows that all forms of C3 are entailed by the C and CB models.

For unidirectional judgments, only form 1 or form 2 of C3 applies (depending on direction of judgment), resulting in only one test case per tetrad, as in the case of C2. Of course, in the unidirectional case, the RBD and RBDS models are not applicable, since the directional factor is not operative.

The complete list of entailments for each model is given in Table 4.

Note the partial nesting of models exhibited by these entailments, with the C model placing the strongest constraints on the structure of the data and the RBDS model, the weakest. A complete nesting would exist if the RBD model entailed all forms of C3.

The bidirectional theory will be tested using two judgmental tasks, ratio estimation and part-sum estimation. It was earlier shown that, for ratio estimation, the response transformation is $J_{ab}^* = R_{ab}$, and, hence, one need only make this substitution to test the bidirectional model for ratio estimation judgments R_{ab} and R_{ba} . However, for part-sum estimation we need to describe the method, state the judgmental model, and find the response transformation J_{ab}^* .

Part-Sum Estimation

The *part-sum estimation* method was apparently introduced by Goude (1962), and is similar to the constant-sum method (Comrey, 1950). In the part-sum method, stimuli are presented in pairs, and the task of the subject is to judge the percent magnitude of *each* member of the pair to the *sum* of the two magnitudes. The basic difference between the part-sum and constant-sum methods is that in the part-

Table 3
Parametric Forms of C3 Entailed by RBDS Model

Form	C3 ($a < b < c < d$)
1 (H)	$J_{ac}^* / J_{bc}^* = J_{ad}^* / J_{bd}^*$
2 (L)	$J_{ca}^* / J_{da}^* = J_{cb}^* / J_{db}^*$
3 (LH)	$J_{ba}^* / J_{ca}^* = J_{bd}^* / J_{cd}^*$
4	$(\alpha_{bL} / \alpha_{bH}) J_{ab}^* / J_{db}^* = (\alpha_{cL} / \alpha_{cH}) J_{ac}^* / J_{dc}^*$
5	$J_{ba}^* / J_{da}^* = (\alpha_{cL} / \alpha_{cH}) J_{bc}^* / J_{dc}^*$
6	$(\alpha_{bL} / \alpha_{bH}) J_{ab}^* / J_{cb}^* = J_{ad}^* / J_{cd}^*$

Note—Deleting all α transforms Table 3 to six forms of C3.

Table 4
Model Entailments

Structural Condition	Forms	Model				
		C	CB	RBD	RBS	RBDS
C1	All	X				
C2	1-2-3	X	X	X		
	4-5	X	X			
C3	1-2-3	X	X	X	X	X
	4	X	X	X	X	
	5-6	X	X		X	

Note—X indicates row condition entailed by column model.

sum method, two ratio judgments are made for each presentation pair, whereas for the constant-sum method (divide 100 points between the two stimuli), only one ratio is entailed by the instructions.

The part-sum judgmental model, assuming no response bias, is simply

$$P_{ab} = \psi_a / (\psi_a + \psi_b), \quad (15)$$

where P_{ab} is the percent judgment of the comparison stimulus a relative to the sum of stimuli a and b, and ψ is the scaling function. Equation 15 can be written in the form

$$\psi_a / \psi_b = P_{ab} / (1 - P_{ab}). \quad (16)$$

Therefore, the required response transformation is

$$J_{ab}^* = P_{ab}^* = P_{ab} / (1 - P_{ab}) \quad (17)$$

and

$$P_{ab}^* = \psi_a / \psi_b. \quad (18)$$

P_{ab} and P_{ba} are empirically distinguishable, and $P_{ba}^* = \psi_b / \psi_a$. Therefore, the reciprocal relation $P_{ab}^* P_{ba}^* = 1$ holds for the C model, and part-sum estimation satisfies the formal requirements of bidirectional judgments. Bias parameters are introduced as in Equations 1-2-3-11-13, and with the substitution $J_{ab}^* = P_{ab}^*$ the theory of bidirectional judgments is directly applicable to P_{ab}^* (not P_{ab}).

In the interpretation of bidirectional theory as applied to ratio estimation and part-sum estimation, two differences should be noted. First, for ratio estimation the standard is explicit, whereas for part-sum estimation the standard is implicit and assumed: In treating P_{ab}^* and P_{ba}^* as bidirectional, the first stimulus is the comparison and the second the assumed standard stimulus. Second, for ratio estimation the bidirectional theory applies directly to the elementary judgments R_{ab} and R_{ba} , whereas for part-sum estimation the theory applies to the transformed responses P_{ab}^* and P_{ba}^* .

There are two reasons why part-sum estimation is worthy of serious attention. First, to try to answer the basic question, is there a magnitude scale (for a given attribute) that mediates a diversity of judgmental tasks such as ratio and part-sum estimation? The second reason refers to a nice property of the method: it is *unit-free*. By this is meant that the judgments do not depend on the particular unit used, as can be seen from Equation 15, since units cancel. Ratio estimation is also unit-free, whereas magnitude estimation (without a standard) is not. Thus, for ratio estimation and part-sum estimation, the theory is indifferent to changes of unit on every trial (or to different subjects using a different unit if a group scale is being constructed), whereas for magnitude estimation, the subject must retain the same unit on every trial or, if subjects are aggregated, all subjects must use the same unit.

RESULTS AND DISCUSSION

Tests of the models were based on the ratio estimation of brightness (gray patches) and weight data of Engen and Levy (1955); and the part-sum estimation of weight data of Goude (1962). In the remaining discussion, ratio estimation of brightness and weight will be abbreviated RE(B) and RE(W), respectively; and part-sum estimation of weight will be abbreviated PS(W).

Engen and Levy (1955) used nine stimuli and three standards for each of the two attributes. The standards were the third (low), fifth (middle), and seventh (high) stimuli. Each of the remaining eight stimuli were judged relative to each of the three standards, twice by each of 15 subjects. Estimates of the R_{ab} were taken as the median of the 15 subjects (30 judgments).

In the PS(W) data of Goude (1962), there were five stimuli and each stimulus served as both comparison and standard. Estimates of the P_{ab} were the arithmetic means of six subjects.

Tests of Structural Conditions

The structural conditions were tested in the form of the following null hypotheses:

$$H_0(C1): J_{ab}^* = J_{a1}^* J_{1b}^* = J_{a2}^* J_{2b}^* = \dots J_{ak}^* J_{kb}^*$$

$$H_0(C2): J_{a1}^* J_{1b}^* = J_{a2}^* J_{2b}^* = \dots J_{ak}^* J_{kb}^*$$

$$H_0(C3): J_{a1}^* / J_{b1}^* = J_{a2}^* / J_{b2}^* = \dots J_{ak}^* / J_{bk}^*.$$

The "equivalents" of the equations of the null hypotheses constituted the levels of the first factor in a two-way repeated measures analysis of variance design. The second factor was "stimulus sets," con-

sisting of triples of stimuli for C1 and tetrads for C2 and C3. Real differences in the levels of the second factor are expected, and are of no interest. The statistical tests were carried out on the first factor, for each structural condition separately. The dependent variable was log observation ($\log J^*$) with one observation per cell. The analysis also provided an estimate of the intraclass correlation coefficient (ICC), which is a measure of the agreement or consistency of the equivalents of the structural condition³ (Bartko, 1976; Shrout & Fleiss, 1979).

For the PS(W) data of Goude (1962), each of the n stimuli served as a standard (and comparison) stimulus. In the case of C1 for the Goude data, $k = n - 2$, and the number of equivalents (levels of the first factor) was $k + 1 = 4$. The number of stimulus sets (levels of the second factor) for C1 was $2\binom{n}{2} = n(n-1) = 20$.

For the RE(B) and RE(W) data of Engen and Levy (1955), only three standards were used and $n = 9$. For C1, $k = 2$, and the number of equivalents was $k + 1 = 3$. The number of stimulus sets for C1 was 18.

In order to choose among the CB, RBD, RBS, and RBDS models, it is necessary to test subsets of the forms of C2 and C3 as defined in Tables 2 and 3. When C2 and C3 were organized by form, the number of equivalents was $k = 2$.

For the complete pair-comparison presentation used in the PS(W) data, the number of stimulus sets was 60 for C2 and 30 for C3. For the RE(B) and RE(W) data, the number of stimulus sets were: for C2, 18 for all forms combined and 12 for forms 1-2-3 (entailed by RBD model); and for C3, 63 for all forms combined and 19 for forms 1-2-3 (entailed by the RBDS model).

The model entailments listed in Table 4 provide a guide to the way in which the structural conditions will be used to evaluate and compare models. For example, a rejection of C1 entails immediately a rejection of the C model; a comparison of the fit of forms 1-2-3 vs. forms 4-5 of C2 provides a basis for choosing between the RBD and CB models; and a compari-

son of the fit of forms 1-2-3 vs. 4-5-6 of C3 provides a basis for choosing between the RBDS and the RBS models.

Three criteria will be used to analyze and compare structural conditions: the analysis of variance test described above, the ICC as a measure of agreement or consistency of the equivalents of a structural condition, and an analysis of systematic errors. The fit of the data to the various reciprocal relations also will be analyzed.

The analysis of variance results, including the ICC measures, are given in Table 5.

Table 5 shows that C1 is rejected for all three sets of data, providing sufficient basis for a rejection of the C model. Of particular interest are the nature of systematic errors in C1, to be considered after analysis of C3.

As Table 4 shows, C2 provides a basis for testing the CB and RBD models, although the RBD model also entails forms 1-4 of C3. Table 5 shows that C2 (all forms) is rejected for PS(W) and RE(B), but not for RE(W). Forms 1-3 of C2 are rejected for PS(W), but not for RE(W) or RE(B). These somewhat equivocal findings are clarified by means of appropriate graphs and an analysis of systematic errors.

Figure 1 shows a plot of $\log R_{ay}R_{yb}$ as a function of $\log R_{ax}R_{xb}$ ($x < y$) for RE(B). The dominant trend is clearly $R_{ax}R_{xb} < R_{ay}R_{yb}$, with proportions $\hat{p} = .889$ for all forms and $\hat{p} = .833$ for forms 1-2-3. A test of $H_0: p = .5$ is rejected at the .01 level for all forms and at the .05 level for forms 1-2-3.

A plot of $\log P_{ay}^*P_{yb}^*$ as a function of $\log P_{ax}^*P_{xb}^*$ for PS(W) (not presented) gives similar results. The proportions for $P_{ax}^*P_{xb}^* < P_{ay}^*P_{yb}^*$ are $\hat{p} = .733$ for all forms and $\hat{p} = .800$ for forms 1-2-3, with significance levels of $p = .001$ in both cases.

For PS(W), the ANOVA test and the analysis of systematic errors agree: both the CB and RBD models must be rejected.

For RE(B), both tests agree that the CB model must be rejected. And, although the ANOVA test is not significant for forms 1-2-3 of C2, Figure 1 is

Table 5
Intraclass Correlation Coefficient (ICC) and Analysis of Variance Test (F)

Con- dition	Entailed by Model:	Forms	Goude (1962)			Engen and Levy (1955)					
			PS(W)			RE(W)			RE(B)		
			ICC	F	df	ICC	F	df	ICC	F	df
C1	C		.996	7.17††	3,57	.938	3.34**	2,34	.925	3.36**	2,34
C2	CB	All	.995	22.58††	1,59	.929	4.41*	1,17	.911	5.13**	1,17
	RBD	1-2-3	.996	21.04††	1,39	.924	2.52*	1,11	.888	1.59*	1,11
C3	RBS	All	.982	.23*	1,29	.947	20.18††	1,62	.924	.23*	1,62
	RBDS	1-2-3	.957	.08*	1,14	1.000	9.05†	1,18	.976	1.28*	1,18
	RBS	4-5-6	.970	.69*	1,14	.861	20.86††	1,43	.818	.00*	1,43
	RBD	1-2-3-4	.990	.13*	1,19	.952	10.96†	1,38	.919	.14*	1,38

*Not significant. ** $p < .05$. † $p < .01$. †† $p < .001$.

Figure 1. C2: $\log R_{ay}R_{yb}$ as a function of $\log R_{ax}R_{xb}$ ($x < y$) for RE(B) with forms 1-2-3 and 4-5 separately identified. The dashed line is the identity line $\log R_{ay}R_{yb} = \log R_{ax}R_{xb}$. Data from Engen and Levy (1955).

persuasive evidence for the rejection of the RBD model.

The status of the CB and RBD models for RE(W) is less clear, although the ANOVA test failed to reject in either case. As in the previous two cases, an analysis of systematic errors helps to clarify.

Figure 2 shows a plot of $\log R_{ay}R_{yb}$ as a function of $\log R_{ax}R_{xb}$ for RE(W).⁴

In spite of the ANOVA test result, Figure 2 shows that the weight data fit poorly for C2, although the systematic trend is not the simple one exhibited in Figure 1. The reverse trend from the lower to upper range in Figure 1 was tested by means of a runs test. Results were (see Siegel, 1956: one-sample runs test, Table F): For all forms, $n_1 = 9$, $n_2 = 9$, $r = 3$; for forms 1-2-3, $n_1 = 6$, $n_2 = 6$, $r = 3$. In both cases, the hypothesis of randomness of order is rejected ($p < .05$). The observed systematic trend of $R_{ax}R_{xb} > R_{ay}R_{yb}$ at the lower end of the range and the reverse trend at the upper end is therefore tenable. It is on the basis of Figure 2 and the consequent runs test that the CB and RBD models may be rejected for RE(W). In conclusion to this part of the analysis, the CB and RBD models are rejected for all three data sets.

Condition C3 provides a means of choosing between the RBS and RBDS models. The RBDS model places the weakest constraints on the structure of the data of all the models considered, requiring only that forms 1-2-3 of C3 be satisfied. The stronger RBS

model demands, in addition, that forms 4-5-6 be satisfied. Hence, to make a choice between the two models, we need to compare results for forms 1-2-3 as against forms 4-5-6.

Consider the results for PS(W) first. Table 5 shows that none of the four cases of C3 may be rejected, providing no basis for a choice between the RBS and RBDS model. Figure 3 shows a plot of the product form of C3 for PS(W): $\log P_{ad}^*P_{bc}^*$ as a function of $\log P_{ac}^*P_{bd}^*$ for forms 1-2-3 (panel A) and forms 4-5-6 (panel B) separately.⁵

Figure 3 for PS(W) shows a reasonable fit in both panels. There is the suggestion of a systematic trend in panel B, but a runs test, as applied to the data of Figure 2, was not significant ($n_1 = 5$, $n_2 = 10$, $r = 6$): $p > .05$. The ICCs are comparable in both panels, in fact, slightly lower for the RBDS model. We conclude that the C3 part-sum data are consistent with the stronger RBS model.

Table 5 also shows that none of the four cases of C3 may be rejected for the RE(B) data. Figure 4 shows a graph of C3 for RE(B) following the same form as Figure 3.

Inspection of Figure 4 indicates a better fit to the data in panel A, consistent with the much higher ICC: .976 for the RBDS model (forms 1-2-3), and .818 for the RBS model (forms 4-5-6). On the basis of Figure 4 and the relative ICCs, the RBDS model is judged superior to the RBS model for the RE(B) data.

Figure 2. C2: $\log R_{ay}R_{yb}$ as a function of $\log R_{ax}R_{xb}$ ($x < y$) for RE(W) with forms 1-2-3 and 4-5 separately identified. The dashed line is the identity line $\log R_{ay}R_{yb} = \log R_{ax}R_{xb}$. Data from Engen and Levy (1955).

Figure 3. C3: $\log P_{ad}^* P_{bc}^*$ as a function of $\log P_{ac}^* P_{bd}^*$ for PS(W). Panel A, forms 1-2-3; panel B, forms 4-5-6. The dashed line is the identity line $\log P_{ad}^* P_{bc}^* = \log P_{ac}^* P_{bd}^*$. Data from Goude (1962).

Table 5 shows that all four cases of C3 are statistically significant for RE(W). Figure 5 shows a graph of C3 for RE(W) following the same form as Figures 3 and 4.

Inspection of Figure 5 shows that the fit of C3 is extremely good for the RBDS model (panel A) but poor for the RBS model (panel B). The statistically significant result for the RBDS model in Table 5 is due to the fact that the mean squared error is extremely small. The mean squared error for forms 4-5-6 is 121 times that for forms 1-2-3. Comparing forms 1-2-3 for RE(W) and RE(B), the mean squared error for RE(B) is 7.3 times that of RE(W), so that the less favorable result for RE(W) is due to very small mean squared error and not large mean squared deviations.⁶ It is to be noted that for RE(W), ICC =

1.000 for forms 1-2-3, whereas ICC = .861 for forms 4-5-6. On the basis of Figure 5 and the ICCs, the RBDS model fares much better than the RBS model for RE(W).

Reciprocal Relations

Figure 6 shows a plot of $1/J_{ba}^*$ as a function of J_{ab}^* for all three data sets. Note that there are only three data points each for RE(W) and RE(B).

The C model (Equation 7) requires that the function be a straight line through the origin with unit slope (solid line). These data do not conform well to this requirement.

The CB model (Equation 8) requires that the function be a straight line through the origin with slope $1/\alpha^2$. This is the dashed line in Figure 6 for PS(W) only, and there is little improvement. Figure 6 is consistent with the rejection of the C and CB models based on failure of C1 and C2, respectively.

Figure 7 shows a plot of α_{aL}/J_{ba}^* as a function of J_{ab}^*/α_{bH} for all three data sets. The RBDS model (Equation 13) requires that the function be a straight

Figure 4. C3: $\log R_{ad} R_{bc}$ as a function of $\log R_{ac} R_{bd}$ for RE(B). Panel A, forms 1-2-3; panel B, forms 4-5-6. The dashed line is the identity line $\log R_{ad} R_{bc} = \log R_{ac} R_{bd}$. Data from Engen and Levy (1955).

Figure 5. C3: $\log R_{ad} R_{bc}$ as a function of $\log R_{ac} R_{bd}$ for RE(W). Panel A, forms 1-2-3; panel B, forms 4-5-6. The dashed line is the identity line $\log R_{ad} R_{bc} = \log R_{ac} R_{bd}$. Data from Engen and Levy (1955).

line through the origin with unit slope (dashed line). Figure 7 shows a good fit, giving further support to the RBDS model.

It might be argued that, because of the large number of parameters for the RBDS model, the good fit in Figure 7 is not compelling evidence. However, the α_{aL} and α_{bH} are *not* estimated to give the best fit to the data in Figure 7 conforming to Equation 13. Rather, the bias parameters were estimated from appropriate *triples* (Table 1), and hence Figure 7 represents a prediction from triples to pairs of stimuli. Nevertheless, the more convincing evidence is satisfaction of a structural condition, since such a

test is parameter-free. In any event, Figure 7 is consistent with satisfaction of forms 1-2-3 of C3 and both give good support for the RBDS model.

Bias Parameter Estimates

Estimates of bias parameters for each model are presented in Table 6. The estimates are based on the geometric means of appropriate triples, as defined in Table 1. The number of triples used in the estimate of each α is indicated in the appropriate column of Table 6 labeled "N."

Since the RBDS model shows the greater consistency with all three data sets, attention is focused on the implications of the estimates for that model. Inspection of Table 6 shows that in most cases $\alpha_{aL} < 1$ and $\alpha_{bH} > 1$, the exceptions being $\alpha_{2H} = .997$ for PS(W), $\alpha_{5L} > 1$ for RE(W), and $\alpha_{5H} < 1$ for RE(B). Both exceptions for ratio estimation are for the middle of the three standards, which does not show consistent biasing effects, although this exception is not present in the part-sum data. Putting aside the implications of the middle-standard exception for the moment, we consider the *dominant bias directions* $\alpha_{aL} < 1$ and $\alpha_{bH} > 1$.

Our first concern is a measure of *consistency* of the dominant directions. Since bias magnitudes on different sides of one signify opposite directional biases, a reasonable measure of consistency is the percent of triples of stimuli that estimate the bias on the dom-

Figure 6. Reciprocal relations Models C and CB: $1/J_{ab}^*$ as a function of J_{ab}^* ($a < b$) for PS(W) (Goude, 1962), RE(B), and RE(W) (Engen & Levy, 1955). The solid line is the identity line $1/J_{ab}^* = J_{ab}^*$ (C model), and the dashed line is the function $1/P_{ab}^* = (1/a)^2 P_{ab}^*$ [i.e., for PS(W) judgments only].

Figure 7. Reciprocal Relation Model RBDS: α_{aL}/J_{ba}^* as a function of J_{ab}^*/α_{bH} ($a < b$) for PS(W) (Goude, 1962), RE(B) and RE(W) (Engen & Levy, 1955). The dashed line is the identity line $\alpha_{aL}/J_{ba}^* = J_{ab}^*/\alpha_{bH}$.

inant side. For all three data sets combined, the results are 86.7% for $\alpha_{aL} < 1$ and 78.3% for $\alpha_{bH} > 1$. If the middle standard for RE is deleted, the percents rise to 97.2% for $\alpha_{aL} < 1$ and 86.1% for $\alpha_{bH} > 1$.

Systematic Errors in C1

Systematic errors in C1 are important, because this structural condition makes the appealing, though strong, measurement assumption that subjective ratios behave like numerical ratios.

Consider form 1 in Table 1, with $\alpha' = J_{ab}^*J_{bc}^*/J_{ac}^*$, where α' may be the bias parameter for any of the models. If (1) $\alpha' < 1$, then $J_{ac}^* > J_{ab}^*J_{bc}^*$, and (2) if $\alpha' > 1$, then $J_{ac}^* < J_{ab}^*J_{bc}^*$. We find that the direction of inequality is almost equally split, with $J_{ab}^* > J_{ax}^*J_{xb}^*$ (all forms) in 55.2% of the cases, combined over the three data sets. Does this mean an absence of systematic errors in C1?

Each model may give a different answer to this question. For example, the CB model does not detect systematic errors, because it fails to take into account the fact that the effects of standards may differ. We now consider how the RBDS model exposes systematic errors in C1 by taking into account the direction of the standard.

Consider form 3 in Table 1. According to the RBDS model, all three judgments are biased and

$$(1/\alpha_{aL})J_{ba}^*(1/\alpha_{cH})J_{ac}^* = (1/\alpha_{cH})J_{bc}^*.$$

The biases on J_{ac}^* and J_{bc}^* are equal and cancel, re-

ducing the parametric form of C1 to

$$(1/\alpha_{aL})J_{ba}^*J_{ac}^* = J_{bc}^*.$$

Hence, even though all three judgments are biased, only the bias on J_{ba}^* may produce systematic errors in C1. Similarly, for form 6 the parametric form is $(1/\alpha_{cH})J_{bc}^*J_{ca}^* = J_{ba}^*$ and systematic errors in C1 occur if $\alpha_{cH} \neq 1$. Given the RBDS dominant directions, (1) $\alpha_{aL} < 1$ and (2) $\alpha_{bH} > 1$, it is entailed that (1) $J_{ab}^* > J_{ax}^*J_{xb}^*$ for low effective standards (forms 2 and 3) and (2) $J_{ab}^* < J_{ax}^*J_{xb}^*$ for high effective standards (forms 1 and 6). The measures of consistency for α_{aL} and α_{bH} given above are, of course, the measures of consistency for the two inequalities in J^* . The most striking result is that with the middle stimulus deleted for RE, $J_{ab}^* > J_{ax}^*J_{xb}^*$ in 97.2% of the cases of low effective standards.

It was pointed out above that the consistent pattern of systematic errors in C1 in relation to high and low effective standards breaks down for the middle standard in the ratio estimation data. However, judgments relative to the middle standard were not unbiased: With respect to the low effective middle standard, RE(B) judgments were consistent with the dominant direction, but RE(W) were not; and with respect to the high effective middle standard, RE(W)

Table 6
Bias Parameter Estimates

BP	Goude (1962)		Engen and Levy (1955)			
	PS(W)	N	RE(W)	N	RE(B)	N
CB						
α	.975	60	.895	42	.979	42
RBD						
α_L	.901	30	.831	21	.983	21
α_H	1.056	30	.963	21	.975	21
RBS						
α_1	.880	12				
α_2	.913	12				
α_3	1.051	12	.744	14	.958	14
α_4	.984	12				
α_5	1.063	12	1.014	14	.935	14
α_6			.950	14	1.047	14
α_7						
RBDS						
α_{1L}	.831	6				
α_{2L}	.834	6				
α_{3L}	.957	5	.592	4	.939	4
α_{4L}	.940	3				
α_{5L}			1.207	5	.978	5
α_{7L}			.957	4	.976	4
α_{2H}	.997	3				
α_{3H}	1.181	5	1.447	4	1.042	4
α_{4H}	1.053	6				
α_{5H}	1.141	6	1.052	5	.909	5
α_{7H}			1.096	4	1.041	4

Note—BP = bias parameter; N = number of triples used in estimation.

judgments were consistent with the dominant direction, but RE(B) were not. The effect of the middle standard in these ratio estimation data is perplexing and warrants further study.

It was noted previously that forms 4 and 5 in Table 1 do not define a unique parameter for the RBDS model, and therefore do not entail a single low or high effective standard determining the direction of inequality in C1. On the other hand, the RBD model does entail a high and low effective standard for forms 4 and 5, respectively. However, the RBD model does not exhibit, in Table 6, a consistent pattern for α_L and α_H ($\alpha_L, \alpha_H < 1$ for RE). In any event, using the empirical values obtained for α_L and α_H to determine direction for each data set separately, the measure of consistency is quite low (62.2%).

Concluding Remarks

The C model assumes that judgments of comparison stimuli are not influenced by the presence of standards. This view is clearly inconsistent with these data for ratio estimation of weight and brightness, and for part-sum estimation of weight, as demonstrated by the rejection of C1 and the C model form of the reciprocal rule.

Fagot's (1978) analysis led to the rejection of the C model for unidirectional (high standard) judgments of each of the following attributes: angles and weight (Goude, 1962); brightness (Fagot, Stewart, & Kleinknecht, 1975); velocity (Mashhour, Note 3); heaviness (Eisler, 1960); and darkness and area (Ekman, Goude, & Waern, 1961). However, C1 was not rejected for size of circular surfaces (Ekman, 1958) and odor intensity (Engen & Lindstrom, 1963), although for the latter the ICC was only .691.

Fagot's (1979) study led to the rejection of C1 for unidimensional similarity judgments of pitch (Eisler & Ekman, 1959), darkness and visual area (Ekman, Goude, & Waern, 1961), and heaviness (Eisler, 1960).

Finally, Svenson and Åkesson (Note 1) found deviations from the simple reciprocal relation entailed by the C model for individual subjects judging area of circles and darkness. It is clear that the simplistic psychophysical view of the C model must be rejected, and its value lies chiefly as a point of reference for models that place weaker constraints on the structure of the data.

The CB model assumes a constant response bias. In the present context, this implies that standards have a constant biasing effect on comparison stimuli independent of the location of the standard relative to the comparison stimulus. This view is not supported by the data for RE(B), RE(W), and PS(W).

Condition C2 has met with mixed success in the analysis of prior research on unidirectional judgments carried out by Fagot (1978, 1979). C2 was satisfied for ratio estimation judgments of size of

circular surfaces (Ekman, 1958), velocity (Mashhour, Note 3), weight (Goude, 1962), area (Ekman, Goude, & Waern, 1961), and odor intensity (Engen & Lindstrom, 1963). C2 was also satisfied for unidimensional similarity estimates of pitch (Eisler & Ekman, 1959), visual area (Ekman, Goude, & Waern, 1961), and heaviness (Eisler, 1960). However, C2 was rejected for ratio estimation judgments of angles (Goude, 1962), brightness (Fagot, Stewart, & Kleinknecht, 1975), heaviness (Eisler, 1960), and darkness (Ekman, Goude, & Waern, 1961). C2 was also rejected for unidimensional similarity estimates of darkness (Ekman, Goude, & Waern, 1961).

In all of the studies cited above analyzed by Fagot (1978, 1979), the judgments were, as stated, unidirectional (high standard). It is not surprising that C2 fares better with unidirectional judgments, since the direction of the standard clearly affects judgments of comparison stimuli. The CB model is best viewed as an approximation to the RBS or RBDS model, in which the bias parameter in the CB model is the average of the bias parameters of the various standards. That approximation is going to be best when judgments are unidirectional and standards are closely spaced, and poor for bidirectional judgments and widely spaced standards.

The RBD model entails that two directional biases are operative, one for comparison stimuli above the standard and another for comparison stimuli below the standard. This is an appealingly simple conception, but one that is not supported by the data on PS(W), RE(W), and RE(B). The data analyzed by Fagot (1978, 1979) were unidirectional, and do not therefore provide test data for the RBD model.

The RBS model does not explicitly take account of the direction of the standard relative to the comparison stimulus, but does permit each standard to have a different biasing effect. The RBS model is supported by the part-sum data, but does not fare as well for the ratio estimation data as the RBDS model. The unidirectional ratio estimation data (Fagot, 1978) and similarity data (Fagot, 1979) were generally supportive of C3 and therefore the RBS model. It would appear that the RBS model fares extremely well with unidirectional judgments, but does not give a good account of bidirectional judgments because of the obvious failure to incorporate assumptions relating to the direction of the standard.

The RBDS model entails that each standard has two biasing effects depending on the location of the standard above or below the comparison stimulus. The PS(W), RE(B), and RE(W) data are strongly supportive of the RBDS model, as can be seen from the fit displayed in the A panels of Figures 3, 4, and 5 and in Figure 7. The unidirectional data (Fagot, 1978, 1979) were also consistent with the RBDS model, but cannot, of course, be used to choose between the RBS

and RBDS models, since both models have precisely the same entailments for unidirectional judgments (form 1 or form 2 of C3, Table 3).

The C model makes the erroneous measurement assumption that subjective ratios behave like numerical ratios, that is, that $J_{ab}^* = J_{ax}^* J_{xb}^*$. The RBDS model shows that, in general, (1) for low effective standards, $\alpha_{aL} < 1$ and $J_{ab}^* > J_{ax}^* J_{xb}^*$, and (2) for high effective standards, $\alpha_{bH} > 1$ and $J_{ab}^* < J_{ax}^* J_{xb}^*$. The unidirectional data (high standard) examined by Fagot (1978, 1979) were highly consistent with Case 2. Only for similarity judgments of pitch (Eisler & Ekman, 1959) was there a clear reversal ($S_{ab}^* > S_{ax}^* S_{xb}^*$).

The bias parameters may be interpreted as measuring the effect on scale values of using an incorrect model. For example, the error in estimating Ψ_a/Ψ_b using the C model, given that the RBDS model is correct, is $\alpha_{bH} - 1$ ($b > a$) or $\alpha_{bL} - 1$ ($b < a$). Table 6 shows that the errors are in some cases quite large. The largest errors are for standard 3, RE(W): 40.8% for the low standard and 44.7% for the high standard.

According to the RBDS model and the dominant bias directions, J_{ab}^* overestimates the sensory ratio Ψ_a/Ψ_b and J_{ba}^* underestimates Ψ_b/Ψ_a (that is, J_{ba}^* overestimates Ψ_a/Ψ_b). Hence, low and high effective standards are consistent in their biasing effects, both effecting overestimation of the sensory ratio Ψ_a/Ψ_b . However, the biasing effect of low standards tends to be larger than that of high standards, since, in the majority of cases, $\alpha_{aL}\alpha_{bH} < 1$. For ratio estimation, this means that fractional judgments (high standard) tend to be less biased than multiple judgments (low standard). The directional biases for low and high effective standards are consistent with the hypothesis of an assimilation effect, which implies that the "true" separation between the two stimuli of a presentation pair is attenuated by bias.

The analysis of ratio estimation with a high standard by Fagot (1978) indicated a tendency for α_{bH} to decrease with the increasing magnitude of the standard, implying, since $\alpha_{bH} > 1$, a decrease in biasing effect as the magnitude of the high standard increases. This tendency is not apparent in the PS(W) data, but the RE(W) and RE(B) data are consistent with this tendency provided the troublesome middle standard is deleted (Table 6).

It is hypothesized that for low effective standards, α_{aL} increases with increasing magnitude of the standard, implying, since $\alpha_{aL} < 1$, a decrease in biasing effect as the magnitude of the low standard increases. This hypothesis receives partial support from the data of Table 6, but for RE(W) data, the ordering is disrupted for the middle standard.

Note that, for both low and high effective standards, the hypothesized ordering on α_{bH} and α_{aL}

implies a decrease in biasing effect as the magnitude of the standard increases (i.e., as the separation between comparison and standard increases). It appears to be like a magnetic attraction for which the effect decreases with distance. In any event, further support for this hypothesis is needed, especially with individual subjects, and the role of the middle standard must be investigated.

Special features of the theory of relative judgment presented in this paper and in Fagot (1978, 1979) are: (1) parameter-free testing of structural conditions, (2) nesting of alternative models, and (3) applicability to a variety of judgmental methods. The nesting of models involves a progressive weakening of the constraints placed on the structure of the data, permitting a search for the strongest model consistent with the data. The theory has the generality to apply to any judgmental method for which an appropriate response transformation can be found, and this has been done for unidimensional similarity judgments (Eisler & Ekman, 1959) and part-sum estimation (Goude, 1962). For ratio estimation, of course, the response transformation is the identity function ($J_{ab}^* = R_{ab}$).

For bidirectional judgments, it was found that only the RBDS model was acceptable for all three data sets—RE(B), RE(W), and PS(W). Given satisfaction of the testable consequences of the RBDS model, a ratio scale may be constructed using Equation 13.

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NOTES

1. Svenson & Åkesson (Note 1, Note 2) used the term "reciprocal asymmetry" for our "reciprocal relation."

2. Note that $H_0(C1)$ and $H_0(C2)$ are expressed in product form, whereas $H_0(C3)$ is expressed in ratio form. $H_0(C3)$ can be stated in ratio form only for $k \geq 3$, but may be stated in the product form for $k=2$, $J_{a1}^* J_{b2}^* = J_{a2}^* J_{b1}^*$. For $k=2$, $H_0(C3)$ was stated in product form for statistical tests and plots to obtain comparability in form to C1 and C2. Since $\log J^*$ was used as the dependent variable, the statistical tests were invariant under change in form (product or ratio), but the plots were made more comparable by using the product form of C3.

3. The ICC measure used was ICC(3,1), corresponding to Case 3 as defined by Shrout and Fleiss (1979). However, for these data the ICCs are high enough for it to make little difference which ICC measure is used.

4. The range of the brightness judgments was much smaller than the range of the weight judgments, and hence, for display purposes, different scales were used in Figures 1 and 2 [since the point of the figures was not to compare RE(W) and RE(B)]. The mean squared deviations about the identity line for weight were, in fact, about 10 times larger than for brightness.

5. To avoid possible confusion brought on by the labeling of the axes in Figure 3, and Figures 4 and 5 to follow, it should be pointed out that no specific ordering on a, b, c, d is implied: The ordering will vary with the form of C3.

6. Figures 4 and 5 should not be compared directly, since the scale of Figure 3 is about six times that of Figure 4 (see Footnote 4). However, the comparability of the ICCs serves to correct for this scale difference.

(Manuscript received January 26, 1981;
revision accepted for publication May 8, 1981.)