PROGRAM ABSTRACTS/ALGORITHMS

A program to test equality of two or more coefficients of variation

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Two tests of the hypothesis that two or more samples are drawn from populations with equal coefficients of variation are reviewed, and their use is illustrated. A program that performs the tests, written in QuickBASIC, is described.

In the testing of hypotheses about group differences in variability, it is occasionally useful to consider Pearson's (1896) coefficient of variation, $V = \sigma/\mu$, instead of more common tests of homogeneity of variance. The coefficient of variation often permits meaningful comparisons of relative variation across groups that have quite different means, or where the variables are measured in different units (Kirk, 1990). If there are K different populations, the population coefficients can be estimated by the respective sample coefficients of variation, $v_i = S_i/\bar{X}_i$, where S_i represents the sample standard deviation in the i th group (using $n_i - 1$), and $\bar{X}_i$ represents the corresponding sample mean.

Although v_i has sometimes been employed in empirical research comparing variability across groups, it is rare for investigators to perform inferential tests to determine the significance of differences in its values (which are necessarily subject to sampling error). It has long been known that when the various n_i s are 30 or more, it is possible to compute a two-sample test on the null hypothesis that $V_1 = V_2$ using a standard normal-curve approximation to estimate an error term (Sachs, 1984, p. 275). However, until the past decade, appropriate tests for small n_i s and for more than two groups have been unavailable. The present paper is a discussion of the implementation of two such tests (in the formulas that follow, all summation is over i , representing groups; K represents the number of groups).

Doornbos and Dijkstra (1983) defined the statistic $T = \sum n_i(b_i - \bar{b})^2$, where $b_i = 1/v_i$, $\bar{b} = (\sum n_i b_i)/N$, and $N = \sum n_i$. Using the noncentral t distribution, they were able to demonstrate that $D = (K-1)T/(L + a^2 C)$ is distributed as χ^2 with $(K-1)$ degrees of freedom; here,

$$L = \sum \frac{(N-n_i)(n_i-1)}{N(n_i-3)}, \quad (1)$$

$$a^2 = \frac{\sum n_i b_i^2 - \sum \frac{n_i-1}{n_i-3}}{\sum \frac{n_i(n_i-1)}{n_i-3}}, \quad (2)$$

$$C = \sum \frac{n_i(N-n_i)(n_i-1)}{N(n_i-3)} + \sum \frac{n_i^2 \epsilon_i^2}{N} - \frac{(\sum n_i \epsilon_i)^2}{N}, \quad (3)$$

and

$$\epsilon_i = \frac{\Gamma[(n_i-2)/2]}{\Gamma[(n_i-1)/2]} \sqrt{\frac{n_i-1}{2}}. \quad (4)$$

An alternative approach has been described by Bennett (cited in, and modified by, Shafer & Sullivan, 1986). This approach involves a likelihood-ratio test based on McKay's (1932) approximation to the distribution of v_i . It yields Equation 5:

$$\chi^2_{(K-1)} \approx \frac{(N-K) \ln \left[\sum \frac{u_i}{N-K} \right] - \sum (n_i-1) \ln \left[\frac{u_i}{n_i-1} \right]}{1 + \left[\frac{1}{3(K-1)} \right] \left[\sum \frac{1}{n_i-1} - \frac{1}{N-K} \right]}, \quad (5)$$

where $u_i = (n_i v_i^2)/(1 + v_i^2)$, and K represents the number of groups. This statistic is much like Bartlett's (1937) better known test for homogeneity of variance (as the numerator evaluates to $-2 \ln \lambda$).

The coefficient of variation is undefined when $\bar{X} = 0$, and both the Doornbos and Dijkstra (1983) and the Bennett-Shafer-Sullivan statistics (see Shafer & Sullivan, 1986) assume a positive value for all v_i , and approximately normal populations. Both of the approaches appear to have adequate power for most situations, although Monte Carlo simulations suggest the superiority of Equation 5, particularly with very small n_i (where D appears to be too conservative). An exception would occur when the distributions depart markedly from normality, in which case D appears preferable (Shafer & Sullivan, 1986; in fact, the consequences of violations of assumptions for these statistics remains generally unexplored).

Program COEFVAR

Although it is possible to compute either test described above by hand, the process is greatly facilitated by implementation on a computer. Shafer and Sullivan (1986) reported a program to compute a slightly modified ver-

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sion of Equation 5 above, but it requires access to the general software package SAS (Ray, 1985) and does not compute the Doornbos and Dijkstra (1983) statistic. In this paper, I describe the program COEFVAR, written in QuickBASIC Version 4.5, for use on IBM-PC-compatible microcomputers. This program computes both the Doornbos and Dijkstra and the Bennett-Shafer-Sullivan approximations to chi-square.

During execution, the user indicates whether to enter raw data or previously computed sample means and standard deviations. Next, the user specifies the number of groups (presently set at a maximum of 10) and is then prompted to enter the n_i , followed by the raw scores (or $\bar{X}_i$ and S_i) for each group. Output consists of v_i for each group (and means and standard deviations, if they have been computed), as well as the obtained values of the two approximations to chi-square, an indication of significance at $\alpha = .05$, and the corresponding degrees of freedom.

Example

By way of illustration of the use of the program, consider cross-sectional data on field dependence, originally obtained by Witkin et al. (1954). These investigators reported that fifty-four 10-year-old males had a mean score of 146.8 ($S = 53.8$) on the Embedded Figures Test, a measure on which scores represent time (in seconds) to locate a target visual figure. A sample of twenty-three 17-year-old males had a mean score of 31.2 ($S = 31.0$). The most common test for homogeneity—an F on the ratio of the two sample variances—would yield $F(53,22) = 3.01$, $p < .005$, with the conclusion that the 10-year-olds showed significantly more variability than the 17-year-olds did. Notice, however, that the means were appreciably different, suggesting examination of the coefficients of variation.

For the 10-year-olds, $v = .37$, and for the 17-year-olds, $v = .99$. Analysis by means of Shafer and Sullivan's (1986) SAS program yielded $\chi^2(1) = 18.72$, $p < .001$. The slightly more refined estimate, suggested by those authors, and given as Equation 5, yielded $\chi^2(1) = 18.40$ when computed by hand, matching the result from the program COEFVAR exactly. As expected, given its gener-

ally weaker power, the Doornbos and Dijkstra (1983) test yielded a smaller estimate, although it too was significant: $\chi^2(1) = 11.26$, $p < .001$ —the same value as that computed by hand.

Thus, examination of the coefficients of variation leads one to conclude that relative to their respective means, the scores of the 17-year-olds were significantly *more* variable than those of the 10-year-olds—just the reverse of the conventional interpretation.

Availability

A listing of the program can be obtained without charge from A. R. Gilpin, Department of Psychology, University of Northern Iowa, Cedar Falls, IA 50614-0505. A diskette (MS-DOS 4.01) containing an executable program is available for \$5 to cover the cost of mailing; specify size desired (3.5 or 5.25 in.). The program is also available without charge via electronic mail (Internet address: gilpin@uni.edu).

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