METHODS & DESIGNS

The design of ballantines

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The shared variance between two variables can be expressed graphically by overlapping circles. A procedure is presented for locating the circles so that the graphical and statistical relations correspond exactly. The procedure does not generalize to the representation of multiple, part, and partial correlations between three or more variables. The reasons for the nonextension are explained, and some alternative graphical representations are suggested.

The widespread availability of graphics programs for personal computers has greatly increased the potential for visual displays of data. The display of pairwise correlations between two and three variables is of special interest to psychologists. To illustrate the motivation for what follows, consider a case that arose in our laboratory. College students participated in three tasks: an auditory detection task (variable x), an arithmetic task (variable y), and a visual detection task (variable z). The correlations between the tasks were

$$\begin{aligned}r_{xy} &= (\text{auditory, arithmetic}) = .47; \\r_{xz} &= (\text{auditory, visual}) = .42; \text{ and} \\r_{yz} &= (\text{arithmetic, visual}) = .30.\end{aligned}$$

Our interest was the extent to which variance was shared between pairs of tasks, with some portion of the variance in the third task held constant. Part and partial correlations may be used to express the statistical relations. However, this method of summarization is not appropriate for verbal presentations of our results, especially to audiences who are not familiar with advanced methods of correlational analysis.

An alternative to the statistical summary is to use a visual display, in which the variance of each variable is represented by a circle. Shared variance is represented by the overlap between two circles. If three variables are represented, the resulting figure is called a *ballantine*. Several authors have advocated its use to represent covariation in three-variable problems (e.g., Cohen & Cohen, 1975). The ballantine is a useful display of shared and

Figure 1. A ballantine representing the correlations between an auditory detection task, a visual detection task, and a test of arithmetic skill.

unique variance because each component of variance can be identified visually in the geometric form. This can be seen in Figure 1, which is a ballantine representation of our data. The ballantine is used in a manner analogous to the use of a Venn diagram. Shared variance between tasks is represented by the regions of overlap (a_1 , a_2 , a_3 , and a_4) shown in the figure.

Obviously, ballantines are generated from simpler two-circle figures that represent the variance-covariance relations between two variables, x and y . This is shown in Figure 2 by overlapping two circles, X and Y , that represent the variances of the respective variables. (Throughout this paper, capital letters are used to signify geometric figures and lowercase letters to signify variables.) If representations such as Figures 1 and 2 are to portray data accurately, the percentage of a circle's area

The preparation of this paper was supported in part by Contract No. N00014-84-K-5553 between the Office of Naval Research and the University of Washington (Earl Hunt, principal investigator). The opinions expressed are entirely the responsibility of the author. Colene McKee's assistance in preparing the graphs is gratefully acknowledged. I am also indebted to an anonymous reviewer of an earlier draft, whose examples led to my developing the alternative representations for part and partial correlations.

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Figure 2. A correlation indicated by an overlap between two circles. For the representation to be exact, the proportion of the area of each circle that falls in region A should be equal to r_{xy}^2 .

lying in the intersection region (Region A in Figure 2) should be exactly equal to r^2 , the squared correlation between the appropriate variables. In fact, the ballantine of Figure 1 does fulfill this condition for our data. Figure 2 exactly represents the correlation between the auditory and visual detection measures. The purpose of this paper is to explain how such figures may be constructed and to explain why multivariate relations may be incorrectly represented in a figure that correctly displays bivariate relations. Some suggestions will be made for the construction of figures that do portray multivariate relationships correctly.

THE UNDERLYING GEOMETRIC RELATIONS

Let the circles X, Y, and Z stand for the variances of three variables, x , y , and z . Let the circles have a con-

stant radius, R . This "visually standardizes" the variables by representing $\text{Var}(x)$, $\text{Var}(y)$, and $\text{Var}(z)$ by circles with area πR^2 . Two circles X and Y are said to be placed correctly with respect to each other if and only if the overlapping area contains the proportion of each circle equal to the squared correlation coefficient. In the case of Figure 2, area A is equal to

$$A = r_{xy}^2 \pi R^2 \quad (1)$$

The area of intersection of circles X and Y, both of radius R , is determined by the length of line L between the center of circle X (C_x) and the center of circle Y (C_y), that is by L_{xy}/R . This is shown in Figure 3. Therefore, for fixed C_x , C_y may be located anywhere on the circle of radius L_{xy} centered on C_x . If we adopt the conventions that L_{xy} be horizontal and that X always lies to the left of Y, the locus of circle Y is thus determined once X is located and L_{xy} is determined.

The position of the third circle of a ballantine can be determined in a similar way. The center of circle Z (representing the variance of variable z) must lie on the circumference of a circle of radius L_{xz} , centered on C_x , and on the circumference of a circle of radius L_{yz} , centered on C_y . Since two nonidentical circles intersect at either two or no points, there are two possible ballantines when the three variables share common variance. In one of these circles, Z lies above the line L_{xy} ; in the other, it lies below it. Either figure would be an appropriate ballantine. Here circle Z will always lie below the horizontal midline. These relations are shown in Figure 4.

For the sake of completeness, two degenerate cases must be mentioned. If $r_{xy}^2 = 1$, then circles X and Y are identical ($L_{xy} = 0$), and similarly for X and Z and Y and Z. If $r_{xy}^2 = 0$, then $L \geq 2R$, so that circles X and Y do not overlap. By convention, the relation $L_{xy} = 2R$ will

Figure 3. The geometric relation used to construct an appropriate ballantine. Angle θ is implicitly defined by r_{xy}^2 . Angle θ , in turn, determines the length of line L_{xy} .

Figure 4. The three lines between the centers of the circles define one of two possible triangles, with circle Z either above or below line L_{xy} . By solving for the interior angle θ at the center of circle X, and given line L_{xz} , the position of circle Z is determined relative to the positions of circles X and Y.

be used, so that the circles for variables that do not share common variance will lie next to each other without overlapping.

TRIGONOMETRIC RELATIONS

An algorithm for determining the length of L_{xy} will now be presented. The identical algorithm, with a change of variable names, applies to L_{xz} and L_{yz} . Developing the algorithm is basically an exercise in high school trigonometry.

The algorithm will be described by referring to the lines and angles shown in Figure 3. Consider the segment bounded by line AB and arc b. This has area $\frac{1}{2}A$, where A is the area of overlap. The value of A is defined by

$$\frac{1}{2}A = \frac{1}{2}R^2 (\alpha - \sin(\alpha)), \quad (2)$$

where α is measured by radians (Burlington, 1948).

For a standard circle, with $R=1$, Equation 1 may be substituted into Equation 2. Then, simplifying,

$$r_{xy}^2 \pi = \alpha - \sin(\alpha). \quad (3)$$

If $r_{xy}^2 = 1$, α has the value of π (in radians). At this point, the two circles will be identical. At the other extreme, if $r_{xy}^2 = 0$, then $\alpha=0$. This establishes limits on α .

Equation 3 defines α implicitly, as a transcendental function of r_{xy}^2 . The value of α for a given value of r_{xy}^2 may be approximated to any desired degree of accuracy. The existence of a unique solution is ensured by the fact that the quantity $(\alpha - \sin(\alpha))$ increases monotonically from 0 to π throughout the range of α . (The first derivative, $1 - \cos(\alpha)$, is nonnegative for $(0 \leq \alpha \leq \pi)$). Once α is found, the value of R can be calculated directly. By inspection of Figure 3,

$$L_{xy} = 2R - 2h. \quad (4)$$

However,

$$R - h = R (\cos(\alpha/2)). \quad (5)$$

Substituting, and letting $R=1$ to establish a scale,

$$L_{xy} = 2(\cos(\alpha/2)). \quad (6)$$

Therefore, the problem is solved if α can be determined. This can be done by finding the value of α that satisfies Equation 3.

COMPUTATION

The computation of α for a given r_{xy}^2 is generally not feasible without a computer. Appendix 1 is a Pascal program that executes the appropriate algorithm. It computes circle positions, given the correlations for a two- or three-variable problem. The heart of the program is the procedure CONVERGE. For any value of α , CONVERGE calculates α by successive approximations until α is within .0001 radians of its true value. The value of L_{xy} is then computed by using Equation 6.

It would be tedious to recompute the relations for every new case of a bivariate relation. Table 1 presents values of L_{xy}/R for an r_{xy}^2 ranging from .00 to 1.00 in steps of .01. If a ballantine is to be drawn by hand, Table 1 can

Table 1
Distance Between Circles of Radius One as a Function of the Squared Correlation (r^2)
Between the Variables Represented by the Circles

r^2	r^2									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	2.00	1.92	1.87	1.83	1.79	1.76	1.72	1.69	1.67	1.64
0.10	1.61	1.58	1.56	1.53	1.51	1.49	1.46	1.44	1.42	1.40
0.20	1.37	1.35	1.33	1.31	1.29	1.27	1.25	1.23	1.21	1.19
0.30	1.17	1.15	1.13	1.11	1.09	1.08	1.06	1.04	1.02	1.00
0.40	0.98	0.97	0.95	0.93	0.91	0.89	0.88	0.86	0.84	0.83
0.50	0.81	0.79	0.77	0.76	0.74	0.72	0.71	0.69	0.67	0.66
0.60	0.64	0.62	0.61	0.59	0.57	0.56	0.54	0.52	0.51	0.49
0.70	0.48	0.46	0.44	0.43	0.41	0.40	0.38	0.36	0.35	0.33
0.80	0.32	0.30	0.28	0.27	0.25	0.24	0.22	0.20	0.19	0.17
0.90	0.16	0.14	0.13	0.11	0.09	0.08	0.06	0.05	0.03	0.02

be used to determine the radii of the circles to be used in the construction.

If the ballantines are to be drawn by computer graphics, let C_x be located at point (X_x, Y_x) in a Cartesian coordinate system. A convenient position for C_y (X_y, Y_y) is

$$\begin{aligned} X_y &= X_x + L_{xy} \\ Y_y &= Y_x. \end{aligned} \quad (7)$$

Locating C_z is slightly more complex. As Figure 4 shows, the three points C_x , C_y , and C_z define a triangle with sides L_{xy} , L_{xz} , and L_{yz} . Let θ be the interior angle of the triangle $\Delta C_x C_y C_z$ at point C_x . For ease of notation, let

$$\begin{aligned} a &= L_{yz} \\ b &= L_{xz} \\ c &= L_{xy} \\ s &= \frac{1}{2} (a + b + c) \end{aligned} \quad (8)$$

and

$$v = ((s-a)(s-b)(s-c)/s). \quad (9)$$

Angle θ obeys the relation

$$\theta = 2 \arctan(v/(s-a)) \quad (10)$$

(Burlington, 1948, p. 20).

The coordinates of the two possible points for C_z are

$$\begin{aligned} X_z &= X_x + \cos(\theta) \cdot b, \text{ and} \\ &= X_x + \cos(\theta) \cdot L_{xz}. \end{aligned} \quad (11a)$$

$$\begin{aligned} Y_z &= Y_x + \sin(\theta) b \\ &= Y_x + \sin(\theta) L. \end{aligned} \quad (11b)$$

The program in the Appendix has an option which locates all circles relative to $C = (0,0)$ using the scale $R=1$, or, as an option, the user may specify the desired scale and origin. The program then locates the ballantine on the user's coordinate system.

What Ails Ballantines: Representing Partial and Multiple Correlations

When ballantines are constructed in the manner just described, there is an exact analogy between the percentage of one circle that intersects another and the squared correlations between pairs of variables. Unfortunately, the analogy between ratios of areas and squares of correlation coefficients does not hold for multiple, partial, and semipartial (part) correlations. The reason is that the analogy for first-order correlations depends upon a special condition that does not hold for the derived variables.

The special condition will now be explained, and alternative graphical displays will be suggested for multivariate relations. The product-moment correlation coefficient is the ratio of the covariance to the geometric mean of the variances. Therefore,

$$r = \frac{\text{Cov}(x,y)}{((\text{Var}(x) \cdot \text{Var}(y))^{1/2})} \quad (12)$$

In the geometric presentation developed here, the standardized variances of x and y have been represented by

the equal-sized circles X and Y , and the circles have been positioned so that

$$r = \frac{XY}{X} = \frac{XY}{Y}, \quad (13)$$

where X and Y are the areas of the circles representing the variances of x and y , and XY is the area of intersection of circles X and Y . Algebraically, Equation 13 is analogous to

$$r = \frac{\text{Cov}(x,y)}{(\text{Var}(x))^{1/2}} \quad (14a^*)$$

or

$$r = \frac{\text{Cov}(x,y)}{(\text{Var}(y))^{1/2}} \quad (14b^*)$$

(Following the linguistic convention for marking ungrammatical sentences, "untrue" equations will be marked by asterisks.) The geometric representation is accurate if both variables are standardized, for in that case, Equations 12 and 14 are identical since $\text{Var}(x) = \text{Var}(y) = 1$. This always is true for the correlation between two variables, represented by Figure 2. A ballantine, such as that in Figure 1, is simply a straightforward extension of the principles used to construct Figure 2.

Now consider the multiple correlation between variable y and two predictor variables, x and z . Abstractly this is the correlation between variable y and a predicted variable

$$\hat{Y} = (a_x x + a_z z). \quad (15)$$

$R^2_{y(xz)}$ could be represented by two intersecting circles, as in Figure 2, but the two circles should represent standardizations of y and $\hat{y}$. Obviously if $\hat{y}$ is to be represented by a circle $\hat{Y}$ of unit size, the circles X and Z cannot also be of unit size. Therefore, the union of the intersections of circles X and Z with Y do not correspond to the percent variance of y that is associated with the combination of x and z . This is the point at which the visual similarity of a ballantine to a Venn diagram may be misleading. Consider Figure 1. The multiple correlation between arithmetic skill and the combination of auditory and visual detection is .48, which is only trivially higher than the .47 correlation between arithmetic skill and auditory detection alone. A glance at Figure 1 shows that the ballantine based on first-order correlations suggests, erroneously, that the visual detection measure makes substantial independent contribution to predicting arithmetic skill.

Although it might be possible to develop a ballantine-like representation of the relation between a dependent variable and its predictors, there is little point in doing so. An alternative, widely used graphic representation is available. The dependent variable can be represented by a rectangle, with contiguous subareas of the rectangle being assigned to the proportion of variance predicted by the first independent variable, the second variable after the first has been considered, the third after the first and second have been considered, and so forth. Each subarea should have a site proportional to the appropriate squared semipartial correlation. This representation is illustrated

in Figure 5, using the same variables that were used to construct Figure 1. It provides a good graphic representation of dependent variable-independent variable reactions, but does not depict the relations between the independent variables.

The partial correlation between two variables, x and y , with respect to a third variable, z , $r_{(x,z)(y,z)}$, is the correlation between the residual in x and the residual in y , after an attempt has been made to predict both of these variables from linear regression on z . Since

$$\text{Var}(x.z) = \text{Var}(x)(1 - r_{xz}^2) \quad (16a)$$

and

$$\text{Var}(y.z) = \text{Var}(y)(1 - r_{yz}^2), \quad (16b)$$

neither of the residual variances can be standardized in a figure in which the variances of the original variables are standardized, except for the uninteresting case in which z is irrelevant to the x - y relation ($r_{xz} = r_{yz} = 0$). Thus, the partial correlation (squared) has no simple relation in a ballantine.

Partial correlations can be represented by a ballantine augmented by subsidiary figures similar to those in Figure 2. In most situations, the investigator's interest centers on the variation in a designated independent variable y that is predictable by x , after variable z has been statistically controlled. The partial correlations can be represented by first drawing a ballantine showing the first order x , y , and z relations, as in Figure 1, and then drawing overlapping circles representing $r_{(y,z)(x,z)}^2$ and $r_{(y,z)(x,z)}^2$. The correspondence between the geometry and algebra of the situation can be retained by reducing the

0 .09 .23

1.0

Y VARIANCE

Figure 5. The representation of multiple and semipartial correlations. Rectangle Y represents the (unit) variance of the dependent variable. Area Z is proportional to the squared correlation r_{xz}^2 , and represents the variance in y that is predictable from knowledge of variable z . Region X.Z represents the variance in Y predictable from knowledge of x after allowing for the prediction of y from z . Area X.Z is proportional to $r_{(y,z)(x,z)}^2$, the squared semipartial correlation of y with x , after allowing for the x . z relation. The combined areas Z and X.Z are proportional to the squared multiple correlation of y with x and z , $r_{y(x,z)}^2$. The figure was constructed using the numerical values used to construct Figure 1.

Figure 6. The depiction of partial correlations. The upper part of the figure is a reduced version of the ballantine of Figure 1. The lower figures represent the two squared partial correlations, drawn at a reduced scale to reflect the covariate's relation to the dependent variable.

size of the subsidiary circles by factors of $(1 - r_{yz}^2)$ and $(1 - r_{xz}^2)$. What this does is to rescale the subsidiary figures, relative to the original ballantine, by an amount proportional to the reduction in the variance of the dependent variable due to the use of a covariate. An example is shown in Figure 6, which depicts Figure 1 and the two subsidiary variables that can be derived from it.

Depicting Suppressor Variables: A Fatal Ailment

Suppressor variables are one of the paradoxical puzzles of multivariate statistics. Variable z is said to suppress the relation between variables x and y if the semipartial or partial correlation between x and y , controlling for z , ($r_{(y,z)(x,z)}^2$ and $r_{(y,z)(x,z)}^2$) is higher than the first-order correlation r_{xy}^2 . Suppression occurs if z is associated with a substantial part of the variation in x or y that is *not* associated with the x - y correlation. Algebraically, the effect of partialling out z is to decrease the variance of x by a factor equal to $1 - r_{xz}^2$ while leaving $\text{Cov}(y, x.y)$ and $\text{Cov}(x.z, y.z)$ relatively untouched.

Geometrically, suppression effects may produce a ballantine that is impossible to draw. The intuitive reason is that circles XY and XZ must be close, to represent the r_{xy}^2 and r_{xz}^2 terms, while circles Y and Z must be distant, to reflect the similar r_{yz}^2 term. More formally, L_{xy} , the distance from the center of circle X to circle Y, is a monotonically decreasing function of r_{xy}^2 , ranging from any value greater than 2, if $r_{xy}^2 = 0$, to 0 if $r_{xy}^2 = 1$. Similar relations hold between circles X and Z and Y and Z. The centers of circles Y and Z will be at most $(L_{xy} + L_{xz})$ units apart, when circles Y and Z are centered on opposite sides of circle X. If $L_{yz} > (L_{xy} + L_{xz})$, the appropriate ballantine cannot be drawn. In some cases, suppression effects can actually change the sign of $r_{y(x,z)}$ or $r_{(y,z)(x,z)}$ relative to r_{xy}^2 . Such effects obviously have no representation in a ballantine, since ballantines are based on squared correlations.

The fact that ballantines cannot represent suppression is clearly a limitation on the graph. The seriousness of the limitation will depend on the application. Although suppression relations are interesting when found, they appear to be rather rare.

CONCLUSIONS

Cohen and Cohen (1975) make a cogent argument for the use of ballantines to represent the relationship between two or more variables. Ballantines are vivid graphic displays of patterns of correlations. In some cases, ballantines can be made to represent the exact relationship between pairs of variables. An algorithm has been presented

for drawing ballantines when this is possible. In general, however, ballantines will not correctly represent relations based on multiple, part (semipartial), and partial correlations. Alternative representations derived from the ballantine can be used to depict most multivariate relations. It is not possible to depict suppression effects by using the ballantine.

REFERENCES

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 COHEN, J., & COHEN, P. (1975). *Applied multiple regression and correlation analysis for the behavioral sciences*. Hillsdale, NJ: Erlbaum.

APPENDIX

```

program ballantine(input,output);
{ Locates circles so that overlap is  $r^2$  of area of each circle }

const pi = 3.14159265;

var cx, cy, radius :real;  com: char;

function a2srqq(consts p1,p2:real4): real4; extern;
{ IBM arctan function }

function converge(r:real):real;
{ computes value of angle theta , and then uses theta to compute the distance between circle centers.
  Input is  $r^2$ .
  Output is distance between centers, assuming radius of 1 }

var high,low, alpha, old, delta, q, z : real;

begin { converge }
if r = 1.0 then converge := 0.0 { identical circles } else
if r <= 0.0 then converge := 2.0 {no intersection } else
begin { intersecting circles. compute overlap }
  low := 0; high := pi; { angle theta from 0 to pi in radians }
  q := r * pi; { sector area of overlap }
  old := 0; alpha := pi/2.0; { 90 degrees, initial guess for angle }

  repeat { converge loop }
    z := alpha -sin(alpha);
    if z = q then delta := 0.0 {exact match} else
    {compute adjustment }
    begin
      old := alpha;
      if z > q then { decrease alpha }
      begin
        alpha := alpha - (alpha-low)/2.0;
        high := old;
      end
      else { increase alpha }
      begin
        alpha := alpha + (high-alpha)/2.0;
        low := old;
      end;
      delta := abs(old-alpha); {size of adjustment }
    end; { of adjustment }
  until delta < 0.0001; { converge to thousandth of a radian }
  { compute the distance between circles }
  alpha := alpha/2.0;
  converge := 2.0 * cos(alpha);
end; { overlap computed }
end; { converge function }

```

APPENDIX (Continued)

```

procedure graphpars(var cx,cy,radius :real);
{Computes the scale and translation factors for a real graph }
begin
  writeln ('Your graph is assumed to have 0,0 at the lower left');
  writeln ('enter maximum value of x and y as integers ');
  readln(cx,cy);
  cx := cx/2.0; cy := cy/2.0;
  if cx < cy then radius :=0.90 * cx/2.0
    else radius := 0.9 * cy/2.0;
end; { Graphpars }

Procedure twocircles(cx,cy,radius:real);
var r,c,l,z,x :real;
begin { twocircles }
  writeln ('What is the value of the correlation ?');
  readln(r); r := r * r;
  l := converge(r) * radius ;
  writeln('Distance between circles is ',l:10:4);
  x := cx-1/2.0; z := cx + 1/2.0;
  writeln( 'Circle X at point ',x:7:2,' ',cy:7:2);
  writeln( 'Circle Y at point ',z:7:2,' ',cy:7:2);
  writeln(' Radius = ',radius:10:4);
end; { twocircles }

Procedure threecircles(cx,cy,radius:real);
const x = 1; y = 2; z = 3; { used for names of circle }
var rxy, rxz, ryz, lxy, lxz, lyz :real ; { Same names as in paper }
    cc : array [1..3,1..2] of real; { centers of circles }
    a,b, c, s, thetha, v : real; {Auxiliary variables named in paper }
    xx, dx, dy : real; { scratch variables for computing }
begin { procedure threecircles }
  {get needed values }
  writeln ('Values of correlations rxy, rxz, ryz (real ) ');
  readln (rxy,rxz,ryz);
  rxy := rxy * rxy; rxz := rxz * rxz; ryz := ryz * ryz;
  { calculate intercircle distances }
  lxy := converge(rxy);
  lxz := converge(rxz);
  lyz := converge(ryz);
  { convert to auxiliary notation to conform to the text}
  a := lyz; b := lxy; c := lxz;
  s := (a + b + c)/2.0;
  v := (s-a) * (s-b) * (s-c) / s;
  v := sqrt(v); xx := s-a;
  { calculate value of interior angle theta at center of circle x and then
  determine the distance center of z falls below the x-y centerline. }
  thetha := 2 * a2srqq(v,xx); { IBM terminology for arctan }
  dy := sin(theta) * lxz; dx := cos(theta) * lxz;
  {determine center points,converting to actual graph }
  cc[x,x] := cx - (lxy/2.0) * radius;
  {x-y symmetric re vertical axis}
  cc[x,y] := cy + (dy/2.0) * radius;
  { x-z symmetric re horizontal axis}
  cc[y,x] := cx + (lxy/2.0) * radius;
  cc[y,y] := cc[x,y];
  cc[z,x] := cc[x,x] + dx * radius;
  cc[z,y] := cc[x,y] - dy * radius;
  {print results }

```

APPENDIX (Continued)

```

    writeln ('Ballantine for rxy = ', sqrt(rxy):5:3,
              ' rxz= ',sqrt(rxz):5:3,' ryz ',sqrt(ryz):5:3);
    writeln;
    writeln ('circle   X   Y');
    writeln (' X   ',cc[x,x]:7:2,' ',cc[x,y]:7:2);
    writeln (' Y   ',cc[y,x]:7:2,' ',cc[y,y]:7:2);
    writeln (' Z   ',cc[z,x]:7:2,' ',cc[z,y]:7:2);
    writeln;
    writeln (' All radii = ',radius:7:1);
end; { Of threecircle procedure }
begin { main program }
    write (' Is a real (r) or abstract (a) graph to be positioned ' );
    readln(com);
    if com = 'r' then graphpars(cx,cy,radius) else
        begin
            writeln ('Abstract graph centered at 0,0 with radius = 1 ');
            cx := 0; cy := 0; radius := 1.0;
            end;
    write(' Is a two (2) or three (3) variable problem to be computed? ');
    readln(com);
    if com = '2' then twocircles(cx,cy,radius) else
    if com = '3' then threecircles(cx,cy,radius)
        else writeln ('undefined problem');
end. {Of main program }

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(Manuscript received August 20, 1985;
revision accepted for publication March 27, 1986.)