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Misperception of exponential growth

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Exponential growth in numerical series and graphs is grossly underestimated in an intuitive extrapolation task. Subjects' extrapolations are well described by a model with two parameters only: one for underestimation of the nonlinear growth, the other for linear compensation. The size of the effect is considerable; it is not unusual that two-thirds of the subjects produce estimates below 10% of the normative value. The effect increases with the exponent of the stimulus series, and with addition of a constant to the stimulus series. Neither special instructions about the nature of exponential growth nor daily experience with growth processes enhanced the extrapolations.

Many worldwide problems of today are related to growth. Economical growth and growth of populations induce shortages of energy, raw materials, and food, and an increase in cost of living and pollution. These processes show a marked exponential character: they are going faster and faster. Any attempt to control these processes will depend on the cooperation of individual citizens; they should first appreciate how fast a growth process will be, before they can reasonably weigh the growth problem against a number of alternative factors, such as religious beliefs or personal comfort.

The present study investigated how subjects perceive exponential growth represented by numbers (Experiment I) and by graphs (Experiments II-IV). Would the average person be able to extrapolate an exponentially growing process? Would he be willing to believe a most likely extrapolation? From the literature (De Zeeuw & Wagenaar, 1974; Peterson & Beach, 1966; Tversky & Kahneman, 1971, 1973), it is clear that man has severe problems in dealing with quantitative data in an intuitive way. Normative rules as provided by mathematics or statistics are usually too complex; rather, subjects simplify the problem by using a repertoire of heuristic strategies which, in some cases, induce deviations from the "optimal" behavior prescribed by normative theory. To what extent do such effects occur in the perception of exponential growth?

EXPERIMENT I

Method

Procedure

Samples of a hypothetical growth process over the years 1970 to 1974 were presented to three groups of subjects, together with one of three alternative questions: If nothing will stop the growth: Group 1—What is your intuitive prediction for 1979? Group 2—What is your intuitive prediction for 1975, 1976, . . . , 1979? Group 3—When will the process reach a certain level? Example: Pollution in the upper air space appeared to be as follows in 5 consecutive years:

Year:	Pollution Index:
1970	3
1971	7
1972	20
1973	55
1974	148

Group 1: How large will the index be in 1979?

Group 3: In which year will the index surpass 25,000?

The starting series used are described by (y = pollution index; x = 1, 2, . . . , 5 = number of years since 1969): a series, $y = a e^x$, a = 1, 2, 4, . . . , 128; b series, $y = e^{bx}$, b = 1.0, 1.1, . . . , 1.7; c series, $y = e^x + c$, c = 0, 100, . . . , 700. Note that there are 22 different series, as the conditions a = 1, b = 1.0, and c = 0 are identical. The level indicated for Group 3 was always reached in 1979 according to the normative extrapolation. All starting series were presented to each group.

In the instruction, it was stressed that the subject should give his best intuitive estimate; application of strict arithmetic rules was not encouraged. Time available for each problem was 1 min. The 22 problems were printed on successive pages of a booklet; order of presentation was randomized over subjects. The experimenter indicated when the subjects could turn to the next page. The three groups were run simultaneously in three different classrooms. At the end of the experiment, individual records (sex, credits in mathematics) were collected.

Subjects

Three groups of 30 subjects each, students at Pennsylvania State University, took part in this experiment in partical fulfillment of the requirements for the introductory psychology course.

Results

As a typical result, consider the outcomes of Groups 1 and 3 in the example used in the previous section (a = 1, b = 1.0, c = 0). Two-thirds of the subjects produced estimates at or below 10% of the value prescribed by exponential extrapolation (25,000). Ninety percent of the subjects estimated

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below half of the normative value. Similarly, it appears that about half of the subjects expected the situation of 1979 not before the year 2000. Two-thirds of the subjects thought that the growth to be expected in the next 5 years would spread over at least 10 years.

The systematic presentation of the results will be within the framework of a simple model. In this model, it is assumed that the gross misperception demonstrated above is caused by misperception of the exponent of the multiplier used for extrapolation to a next year. Specifically, it is hypothesized that the exponent is underestimated with a constant factor, β. The subject may compensate the underestimation by increasing the multiplier with a factor, a, which is independent of the exponent. Thus, in b series, the multiplier would be $ae^{\beta b}$. The responses of Group 1 in the b series would be described by

$$\hat{y} = e^{5b}(ae^{\beta b})^5 \quad (1)$$

($\hat{y}$ = predicted index; e^{5b} = last number of the starting series).

From Expression 1, it follows that

$$\ln \hat{y} = 5 \ln a + 5b(1 + \beta), \quad (2)$$

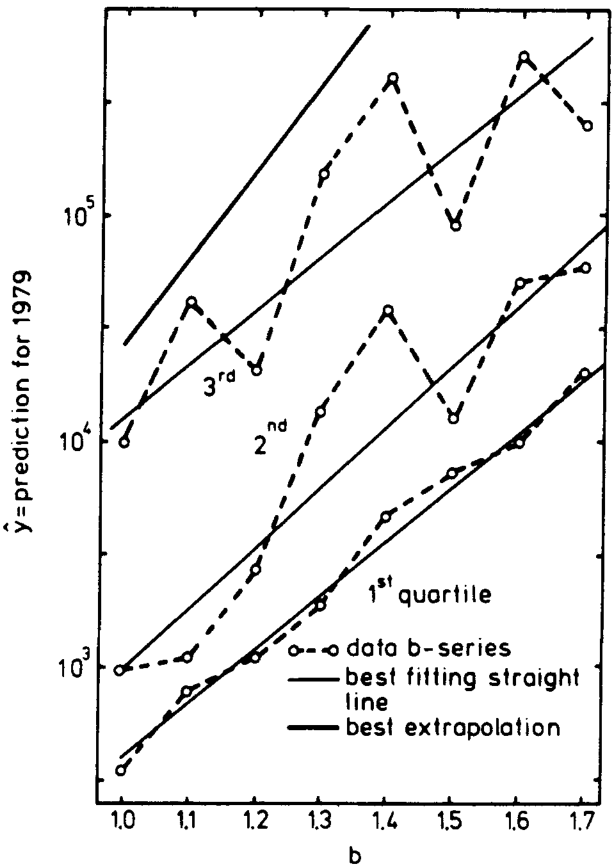

Figure 1. Results of Group 1 in b series. Starting numbers, $y = e^{bx}$.

which means that a plot of $\ln \hat{y}$ vs. b should yield linear functions with slope $5(1 + \beta)$ and intercept $5 \ln a$.

In a series, the model prescribes multipliers of the form (ae^{β}). The responses should be described by

$$\hat{y} = ae^5(ae^{\beta})^5 \quad (3)$$

or

$$\ln \hat{y} = \ln a + 5 \ln a + 5(\beta + 1), \quad (4)$$

which implies that plots of $\ln \hat{y}$ against $\ln a$ should yield linear functions with unity slope and intercept $5 \ln a + 5(\beta + 1)$.

This model does not specify quantitative predictions for the c series. It is not likely that the subjects could identify and subtract the additive constant; rather, they may attempt to estimate the multiplier from the numbers directly, which will result in an increased underestimation.

No differences are predicted between the 1979 data of Groups 1 and 2. The model specifies that in Group 2 a constant multiplier will be used for successive predictions. It follows, from Formulas 1 and 2, that in Group 2 plots of $\ln \hat{y}$ vs. years should yield linear functions. In b series, these functions would have a slope $(\ln a + b\beta)$; in a series, the slope will be $(\ln a + \beta)$.

If the subjects in Group 3 behave according to the same model, their estimates in b series are described by

$$\hat{y} = 1974 + 5b/(\ln a + b\beta), \quad (5)$$

which follows from equating $e^{5b}(ae^{\beta b})^x$ to e^{10b}. A plot of $1/(\hat{y} - 1974)$ vs. $1/b$ should be linear with slope $\ln a/5$ and intercept $\beta/5$. In a similar way, it is predcicted for a series that

$$\hat{y} = 1974 + 5/(\ln a + \beta). \quad (6)$$

Group 1

b series. The model specified that a plot of $\ln \hat{y}$ vs. b should yield linear functions with slope $5(1 + \beta)$ and intercept $5 \ln a$. Such plots are presented in Figure 1 for three quartiles of the response distributions within groups. This way of presentation is meaningful only if something like first, second, or third quartile subjects really exist. This was tested by computation of coefficients of concordance (Siegel, 1956) for all groups in all series. The coefficients, which vary from 0.51 to 0.81 (mean value: 0.65), were all significant at the .01 level. Thus, it is shown that rank ordering of subjects is roughly the same in all conditions.

The results presented in Figure 1 suggest the required linear relationship, which is further substantiated by computing the variance accounted

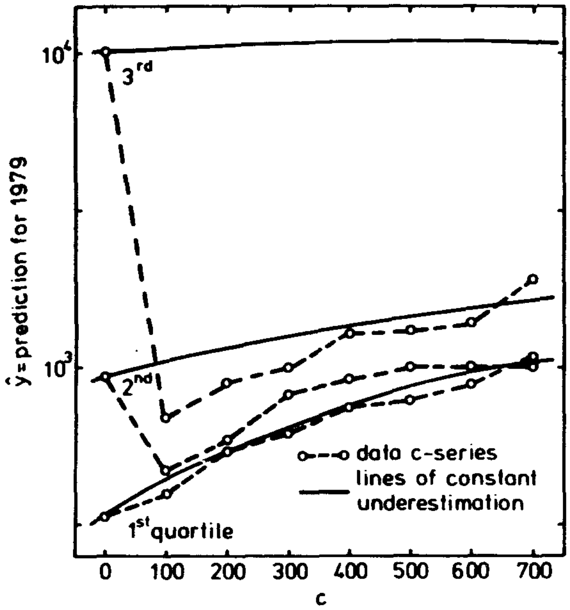

Figure 2. Results of Group 1 in c series. Starting numbers, $y = e^x + c$.

for by the linear components, which is 96%, 82%, and 69% for first, second, and third quartiles. It appears that underestimation of the exponent is considerable and about equal for all subjects: β = 0.12, 0.24, and 0.11 for first, second, and third quartiles. Individual differences are largely due to a, the amount of compensation of the underestimated nonlinearity: a = 1.08, 1.16, and 2.19 for first, second, and third quartiles.

a series. The model requires that plots of ln ŷ against ln a yield linear functions with unity slope and intercept $5\ln a + 5(\beta + 1)$. The functions are again quite linear: the linear components account for 98%, 99%, and 94% of the variance in, respectively, first, second, and third quartile plots. The slopes (1.06, 1.11, and 1.26 for first, second, and third quartiles) do not differ from unity to a significant degree (t = 1.23, 2.22, 1.57; df = 6). Since both a and β define the intercept, it is not possible to estimate these parameters again. The results indicate that the absolute magnitude of the starting series (the a factor) does not affect the underestimation of growth markedly.

c series. The data presented in Figure 2 show the predicted increase of underestimation when a constant is added. Thus, paradoxically, extrapolations for $y = e^x + 100$ fall, on the average, below the extrapolations of $y = e^x$. For each quartile, the line of constant underestimation is added; this line presents the response that would emerge if the subject estimated after subtraction of c. The difference between the conditions c = 0 and c = 100 is significant (Wilcoxon matched-pairs signed-ranks test: z = -2.98, $p < .01$). The increase of ŷ when c goes from 100 to 700 is not merely due to the additive constant: the effect remains even when c is subtracted from all scores (Friedman test: $\chi^2 = 14.54$, df = 6, $p < .05$).

Personal data. The median rank in all conditions for male and female subjects did not differ significantly (Mann-Whitney U test: z = 0.62). The rank correlation between median rank and the number of credits in mathematics courses (ranging from 0 to 40) was also not significant ($\tau = 0.24$).

Group 2

b series. The extrapolations for 1979 of Groups 1 and 2 can be compared directly. Mann-Whitney U tests on these data reached significance only once (z = 1.26, 1.27, 1.84, 0.56, 1.32, 2.11, 1.24, 1.77; for b = 1.0, 1.1, . . . , 1.7). The third-quartile subjects of Group 2 tended to underestimate less than those in Group 1: the slopes of best fitting straight lines in Groups 1 and 2 differed significantly (t = 16.01, df = 14, $p < .01$).

If the same multiplier is used for each successive prediction, plots of ln ŷ against years will yield straight lines with slope $\ln a + b\beta$; the slope of the starting functions is b. For the conditions b = 1.0 and b = 1.7, such plots are shown in Figure 3. The data clearly suggest the deflection immediately at the prediction for 1975. The difference between the slopes of starting series and estimates was significant for all quartiles and all values of b (t ranged from 3.68 to 78.80 with 3 df). In the formula for slope, the effect of b is weighted by β; the data reaffirm that first and

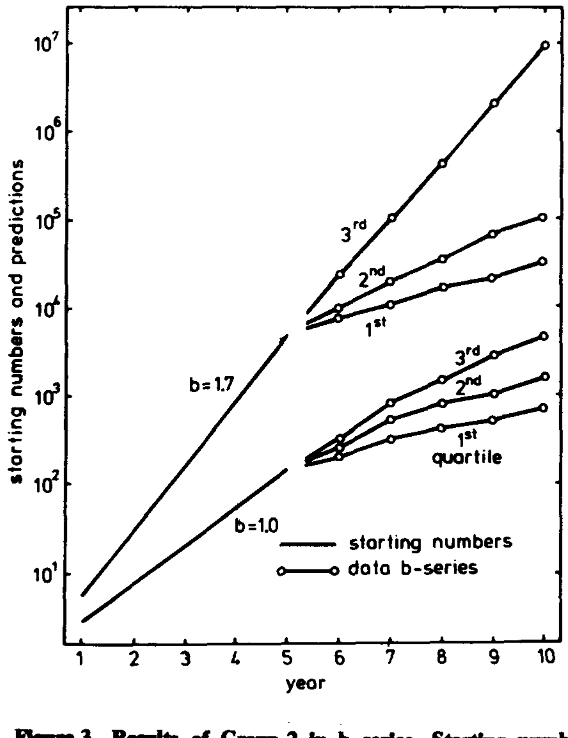

Figure 3. Results of Group 2 in b series. Starting numbers, $y = e^{bx}$.

second quartile subjects have smaller βs (slope relatively independent of b) than third quartile subjects, who show a marked relation between b and slope.

a series. The 1979 estimates in the a series of Group 2 never differed significantly from the results in Group 1 (Mann-Whitney U test, z = 0.06, 0.42, 0.84, 0.00, 1.37, 0.87, 0.95 for a = 2, 4, . . . , 128). Plots of ln $\hat{y}$ vs. ln a were again linear with unity slope.

Again, plots of ln $\hat{y}$ vs. years should be linear, and with a smaller slope than the starting series. In three instances, the difference between observed slope and the slope of the starting series did not reach significance; this occurred in the third quartile for a = 2, 32, and 128 (t = 0.12, 0.37, and 1.60; df = 3). In the 18 other cases, the deflection was significant (t ranged from 4.34 to 98.43; df = 3).

c series. The 1979 data of Groups 1 and 2 were generally quite similar, but the overall level of prediction was somewhat higher in Group 2. Again, estimates dropped dramatically between c = 0 and c = 100 (Wilcoxon matched-pairs signed-ranks test: z = -2.64, $p < .01$). The increase of $\hat{y}$ with c was again significant, even after subtraction of c (Friedman test: $\chi^2 = 12.50$; df = 6, p = .05). This effect is not attributable to a better perception of the true multiplier, since the slopes of the estimated series do not increase with c; the constant multiplier strategy leads to higher estimates when c increases.

Group 3

b series. The responses (Table 1) consistently exceed the dates expected on the basis of Formula 5 and αs and βs in Group 1. This effect may have two causes: (a) the reverse problem formulation elicits different values of α and β, or (b) subjects use a different strategy in this condition. The variance explained by the linear components in plots of $1/(\hat{y} - 1974)$ vs. 1/b was 95%, 68%, and 35% for first, second, and third quartile. This suggests that the model becomes less valid when the subjects predict more accurately. However, the deviations from linearity did not present themselves as a systematic trend; hence, no specific change of the model is suggested by these data. For the first quartile subjects, we find $\alpha = 1.12$ and $\beta = -0.06$.

a series. Again, actual responses are consistently higher than the dates expected on the basis of Formula 6. The estimates should remain constant when a increases, as a is not present in Formula 6. The correlations between a and $\hat{y}$ were 0.16, -0.22, and 0.02 for the three quartiles, which never reaches significance (t = 0.39, 0.55, 0.05; df = 6).

c series. The estimates showed tendencies similar to the results of the other groups: stronger underestimation as soon as a constant value is added to the starting series.

Table 1
Results of Group 3, b Series

	Quartile					
b	1		2		3	
1.0	2074	(1999)	1997	(1987)	1981	(1980)
1.1	2100	(2000)	1996	(1987)	1984	(1980)
1.2	2100	(2001)	2000	(1988)	1985	(1981)
1.3	2200	(2002)	1996	(1988)	1984	(1981)
1.4	2210	(2002)	2006	(1988)	1985	(1981)
1.5	2310	(2003)	2065	(1989)	1998	(1982)
1.6	3500	(2003)	2089	(1989)	1983	(1982)
1.7	3200	(2004)	2030	(1989)	1986	(1983)

Note—Values expected on the basis of the results in Group 1 are in parentheses.

Discussion

The most prominent result is the considerable underestimation of growth by all groups in all conditions. The responses are quite well described by a model with two parameters only: β, the underestimation of the nonlinear element (the exponent), and α, a linear compensation factor. Underestimation of the nonlinearity appears to be almost identical for all subjects. Individual differences are introduced mainly by different amounts of linear compensation. Both parameters are independent of the absolute size of the numbers (the a factor) and of the rate of growth (b factor). The model provided a less satisfactory description in a few cases only. These cases are (a) a larger β for the third quartile subjects of Group 2 in b series, (b) the higher overall level of prediction in c series of Group 2, (c) nonsystematic noise in the responses of Group 3 in b series. None of these deviations suggest a specific change in the model.

One possible complication is the use of numbers as responses. It is well known (Schneider, Parker, Ostrosky, Stein, & Kanow, 1974) that subjects use a subjective scale for number which is described by a power function with exponent < 1. In our case, this factor poses a problem only when it is assumed that the transformation from stimulus to subjective number is not the reciprocal of the transformation from subjective number to response. Such an effect has never been demonstrated. As an extra precaution, mostly nonparametric statistical tests have been applied.

Another possible pitfall is the occurrence of range effects: since all subjects judged all series, responses in extreme series could have shifted towards the median. In that case, however, the absolute magnitude of the starting series (the a factor) would become a major determinant of the underestimation, which was not the case.

It should be realized that intuitive judgment of growth on the basis of numbers will occur only in a limited number of cases. Presentation of data in graphs is much more usual; the next experiments are devoted to that problem. The stimulus series employed were of the form $y = e^{bx}$; effects of other constants, as in $y = ae^x$ and $y = e^x + c$, were not studied, since they affect merely the numbers along the axes and not the shape of the curves. A new variable was the length-to-width ratio of the graphs. It was hoped that steeper graphs would elicit less conservative responses. Another new factor was the level of sophistication of the subjects. In Experiment I, no correlation with credits in mathematics was found. In the next experiments, effects of specific instruction about exponential growth and of practical experience with such processes are investigated.

EXPERIMENT II

Method

Procedure

The stimulus series were of the form $y = e^{bx}$; $b = 1.0, 1.1, \ldots, 1.7$; $x = 1, 2, \ldots, 5$. In the instruction was mentioned that the data represented indices of pollution in the upper air space, measured in the years 1970, 1971, ..., 1974. The graphs were presented with three length-to-width ratios: 3:1, 1:1, and 1:3 (actual sizes: 24.0 x 8.5 cm, 14.3 x 14.3 cm, 8.5 x 24.0 cm; see also Figure 4). All graphs were printed on separate sheets of paper. Each subject solved eight problems with different values of b. Order of presentation was randomized for all subjects. Length-to-width ratio was a between-subjects variable. The task was to answer the question: if nothing will limit this process, which index do you predict for the year 1979? Scribbling or drawing curves was not allowed. Time alloted to each problem was 30 sec. The three groups of subjects (one for each length-to-width ratio) were run simultaneously in three different classrooms.

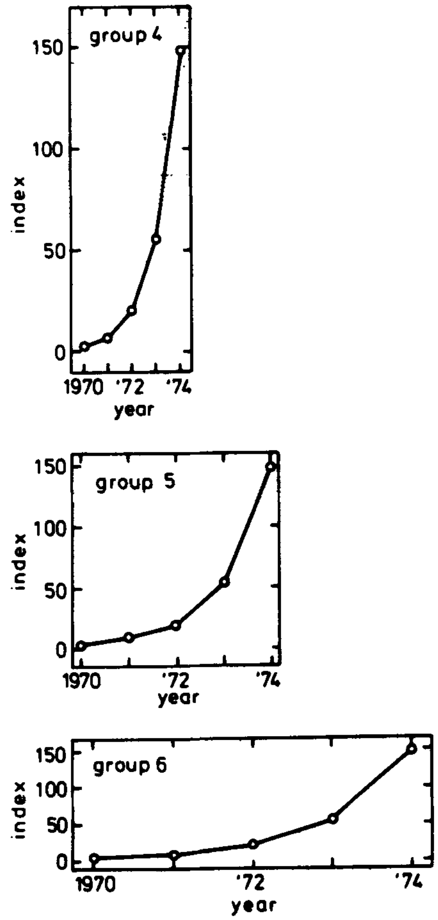

Figure 4. $y = e^x$ **plotted with three length-to-width ratios of the graph.**

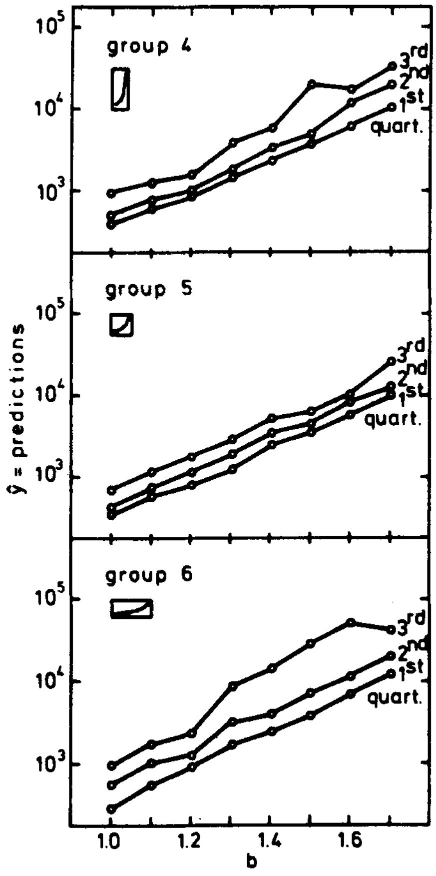

Figure 5. Results of Experiment II. Starting numbers, $y = e^{bx}$.

Subjects

Three groups of subjects (Groups 4 to 6), students at Pennsylvania State University, took part in the experiment in partial fulfillment of the requirements for the introductory psychology course. There were 23 subjects in Group 4 (3:1 ratio), 24 in Group 5 (1:1 ratio), and 20 in Group 6 (1:3 ratio).

Results

Plots of $\ln \hat{y}$ vs. b are presented in Figure 5. Reduction of the data by presentation of first, second, and third quartiles is justified since the rank ordering of subjects was about the same in all conditions; coefficients of concordance were 0.76, 0.69, and 0.80 for Groups 4, 5, and 6 ($p < .01$ in all cases). All functions are extremely linear, which is substantiated by the high proportions of variance explained by the linear components (lowest proportion, 95.0%; highest proportion, 99.8%; mean, 98.6%). Values of β

estimated with the use of Formula 2 appear to be close to zero in all cases but one, which means that the different rates of growth were almost not discriminated (β ranged from -0.07 to 0.28; mean values was 0.04). The slopes were never over 5.00 to a significant degree.

The differences among the three groups never reached significance (Kruskal-Wallis one-way analysis of variance: χ^2 = 1.09, 1.02, 1.19, 4.91, 0.50, 2.99, 3.87, 3.21, for b = 1.0, 1.1, . . . , 1.7; df = 2). In all conditions, the responses of Group 6 tended to be higher.

Responses of Group 6 were tested against the responses obtained with numerical stimuli in Group 1 (Experiment I). With graphical stimuli, responses were lower for all values of b; the differences reached significance in three cases (Mann-Whitney U test: z = 1.65, 1.68, 2.03*, 1.79, 2.87*, 1.98*, 1.85, 1.46, for b = 1.0, 1.1, . . . , 1.7). The slopes of the plots of ln $\hat{y}$ vs. b were smaller with graphical stimuli for first and second quartiles only (df = 14; t = -5.73, p < .01; t = -3.41, p < .01); for the third quartile, numerical stimuli yielded the smaller slope (df = 14, t = 2.48, p < .05).

Discussion

Misperception of exponential growth is not decreased by presenting data graphically instead of numerically. In this sense, a picture is not worth a thousand words! Rather, the evidence suggests that graphs elicit even more conservative extrapolations, irrespective of the length-to-width ratio of the graphs.

EXPERIMENT III

This experiment tested whether prior instruction about exponential curves and underestimation might improve extrapolation.

Method

Procedure

A group of 20 subjects, students at Pennsylvania State University (Group 7), attended a 75-min lecture on misperception of exponential growth. First, they were confronted with a number of numerical examples and requested to extrapolate intuitively; immediate feedback of the best extrapolation was given. Then the general results of Experiment I were discussed. At the end of the lecture, they were asked to participate in a related experiment. The stimuli presented were the eight problems of Group 5 (b series, square graphs). Instructions were the same as before.

Results

The coefficient of concordance among the eight problems was 0.68 (p < .01). The plots of ln $\hat{y}$ vs. b are presented in Figure 6a. The responses were much higher than those produced by Group 5 (Mann-Whitney U test: z = 3.84, 3.69, 5.01, 4.99, 4.54, 4.14, 3.93, 3.45, for b = 1.0, 1.1, . . . , 1.7; p < .01 in all cases). The improvement is due at least partly to a better discrimination of the exponents, as expressed

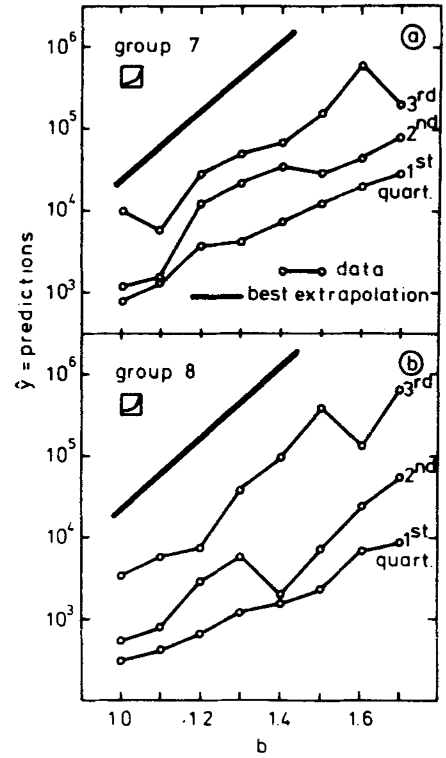

Figure 6. Results of Experiments III(a) and IV(b). Starting numbers, y = e^{bx}. The best extrapolation is the line y = e^{10b}.

by the weights β which are inferred from the slopes of the best fitting straight lines (median = 0.16). The slopes were significantly larger than in Group 5 (df = 14; t = 3.21, 2.96, 3.37, for the first, second, and third quartile; p < .01 in all cases). The differences between the slopes in Group 7 and those obtained with numerical stimuli (Group 1) were significant only for the first quartile; the slope with graphical stimuli was smaller in that case (df = 14; t = -4.44, p < .01; t = -1.19, n.s.; t = 1.92, n.s.; for first, second, and third quartile).

The compensation parameter a in Group 7 was larger than in Group 5. Since the variance of the responses of a subject contains a factor a^{10}, the difference between two values of a can be tested by F = $(a_1/a_2)^{10}$, with df = 7,7. The test revealed that values of a in Group 7 were consistently higher than in Group 4 [F(7,7) = 4.00, p < .05; F(7,7) = 2.25, n.s.; F(7,7) = 10.87, p < .01; for first, second, and third quartile, respectively].

Discussion

The effect of sophistication is twofold. Discrimination among the various exponents is improved slightly, but not above the level obtained with numerical stimuli. On the other hand, a, the compensation by increase of the magnitude of the responses irrespective of the exponent, is also larger;

this reflects the subjects' willingness to produce larger numbers. Thus, the general picture is that sophisticated subjects attempt to evade the effect of underestimation; but, because of the limited discriminability of exponents, they cannot help increasing their responses more or less to the same extent in all problems.

EXPERIMENT IV

In this experiment, extrapolations are collected of subjects whose jobs encompass making important decisions that should be based on correctly perceived growth.

Method

Procedure

To a group of subjects (Group 8), the eight problems of Group 5 were presented (b series, square graphs). Instructions were the same as before.

Subjects

The eight subjects were members of the Joint Conservation Committee of the Senate and House of Representatives of the Commonwealth of Pennsylvania.[1]

Results

The plots of $\ln \hat{y}$ vs. b are presented in Figure 6b. The coefficient of concordance among the eight problems was 0.91 ($p < .01$). Underestimation was generally less pronounced than in Group 5, but never to a significant degree (Mann-Whitney U test: $z = 0.57, 0.47, 0.44, 1.00, 0.98, 0.28, 1.50, 0.46$, for $b = 1.0, 1.1, \ldots, 1.7$). A quantitative analysis is made for the second quartile only, as the number of subjects in the group was small. The variance explained by the linear component was 87%. The slope was significantly higher than in Group 5 ($t = 3.92$, $df = 14$, $p < .01$) and about equal to the slope of Group 1, Experiment I ($t = 0.65$, $df = 14$, n.s.).

Discussion

This group of professional decision makers did not show less underestimation than naive subjects; β remained within the range observed before. Underestimation appears to be a general effect which is not reduced by daily experience with growing processes.

GENERAL DISCUSSION

The model proposed provides a satisfactory description of the data. All plots of $\ln \hat{y}$ vs. b were essentially linear; values of β were generally within the range from 0.00 to 0.20, both for numerical and graphical stimuli.

The model does not specify *why* subjects underestimate as they do. Reference to real-life experience with processes that damp out sooner or later does not seem too relevant, as the subjects in Group 2 of Experiment I do not produce damped functions; they produce exponential functions with constant but too small exponents. Also, the results of Experiment III suggest that we are faced with a real functional impossibility: subjects cannot discriminate between functions with different exponents, Discrimination would probably be helped by presenting stimulus series on a logarithmic scale. However, the final objective of this research program is to find a means of presenting growth processes in such a way that the average man in the street will grasp it. Logarithmic representation would not serve this aim. Other transformations, like the inverse, would be more likely candidates (cf. square miles per individual instead of individuals per square mile; average time between two robberies instead of robberies per hour). Actually, it will be reported in a subsequent paper that β may reach values up to 70% in the case of inversely represented processes.

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