Spectral effects in the perception of pure and tempered intervals: Discrimination and beats

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Thresholds for discrimination between pure and tempered musical intervals consisting of simultaneous complex tones have been investigated previously (Vos, 1982). The aim of the present research was to answer three questions: (1) To what extent are these thresholds determined by the interference of just-noncoinciding harmonics? (2) Is the beat frequency of the first pair of these adjacent harmonics equal to the perceived beat frequency of the tempered intervals? (3) Are differences in discriminability at threshold representative of differences between the perceived strength of beats in supraliminal conditions? In the first three experiments, the answer to the first question was sought by investigating the effect of spectral content of the tones on the discrimination thresholds (DTs) for the fifth (Experiment 1) and the major third (Experiments 2 and 3). The results showed that, for moderately tempered fifths, DTs are determined mainly by the interference of the first pair of adjacent harmonics. For major thirds, however, the data suggest that the interference of other harmonics plays a role as well. In Experiment 4, attempts were made to answer the second and third questions: In supraliminal conditions, both the dominantly perceived beat frequency and the perceived beat strength were determined for various spectral conditions and degrees of tempering. For both the fifth and the major third and for all spectral conditions, the dominantly perceived beat frequency was in most cases equal to the frequency difference of the first pair of adjacent harmonics. Comparison of the DTs and the perceived strength of the beats for corresponding spectral conditions revealed that, especially for the fifth, perceived strength of beats and sensitivity to moderately tempered intervals are highly correlated.

From a classical point of view, a consonant interval is a musical interval in which the ratio of the fundamental frequency f_1 of the lower tone and the fundamental frequency f_2 of the higher tone is equal to $p:q$, with p and q ($p < q$) being small integers. For two simultaneously sounding complex tones, slightly tempered consonant intervals ($f_1:f_2 \sim p:q$) are characterized by small frequency differences between those harmonics which coincide completely in pure intervals. The interference of these just-noncoinciding harmonics gives rise to the perception of beats.

The occurrence of beats must have been known for a long time, and attempts to explain this phenomenon were first made in the 17th century. As will be elucidated below, beats play an important role in

tuning fixed-pitch keyboard instruments, and are essential in models on tonal or sensory consonance. It has recently been demonstrated (Vos, 1982) that thresholds for discrimination between pure and moderately tempered musical intervals are mainly determined by sensitivity to the beats. This same study showed that when the tempered intervals deviated from purity by more than 20 to 30 cents, perception of interval width could also become a relevant aspect in discrimination.

It was the aim of the present study (1) to investigate which interfering harmonics might have determined the discrimination threshold in the experimental conditions described by Vos (1982), and (2) to determine, for supraliminal conditions, the effect of the spectral content of the tones on both the perceived frequency and the perceived strength of the beats.

Beats and Tuning

Beats have been and still are used very often in the process of tuning musical instruments (see, e.g., Barnes, 1959; Ellerhorst, 1936; Elliston, 1916; Scheibler, 1834, 1838; Smith, 1749/1966). Pure consonant intervals can be obtained by freeing them of beats, for example, the fifths ($p=2$; $q=3$) in the Pythagorean system or the major thirds ($p=4$; $q=5$)

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in the mean-tone system for tuning keyboard instruments. Only after the quantitative relations between fundamental frequency, musical intervals, temperaments, and beat frequencies were known was it possible to set the bearings of more complex tuning systems in a reliable way. This was the case in 1749 when Robert Smith published his *Harmonics*, which included tables with the numbers of beats of the fifths in the mean-tone system and in his own system of equal harmony (different from equal temperament). More successful in the propagation of his tuning method, however, was Scheibler (1834, 1838), whose procedure was based on beats from tempered unisons. More specifically, Scheibler's method of tuning instruments consisted of tuning a series of forks on his tonometer 4 beats/sec flatter than the frequencies required. The string, pipe, or reed was then tuned sharper than the fork by 4 beats/sec.

Beats have even been used to determine the fundamental frequencies of two low harmonium reeds, which formed a known and slightly tempered musical interval and sounded simultaneously (Rayleigh, 1878-1879/1966). Frequencies were determined with very high precision by counting for 10 min those beats that resulted from interference of different pairs of higher harmonics.

Beats and Consonance

In models on tonal or sensory consonance (Kameoka & Kuriyagawa, 1969a; Plomp & Levelt, 1965), the total dissonance of musical intervals has been assumed to be equal to the sum of the dissonance of each pair of adjacent harmonics. According to Plomp and Levelt (1965), these subdissonances are roughly about maximum at 25% and minimum at 0% and 100% of the critical bandwidth. Results from Kameoka and Kuriyagawa (1969b) suggest that the subdissonances not only depend on (1) the frequency of the tones and (2) their frequency distance, but also on (3) their sound-pressure levels. The intervals at which the maximum dissonances were found to occur increased with increasing tone level. For example, for two tones at a level of 57 dB SPL, the maximum values of the subdissonances were reached at interval sizes of 300 to 40 cents for frequencies of the lower tone of between 100 and 7000 Hz. According to Kameoka and Kuriyagawa (1969b), these interval sizes range from 460 to 60 cents for two tones at a level of 80 dB SPL.

The essential power of these consonance models is that, from the relationship between beats or roughness and consonance, they can demonstrate that octaves and fifths are much more sensitive to a deviation from their simple frequency ratio than are the major thirds. In widely accepted equal temperament, which permits modulation among keys, the major thirds are tempered considerably (14 cents) in favor of the slightly

tempered fifths (2 cents). The greater tolerance for tempered major thirds, as predicted from the consonance models, may explain why equal temperament has not been rejected.

The generality of these models has been demonstrated by Pierce (1966), who showed that consonance can be obtained with intervals comprising tones from an unconventional 8-note scale, having spectra with nonharmonic partials, provided that the distance between the partials remains larger than the critical bandwidth of the ear. Informal listening tests, reported by Slaymaker (1970), illustrate that chords from complex tones with stretched partials and with frequencies derived from correspondingly stretched scales are perceived as consonant, which is consistent with models on sensory consonance.

Beats of Complex Tones

For simple tones constituting a tempered consonant interval, attempts to explain the beats have yielded a vast number of studies (see Plomp, 1967, 1976, for reviews). For complex tones, the explanation of beats in terms of interference between adjacent harmonics with frequencies of pf_2 and qf_1 Hz was so plausible that, apparently, this case was reduced to beats of tempered unisons. It was recognized long ago, however, that beats of the first-order combination tones can be heard very well at the same time as those due to the higher harmonics of complex tones by an ear accustomed to hearing combination tones (Helmholtz, 1877/1954, chap. 11). Especially for very slightly tempered intervals, it has been noted that it is easy to make a mistake in counting the beats, since, in addition to the first pair of just-noncoinciding harmonics, the higher pairs beat as well (Helmholtz, 1877/1954, chap. 8). However, from more formal experiments, with two simultaneous complex tones that deviated slightly from unison ($p = 1$; $q = 1$), in which moderately well-trained listeners had to count the resulting beats, it was concluded that the beat frequency of the first pair of adjacent harmonics $pf_2 - qf_1$ Hz, was perceptually dominant (Warren, 1978). This was also the case when the fundamental components were absent. The dominantly perceived beat frequency equaled the fundamental beat frequency of the complex beats even when, in contrast to normal situations, the beat frequency of higher pairs of interfering harmonics was lower than that of the first pair of adjacent harmonics (Warren, personal communication, 1981).

Beats and Discrimination

In a previous paper, thresholds for discrimination between pure and tempered fifths and major thirds were presented in terms of level difference between the tones (Vos, 1982). Basically, level differences were introduced to manipulate the depth of the level

Figure 1. Replotted thresholds for discrimination between pure and tempered intervals (Vos, 1982), given as a function of beat frequency with tone duration as parameter. Thresholds are given separately for the fifth and major third in panels a and b, respectively.

variation of the beating harmonics with frequencies of npf_2 and nqf_1 Hz. In Figure 1, these discrimination thresholds (DTs) are replotted in terms of level variation depth and given as a function of beat frequency, $f_b = |pf_2 - qf_1|$, with tone duration as a parameter. It was the purpose of the research reported here to answer three questions: (1) Are the DTs determined mainly by the interference of the first pair of just-noncoinciding harmonics with frequencies of pf_2 and qf_1 Hz? (2) Does the beat frequency, $|pf_2 - qf_1|$, representing at least an objective measure of the amount of tempering also match the *perceived* beat frequency? And (3) are differences between discriminability at threshold representative of differences between the perceived strength of beats in supraliminal conditions? If these questions can be answered positively, the conclusion may be drawn that the DTs in Figure 1, with respect to both the abscissa and the ordinate, are also adequately represented from a perceptual point of view.

Experiments 1, 2, and 3 were designed to answer the first question. In these experiments, the relative importance of various harmonics for discrimination between pure and tempered fifths (Experiment 1) and pure and tempered major thirds (Experiments 2 and 3) was determined by investigating the effect of spectral content on the DTs. In Experiment 4, at-

tempts were made to answer the second and third questions. In supraliminal conditions, both the most dominantly perceived beat frequency and the perceived strength of the beats were determined for various spectral conditions.

EXPERIMENT 1

In a tempered consonant interval, in which the ratio of the fundamental frequency f_1 of tone 1 and the fundamental frequency f_2 of tone 2 departs slightly from $p:q$, the n^{th} pair of just-noncoinciding harmonics comprises the nq^{th} harmonic of tone 1 and the np^{th} harmonic of tone 2 ($n=1, 2, \dots$). Therefore, for the fifth ($p=2$; $q=3$) investigated here, the first pair of just-noncoinciding harmonics comprises the third harmonic of tone 1 and the second harmonic of tone 2. However, for the benefit of a unifying terminology, which will be especially useful in Experiment 4, where both fifths and major thirds ($p=4$; $q=5$) were investigated, it will be maintained throughout this paper that the first pair of just-noncoinciding harmonics comprises the q^{th} and p^{th} harmonics of tones 1 and 2, respectively.

The various spectral conditions presented in Experiment 1 are depicted in Figure 2. In this figure, the first 12 harmonics of tone 1, represented by

Figure 2. Schematic of the spectral conditions which were investigated in Experiment 1. The first 12 harmonics of tone 1, represented by squares, and various harmonics of tone 2, represented by circles, are given on a linear frequency scale. The just-noncoinciding harmonics are indicated by crosses.

squares, and various harmonics of tone 2, represented by circles, are given on a linear frequency scale. The just-noncoinciding harmonics are indicated by crosses. For five of six conditions, the spectrum of tone 2 always included the first harmonic or all the odd harmonics. As a result, it was guaranteed that in these conditions the perceived interval size equaled that of the fifth. In the third spectral condition, which comprised only the odd harmonics, interference between just-noncoinciding harmonics was not possible because the n th harmonics of tone 2 were deleted. In Conditions 4 and 5, the p th or $2p$ th harmonics were selectively added to the odd harmonics. The sixth condition, comprising all harmonics, served as a base rate. Comparison of the DTs from Spectral Conditions 2 to 6 should reveal the relative importance of the first, second, and higher pairs of adjacent harmonics for discrimination. To ascertain to what extent the combination of various triads of harmonics, such as harmonics with their relative frequencies in the ratios of 2:3:4, 8:9:10, and 14:15:16, may have contributed to DT, it was decided to include Spectral Condition 1 as well.

Method

Stimuli. For all fifths, tone 1 was a complex tone with amplitudes a_n equal to $1/n$. Therefore, the spectral envelope slope of the stimulus was -6 dB/octave. The waveform is given by:

$$p(t) = \sum_{n=1}^{n=20} \frac{1}{n} \sin(2\pi nft + \phi_n). \quad (1)$$

The phase (ϕ_n) of the individual harmonics was chosen randomly. The spectral content of tone 2 was varied. Again, the amplitudes, a_n , of the particular harmonics were equal to $1/n$ and the phases (ϕ_n) of the various harmonics were random.

The overall level of tones 1 and 2 together was 85 dB SPL. This level was measured with the help of an artificial ear (Brüel & Kjaer, Type 4152). The duration of the tone bursts was 500 msec. Rise

and decay times of the tones, defined as the time interval between 10% and 90% of the maximum amplitude, were 40 and 20 msec, respectively. Level variation depth of the beating harmonics was manipulated by attenuation of tone 2 by ΔL dB. When the spectral envelopes of the two tones coincide, that is, when the respective amplitudes a_1 and a_2 of the just-noncoinciding harmonics, with frequencies of qf_1 and pf_2 Hz, are equal, then ΔL is defined to be 0 dB. Level variation depth, D , in decibels, being maximal in this condition, is given by:

$$D = 20 \log \left(\frac{a_1 + a_2}{a_1 - a_2} \right) = 20 \log \left(\frac{1 + 10^{\Delta L/20}}{1 - 10^{\Delta L/20}} \right), \quad (2)$$

in which ΔL is the level difference between a_1 and a_2 in decibels [$\Delta L = 20 \log(a_2/a_1)$]. D approaches ∞ for $a_2 \rightarrow a_1$.

Coincidence of spectral envelopes occurs when the overall level of tone 2 is $20 \log(q/p)$ dB lower than the overall level of tone 1. Increments of ΔL from this base condition always result in smaller level variation depths.

Apparatus. The experiment was run under the control of a PDP-11/10 computer. Tones 1 and 2 were generated in the following way. One period of the waveforms of tones 1 and 2 was stored in 256 discrete samples (with 10-bit accuracy) in external revolving memories. These recirculators could be read out by digital-to-analog converters. Sampling rates were determined by pulse trains, derived from two frequency generators. After gating, the tones were filtered by Krohn-Hite filters (Model 3341) with the function switch in the low-pass RC mode and with a cutoff frequency of 8 kHz. The sound-pressure levels of the tones were controlled by programmable attenuators. After appropriate attenuation, the tones were mixed and fed to two headphone amplifiers. The signals were presented diotically (same signal to both ears) by means of Beyer DT 48S headphones. The subjects were seated in a sound-proof room.

Subjects. Four musically trained subjects were tested over seven sessions, each session on a different day. Three of them were conservatory students. They were paid for their services. The author also participated in the experiment.

Experimental design. The independent variables were: (1) spectral content of tone 2 (the spectrum of tone 2 could comprise the following harmonics: first, p th, odd, odd + p th, odd + $2p$ th, and all); (2) frequency difference $|pf_2 - qf_1|$ (2, 4, 8, 16, and 32 Hz); and (3) direction of tempering (stretched vs. compressed interval).

Scoring of data. The task for the subject was to discriminate between pure and tempered musical intervals in a 2-AFC paradigm.

By means of the method of constant stimuli, all stimulus combinations were presented 20 times for seven different values of ΔL in equal steps of 3, 4, or 5 dB, depending on stimulus condition. For every stimulus combination, the raw scores for different ΔL values were transformed to percentage correct. Since the response patterns of the stretched and compressed intervals were similar in almost all cases, the percentages of these two conditions were combined. On the assumption that the scores, for a given set of ΔL values, are cumulative normal distributions (Kling & Riggs, 1972), these percentages were converted to z-scores. The discrimination threshold, that is, the value of ΔL at which a subject responded correctly in 75% of the cases, was obtained by solving the linear regression function $z = a + b\Delta L$ for ΔL when z equals .67. Computation of coefficients of determination (r^2) for all linear regression functions showed that the explained variability in z by ΔL and the linear rule was more than .75 in 86% of the cases. Moreover, r^2 did not depend on either the kind of spectrum or beat frequency. Mean r^2 was .87.

Procedure. Every trial consisted of a comparison of a pure interval and a tempered interval. The tempered interval was presented first in half of the trials and second in the other half of the trials. Between the intervals there was a silent period of 1 sec. By means of two response buttons, the subject indicated whether the first or the second interval was tempered. Feedback was given by a red light above the correct button. The next trial was presented 1.5 sec after the response to the preceding trial. Within a series, three different spectra of tone 2 were presented, together with seven values of ΔL , two directions of tempering, and five beat frequencies ($3 \times 7 \times 2 \times 5 = 210$ trials in a series). There were four series types, each with a different combination of three spectra within a series. Thus, the six experimental spectra were assigned to four kinds of series in different combinations. Two subjects were presented first with series types 1 and 2; the other two subjects started the experiment with series types 3 and 4. Within a series, which lasted about 15 min, the stimuli were presented in a random order. At the beginning of a series, new waveforms of tones 1 and 2, with different ϕ_n values, were determined. The fundamental frequencies were varied from trial to trial in such a way that the central frequency f_c [= the geometric mean $(f_1 f_2)^{1/2}$] of the pure interval was equal to 370 Hz ± 50 , ± 100 , ± 150 , or ± 200 cents, in a random order. As a result, all fundamental frequencies fell within a range of 261.5 Hz (musical C4) and 523 Hz (musical C5). The f_1 of the pure interval was equal to:

$$f_1 = f_c / (q/p)^{1/2}. \quad (3)$$

In the tempered interval, one of the fundamental frequencies equaled the corresponding fundamental frequency in the pure interval and the other one was altered. For the tempered interval, either a new f_1 was calculated by means of:

$$f_1(\text{tempered}) = (p \cdot f_2 \pm f_b) / q, \quad (4)$$

or a new f_2 was calculated by:

$$f_2(\text{tempered}) = (q \cdot f_1 \pm f_b) / p. \quad (5)$$

Within a series, the frequencies of tones 1 and 2 were altered equally often, the particular order being determined by sampling without replacement. Similarly, the particular order of stretched versus compressed intervals (signs in Equations 4 and 5) was randomized. A session comprised six or seven series. During the first session, the subjects were trained. For most conditions, a rather broad range of ΔL from 7 to 31 dB, in steps of 4 dB, adequately covered the subjects' transition range between chance performance and 100% correct responses. For the spectra comprising the first or the odd harmonics, however, ΔL ranged from 4 to 22 dB, whereas for a beat frequency of 32 Hz, the range of ΔL was from 7 to 37 dB. In the $\Delta L < 7$ dB conditions, L_1 and L_2 were attenuated to a small extent to satisfy our criterion that the overall level of the interval remain at 85 dB SPL.

Figure 3. Thresholds for discrimination between pure and tempered fifths, plotted as a function of beat frequency, for various spectral conditions. Pth and 2pth refer to the second and fourth harmonic, respectively. The spectrum of tone 1 always comprised the first 20 harmonics. For both tone 1 and tone 2, waveforms were determined using Equation 1.

Results

Discrimination thresholds (DTs) are presented in terms of level difference ΔL . DTs were subjected to an ANOVA [4 (subjects) $\times$ 6 (spectra of tone 2) $\times$ 5 (beat frequencies), all repeated measures]. Although individual differences occurred [$F(3,60) = 83.7$, $p < .000001$], the experimental variables affected our subjects' DTs in similar ways. DTs were lowest for those spectra in which the second harmonic of tone 2 was present [$F(5,15) = 28.2$, $p < .00001$]. In general, DTs decreased with increasing beat frequency [$F(4,12) = 14.5$, $p < .005$], especially from 2 to 8 Hz. This can be seen in Figure 3, where DTs from the various spectral conditions are plotted as a function of beat frequency. For beat frequencies higher than 8 Hz, a further threshold decrement can be seen only for the odd and the odd + 2pth harmonics, whereas for the other conditions, the DTs remain the same or even tend to increase again for the beat frequency of 32 Hz [$F(20,60) = 3.45$, $p < .0005$].

Discussion

It was the purpose of Experiment 1 to test the hypothesis that for intervals with complex tones DTs are determined *mainly* by the sensitivity to those beats, which result from the interference of the first pair of just-noncoinciding harmonics with frequencies of pf_2 and qf_1 Hz. The results demonstrate that, for fifths, this is true: in conditions in which the p^{th} harmonic of tone 2 is present and can interfere with the q^{th} harmonic of tone 1, DTs are considerably lower than they are in conditions in which the p^{th} harmonic of tone 2 is absent. For the conditions in which the beat frequency is 32 Hz, corresponding to a mean tempering of about 60 cents, differences between the thresholds are strongly diminished. The DTs for the odd + $2p^{\text{th}}$ condition indicate that interference of the second pair of just-noncoinciding harmonics, with frequencies of $2pf_2$ and $2qf_1$ Hz, can also contribute to DT, since these DTs are also lower than the DTs for the spectra that comprise the first, or the odd, harmonics, the effect being more pronounced for the lower beat frequencies. Moreover, for beat frequencies of 2 and 4 Hz, DTs for spectra with odd + p^{th} and odd + $2p^{\text{th}}$ harmonics are about the same, suggesting that, within certain limits, beat frequency can compensate for a lower absolute level of interfering harmonics. DTs for spectra with only the first or the odd harmonics show that discrimination between pure and slightly tempered fifths is possible even *without* just-noncoinciding harmonics.

For beat frequencies higher than 8 Hz, further threshold decrements are found for two of the six spectral conditions. These decrements cannot be explained by an increased sensitivity to beats, since it has been shown that beat thresholds for complex tones in comparable $L_1 > L_2$ conditions (Vos, 1982) remain at about the same level for beat frequencies higher than 4 Hz. From experimental data on beats resulting from two interfering sinusoids or from sine-wave amplitude modulation of one sinusoid, even a threshold *increase* would be predicted for these beat frequencies (see Vos, 1982, for a review).

It is likely that the perception of interval size is responsible for the further decrement of the DT. For a performance criterion equal to that for the DTs, and for stimulus conditions in which ΔL is equal to 20 or 25 dB, mean thresholds at which the *direction* of tempering was consistently identified were found to be equal to 32 and 47 cents, respectively (Vos, 1982). These temperings correspond to beat frequencies of about 15 and 23 Hz, respectively. Considering these results, it is plausible to explain the decrement of the DTs for beat frequencies higher than or equal to 16 Hz as the result of the contribution of the perception of differences in interval size.

One could object that for the spectral conditions in which the p^{th} harmonic of tone 2 is present, perception of interval size should have caused a further

threshold decrement. Especially for $L_1 > L_2$ conditions, however, the higher tone is masked when level differences are greater than about 25 dB.

The condition in which only the first harmonic of the higher tone is present indicates that at least for small amounts of tempering, DTs can be determined by the sensitivity to beats, due to the combination of harmonics that are more than a critical band apart. It has already been observed by ter Kuile (1902) that beats can be heard for chords consisting of three tones with frequencies in the ratio of 2:(3-g):4, g representing a small amount by which the second tone is tempered. These beats are probably due to waveform variations (see Plomp, 1967). We cannot conclude from our experiment that these beats result from the combination of either two harmonics (with frequencies of f_1 and f_2 or f_2 and $2f_1$ Hz) or three harmonics (with frequencies of f_1 , f_2 , and $2f_1$ Hz).

Equal sensitivity to tempering for conditions that comprised only the p^{th} harmonic or both the odd and p^{th} harmonics suggests that when interference between just-noncoinciding harmonics takes place, added interference due to the combination of other harmonics does not really increase sensitivity to tempering.

EXPERIMENT 2

For the major third investigated here, the five spectral conditions presented are depicted in Figure 4. As in Figure 2, the first 12 harmonics of tone 1 and various harmonics of tone 2 are given on a linear frequency scale. The spectrum of tone 2 included the first harmonic, which ensured for four of the five conditions that the intervals would be perceived as major thirds. In the second spectral condition, comprising only the odd harmonics, interference between just-noncoinciding harmonics was not possible. Comparison of the DTs from the first and third conditions with those from the complete spectral condition should reveal the relative importance of the interference of the first pair of just-noncoinciding harmonics. Condition 4 was included to test whether, in addition to interference of the first pair of adjacent harmonics, sensitivity to tempering could be increased even further by simultaneous interference due to the presence of other even harmonics. Moreover, comparison of the DTs from Conditions 4 and 5 should reveal whether for a more complex spectral condition in which interference between various pairs of just-noncoinciding harmonics takes place, added interference due to the presence of odd harmonics would be able to increase sensitivity to tempering.

Method

Stimuli. The intervals were musical major thirds ($p=4$; $q=5$). Apart from this, the stimuli were the same as those in Experiment 1.

Figure 4. Schematic of the spectral conditions investigated in Experiment 2. The first 12 harmonics of tone 1, represented by squares, and various harmonics of tone 2, represented by circles, are given on a linear frequency scale. The just-noncoinciding harmonics are indicated by crosses.

Apparatus. The apparatus was identical to that of Experiment 1.

Subjects. Five musically trained subjects were tested over three sessions, each session on a different day. The subjects were paid for their services. None of them had participated in Experiment 1.

Experimental design. The independent variables were: (1) spectral content of tone 2 (the spectrum of tone 2 could comprise the following harmonics: p^{th} , odd, odd + p^{th} , first + even, and all); and (2) frequency difference $|pf_2 - qf_1|$ (2, 4, 8, 16, and 32 Hz). For each of the 25 different conditions, all of which were presented to five subjects, two DTs were determined.

Procedure. As in Experiment 1, the subjects had to discriminate between pure and tempered major thirds in a 2-AFC paradigm. Instead of the method of constant stimuli, however, DTs were determined by an adaptive procedure in which ΔL was increased by 2 dB after three correct responses and decreased by the same amount after an incorrect response. Each reversal of the direction of change in ΔL over successive trials was counted as a turnaround. The effective DT was taken as the mean value of ΔL on the trials that resulted in the last four turnarounds. This value estimates the value of ΔL required to produce 79.4% correct responses in a nonadaptive 2-AFC procedure (Levitt, 1971). The initial value, that is, the value of ΔL on the first "experimental" turnaround was determined as follows. In the first run, ΔL , being 0 dB at the very first trial, was increased by 3 dB after each correct response. After the first incorrect response, ΔL was decreased after each incorrect response and increased after two correct responses, in steps of 2 dB. The initial value equaled ΔL on the trial that resulted in the fourth turnaround. Thus, each DT was based on a block of eight runs (series of trials); the first four runs resulted in an adequate initial value, and the values of ΔL on the peaks and valleys of the last four runs provided an estimate of the DT.

A series consisted of five blocks, one for each frequency difference $|pf_2 - qf_1|$. Presentation order of the different blocks was randomized. Presentation order of spectrum, which was held constant within a series, was balanced according to two (5×5) Latin squares, one for each replication. At the beginning of a series, new waveforms of tones 1 and 2, with different ϕ_n values, were determined. Fundamental frequencies of the pure and tempered intervals were computed in the same way as in Experiment 1. The central frequency, f_c , of the pure interval, however, was equal to 370 Hz ± 25 , ± 75 , ± 125 , ± 175 , or ± 225 cents, in a random order. The direction of tempering (signs in Equations 4 and 5) was randomized. On trials in which ΔL was smaller than 10 dB, L_1 and L_2 were attenuated to a small extent to keep the overall level of the

intervals at about 85 dB SPL. Five series (25 blocks of trials) were completed during each session. The subjects were trained in the first session.

Results

In none of the conditions were there any systematic differences between the two replications. Therefore, DTs were subjected to an ANOVA [5 (subjects) $\times$ 5 (spectra of tone 2) $\times$ 5 (frequency difference), all repeated measures], with replication as a within-cell variable. Two subjects had significantly lower thresholds than the other three subjects [$F(4,125) = 9.17$, $p < .00005$].

DTs were lowest for those spectra in which at least the even harmonics were present and highest for the spectrum in which only odd harmonics were present [$F(4,16) = 58.9$, $p < .000001$]. A Newman-Keuls paired comparison test showed that except for the two conditions which rendered the lowest thresholds (largest values of ΔL), the DTs for the other spectra were significantly different at the .05 level or better. In general, DTs decreased with increasing beat frequency [$F(4,16) = 9.65$, $p < .0005$]. However, this effect was most pronounced for those conditions in which tone 2 contained only the odd, odd and p^{th} , or p^{th} harmonics [$F(16,64) = 2.44$, $p < .006$]. This is shown in Figure 5, where DTs for the various spectra are plotted as a function of beat frequency.

Discussion

Relative to the condition in which the spectrum of tone 2 consists of only the odd harmonics, sensitivity to tempering is higher when the p^{th} harmonic of tone 2 is present or added to the odd harmonics. However, our data suggest that since the DTs from conditions in which only the p^{th} harmonic or the odd and p^{th} harmonics are present are not as low as the

Figure 5. Thresholds for discrimination between pure and tempered major thirds, plotted as a function of beat frequency, for various spectral conditions. Pth refers to the fourth harmonic. The spectrum of tone 1 always comprised the first 20 harmonics. For both tone 1 and tone 2, waveforms were determined using Equation 1.

conditions in which all the harmonics are present, interference between other harmonics must have played a role as well. Given that the conditions with the first and even harmonics and those with all harmonics rendered similar DTs, we must conclude that in addition to the pth harmonic, other even harmonics also contribute significantly to discrimination, especially for the lower beat frequencies. One of these even harmonics responsible for the low DTs might be the 2pth harmonic. Results from Experiment 4, however, which were obtained before those of Experiment 3, suggested that the second and sixth harmonics of the higher tone may also have influenced the DTs. Most likely, the contribution of the second harmonic of the higher tone to the DT is important and is a result of the interference between three harmonics with frequencies of $2f_1$, $2f_2$, and $3f_1$ Hz, respectively. This is supported by observations by ter Kuile (1902), who found that beats for chords consisting of three tones with frequencies in

the ratio of 4:(5-g):6 could easily be heard, even when the tuning forks were struck weakly.

In accordance with the results from Experiment 1, the DTs for the spectrum with only the odd harmonics show that discrimination between pure and slightly tempered major thirds is possible *without* just-noncoinciding harmonics. However, a pilot experiment revealed that, in contrast to DTs for fifths, those for major thirds in which the higher tone contained only the first harmonic could, in general, not be determined. This finding agrees with experimental data for sinusoidal tones (Plomp, 1967), which show that beats for slightly tempered major thirds with a frequency of the lower tone higher than or equal to 250 Hz could not be heard by all subjects.

For the various spectral conditions in which the beat frequency was 32 Hz, which corresponds to a mean tempering of about 34 cents, differences between the DTs were diminished. The same mechanism put forward to explain further threshold decrements for the fifth might have contributed to the DTs for the major thirds. For a performance criterion equal to that for the DTs, and for stimulus conditions in which ΔL is equal to 0 or 20 dB, mean thresholds at which the *direction* of tempering was consistently identified were found to be equal to 28 and 30 cents, respectively (Vos, 1982).

EXPERIMENT 3

We have demonstrated that DTs for moderately tempered fifths are determined mainly by the sensitivity to beats that result from the interference of the first pair of just-noncoinciding harmonics. For the major thirds, however, the data suggest that interference of other even harmonics must have played a role as well. Although the second pair of just-noncoinciding harmonics can be considered to be the second best contributor, results from Experiment 4 show that when both the pth and 2pth harmonics are removed, the perceived strength of the beats for tempered major thirds remains relatively high. Therefore, Experiment 3 was designed to determine the relative contributions of various even harmonics, either on their own or in combination, to discrimination. To provide a baseline, all spectra of the higher tone included the odd harmonics. For reference, the spectral condition that comprised all harmonics was investigated as well.

Method

Stimuli. The stimuli were the same as those in Experiment 2. The duration of the tone bursts, however, was 1 sec.

Apparatus. The apparatus was identical to that of Experiment 1.

Subjects. Eight trained listeners were tested over three sessions, each session on a different day. Seven of them had not participated in any of the other experiments reported in this paper. The author also served as a subject.

Experimental design. The independent variables were: (1) spectral content of tone 2 [the spectrum of tone 2 could comprise the following harmonics: odd, odd and second, odd and p^{th} , odd and sixth, odd and $2p^{\text{th}}$, odd and second + p^{th} , odd and p^{th} + $2p^{\text{th}}$, and all]; and (2) frequency difference $|pf_2 - qf_1|$ (2 and 4 Hz).

Procedure. In general, the procedure was similar to that of Experiment 2. The adaptive procedure by which DTs were determined was slightly different from that used in Experiment 2. Again, ΔL was increased by 2 dB after three correct responses and decreased by the same amount after an incorrect response. Each reversal of the direction of change in ΔL over successive trials was counted as a turnaround. There were eight turnarounds. In the first run, however, ΔL was increased by 3 dB after each correct response. The effective DT was taken as the mean value of ΔL on the trials that resulted in the last six turnarounds. A series comprised two blocks, one for each frequency difference presented. Presentation order of the different blocks was randomized. Presentation of spectrum, which was held constant within a series, was balanced according to an 8×8 Latin square.

Results and Discussion

For all spectral conditions, thresholds are plotted in Figure 6 as a function of beat frequency. As can be seen in this figure, sensitivity to tempering was highly dependent on the spectral content of the higher tone [$F(7,49) = 37.9$, $p < .00001$], and it increased with increasing beat frequency [$F(1,7) = 72.1$, $p < .0005$]. Thresholds from five of the eight spectral conditions were similarly influenced by beat frequency. As discussed below, the divergence of the DTs caused by beat frequency for the three conditions in which, in addition to the odd harmonics, the higher tone comprised the p^{th} , p^{th} and $2p^{\text{th}}$, or second and p^{th} harmonics is rather vital to our proposed model. Two Newman-Keuls paired comparison tests, one for each beat frequency, showed that many DTs were significantly different from each other. In fact, the size of the effects was such that both tests revealed that the DTs from the condition in which the higher tone comprised the odd and p^{th} harmonics were significantly different ($\alpha = .01$) from those in which this tone comprised the odd + sixth or all harmonics.

Apart from being more complicated than for the fifth, the results are straightforward and can be summarized as follows: (1) While the contribution of the sixth harmonic to the DT is negligibly small, that of the second harmonic is substantial; however, (2) both the p^{th} and the $2p^{\text{th}}$ harmonics contribute more to the DT than does the second harmonic. (3) For the beat frequency of 4 Hz, the DTs from the spectral conditions in which, in addition to the odd harmonics, either the p^{th} and $2p^{\text{th}}$ or the second and p^{th} harmonics have been included suggest that, for major thirds, the lowest DTs are determined by the combined effect of different interfering sources. This model of additivity of interference is consistent with the results of Experiment 2, which showed that only when all relevant even harmonics are included are the DTs as low as those for the condition that contained all harmonics.

Figure 6. Thresholds for discrimination between pure and tempered major thirds, plotted as a function of beat frequency, for various spectral conditions. The spectrum of tone 1 always comprised the first 20 harmonics. For both tone 1 and tone 2, waveforms were determined using Equation 1.

EXPERIMENT 4

The purpose of this experiment was twofold: (1) to investigate whether for tempered intervals the beat frequency of the first pair of adjacent harmonics is equal to the *perceived* beat frequency, and (2) to determine to what extent differences in discriminability at threshold, as found in Experiments 1, 2, and 3, are representative of differences between the perceived strength of the beats in supraliminal conditions. Therefore, both the most dominantly perceived beat frequency and the perceived strength of these beats were determined for various spectral conditions and various amounts of tempering. With respect to the choice of the spectral contents, the presence or absence of the p^{th} and $2p^{\text{th}}$ harmonics was emphasized. Rather than count the beats and estimate the magnitudes of their strengths, it seemed preferable to imitate the rates and the strengths of the beats. This was achieved by adjusting the frequency and the level of a sinusoidal tone relative to the second sinusoidal tone with a fixed frequency of f_2 Hz and a fixed level.

Method

Stimuli. The intervals were major thirds ($p=4$; $q=5$) and fifths ($p=2$; $q=3$). Again, the low tone (tone 1) consisted of the first 20 harmonics, and the spectral content of the high tone (tone 2) was varied. The amplitudes, a_n , of the respective harmonics were equal to $1/n$. Sound-pressure levels of tones 1 and 2 were fixed in such a way that ΔL equaled 0 dB. The overall level of the two tones together was 85 dB SPL. In this experiment, presentation of the complex-tone intervals was alternated with presentation of two simultaneous sinusoidal tones as a comparison sound. To accomplish equal loudness for the complex-tone and sinusoidal-tone intervals, the overall level of the two sinusoids, which had different frequencies, was set at 89 dB SPL. The duration of both the complex tones and the sinusoidal tones was 1 sec. Rise and decay times of the tones were 30 and 20 msec, respectively.

Apparatus. The apparatus was identical to that of Experiment 1. In addition, two potentiometers with blinded knobs were used. With the first potentiometer, the level of one of the sinusoidal tones could be controlled. The frequency of the same tone could be varied by means of the second potentiometer. The voltage across each potentiometer was read out by an analog-to-digital converter and transformed into the appropriate frequency or level measure.

Subjects. Five subjects were tested over four sessions, each session on a different day. Three of the subjects had participated in Experiment 1.

Experimental design. The independent variables were: (1) interval (fifth and major third); (2) spectral content of tone 2 (the spectra could comprise the following harmonics: odd, odd and $2p^{\text{th}}$, odd and p^{th} , all, and all minus p^{th} and $2p^{\text{th}}$; and (3) frequency difference ($pf_2 - qf_1$) (2, 3, 4, 5, and 6 Hz). The stimulus combinations were presented four times to the five subjects. There were two dependent variables: (1) the adjusted beat frequency of the two interfering sinusoidal tones, and (2) the level variation depth of the two interfering sinusoids.

Procedure. The subjects were instructed to listen for beats in the intervals. If beats were perceived, they were to focus on two aspects of these beats: (1) the most dominant beat frequency, and (2) the subjectively experienced strength of the beats. The first part of their task was to imitate the rate of the beats in the musical intervals by adjusting the frequency of one of the sinusoidal tones. The other sinusoidal tone had a constant frequency of f_1 Hz (in a pilot investigation it had been found to be most natural to fix the tone at this frequency). The frequency of the variable tone was equal to f_2 Hz only at the beginning of a trial. During the adjustment, the complex-tone intervals and the sinusoidal-tone intervals were presented alternately in a long sequence. The time interval between the successive tone bursts was 700 msec. The second part of the task was to imitate the strength of the beats by adjusting the level of one of the sinusoidal tones. The level of the variable sinusoid with amplitude a_2 could only be decreased, relative to the level of the fixed tone with amplitude a_1 ($a_2 \leq a_1$). The level variation depth D , in decibels, of the beating sinusoids is given by Equation 2.

When the subject was satisfied that the beats of the two interfering sinusoidal tones reflected the beats of the two interfering complex tones in both respects, he pressed a ready-button. If the subject felt that perception of the beats of the complex-tone interval was hindered by the sinusoidal tones, he had the option of skipping the presentation of the sinusoidal tones. Presentation of the next trial was postponed until both knobs had been returned to their starting positions, that is, to the state that resulted in a frequency difference of 0 Hz and a level difference of 0 dB. The sensitivity of the potentiometers (ΔL or Δf per degree of rotation), however, was varied from trial to trial. Therefore, the maximum frequency difference and the maximum value of ΔL ranged from 20.5 to 32.0 Hz and from 25 to 36 dB, respectively. During the experiment, these extreme values were never used.

A series comprised 25 experimental trials (5 spectra $\times$ 5 beat frequencies) and 5 catch trials, in which pure intervals, one for each

spectrum, were presented. Since all pure intervals were immediately recognized by our subjects, these data will be dropped from further analysis. Within a series, which lasted about $\frac{1}{2}$ h, stimuli were presented in a random order. The musical interval was varied between series. The different intervals were presented alternately; three subjects started with the musical fifth, and two started with the major third. At the beginning of a series, new waveforms of tones 1 and 2, with different ϕ_n values, were determined. The fundamental frequency f_1 of the tempered musical interval was computed by means of Equation 3, with f_c always at 370 Hz. The fundamental frequency f_2 was computed using Equation 5. The tempered intervals were always stretched. During the first session, subjects received two short training series, one for each interval. If the level difference between the two sinusoids was below 10 dB, the levels of both tones were attenuated to a small extent in order to keep the overall level of the tones at 89 dB SPL.

Results

Beat frequency. For all conditions, the adjusted frequency differences between the two sinusoidal tones, representing the frequencies of those beats that were most dominantly perceived, are divided by their corresponding "theoretical" beat frequencies of $(pf_2 - qf_1)$ Hz. The resulting fractions were subjected to an ANOVA [5 (subjects) $\times$ 2 (tempered fifth and major third) $\times$ 5 (spectra of tone 2) $\times$ 5 (frequency difference), all repeated measures]. Although individual differences occurred [$F(4,750) = 5.0$, $p < .001$], a number of experimental variables affected our subjects' adjusted beat frequencies in similar ways. The spectrum with only the odd harmonics rendered lower mean beat frequencies than did the other spectra [$F(4,16) = 6.0$, $p < .005$]. This effect, however, was more prominent for the major third than for the fifth [$F(4,16) = 3.6$, $p < .03$]. Mean adjusted beat frequencies were relatively higher in the conditions in which $(pf_2 - qf_1)$ equals 2 Hz [$F(4,16) = 19.4$, $p < .00005$]. For both the fifth and the major third, this effect was most prominent for the two spectral conditions in which only the odd harmonics, or the odd and $2p^{\text{th}}$ harmonics were present [$F(16,64) = 2.6$, $p < .005$]. The interaction effect is depicted in Figure 7, where the adjusted beat frequency for the different spectra is plotted as a function of the amount of tempering expressed in terms of the theoretical beat frequency of $(pf_2 - qf_1)$ Hz.

It is interesting to look at the distributions of the relative adjusted beat frequencies, because, depending on spectral condition, peaks are expected at .0, 1.0, and 2.0 times the beat frequency of $(pf_2 - qf_1)$ Hz: (1) spectral conditions in which beats can be only faintly perceived should render a high proportion of trials in which no beats at all are heard, (2) spectral conditions in which the p^{th} harmonic is present might render fractions with values around 1 most of the time, and (3) spectral conditions in which, in addition to the odd harmonics, the $2p^{\text{th}}$ rather than the p^{th} harmonic is given, might render more fractions with values around 2 than the other conditions. For the fifth and the major third, and for all spectra sep-

Figure 7. Adjusted beat frequency, plotted as a function of the amount of tempering of musical intervals with fundamental frequencies f_1 and f_2 ($f_1:f_2 \sim p:q$). Data are given for the five spectra of the higher tone separately, and are averaged across the fifth ($p=2$; $q=3$) and the major third ($p=4$; $q=5$).

arately, grouped frequency distributions are given in Figure 8. For all histograms (the class-interval size is equal to .25), midpoints of the class intervals are indicated. The shapes of the distributions are clearly unimodal for spectral Conditions C and D, which contained the p^{th} harmonic. To some extent, the distributions are positively skewed for spectrum B, where the $2p^{\text{th}}$ harmonic may have determined the most dominantly perceived beat rate in some of the

trials. The shapes of the distributions tend to be bimodal for the fifth in Conditions A and E. Spectrally, these conditions are similar up to the sixth harmonic of the higher tone. For the major third, relative beat frequencies are clearly bimodally distributed in Condition A and to a smaller extent in Conditions B and E. In Condition A, the number of times that no beats were perceived at all exceeded those of Condition E by more than 20%.

Level variation depth. The adjusted level variation depth, D , representing the perceived strength of the beats, was subjected to an ANOVA with a design similar to that used for the adjusted beat frequency. The effect of subjects, tested against within-cell variance, was significant [$F(4,750) = 49.8$, $p < .00001$]. Perceived strength of the beats for the fifths was 1 dB higher than for the major thirds [$F(1,4) = 12.2$, $p < .02$]. The spectral content of the higher tone affected the perceived strength of the beats considerably [$F(4,16) = 21.5$, $p < .00003$]. Mean perceived strength of the beats is presented in Figure 9, for each spectrum and for each musical interval separately. Perceived strength of the beats for fifths and major thirds was the same for only the spectrum in which the p^{th} and $2p^{\text{th}}$ harmonic had been removed [$F(4,16) = 3.9$, $p < .02$]. For four of the five spectra, perceived beat strength did not depend on beat frequency. For the spectrum that comprised the odd and p^{th} harmonics, there was a tendency to judge the beats as stronger for beat frequencies of 3 and 4 Hz [$F(16,64) = 1.8$, $p < .05$]. For tempered fifths, a Newman-Keuls paired comparison test showed that perceived strength of beats was significantly higher

Figure 8. Grouped frequency distributions of the relative adjusted beat frequency. This frequency corresponds to that of the most dominantly perceived beats of tempered fifths (upper panels) and tempered major thirds (lower panels). Distributions are given for five different spectral conditions in panels A, B, C, D, and E, respectively. For each label, the corresponding spectral content of the higher tone is indicated. For the fifth and the major third, the values of p are 2 and 4, respectively.

Figure 9. Adjusted level variation depth of two beating sinusoidal tones, which corresponds to the perceived strengths of the beats of tempered fifths ($p=2$) and major thirds ($p=4$). Mean values are given for five different spectral conditions, labeled A, B, C, D, and E, separately. Spectral conditions are specified at the top of the figure.

for the two spectra that contained the p^{th} harmonic than for the other spectra ($\alpha = .01$). For the major thirds, a similar test revealed that the perceived strength of beats was even higher for the spectrum from which the p^{th} and $2p^{\text{th}}$ harmonic had been removed than for the spectra that comprised the odd or the odd and $2p^{\text{th}}$ harmonics ($\alpha = .05$).

One could object that differences between the mean adjusted depth of the beating sinusoidal tones are due to the fact that the trials in which no beats are perceived at all are unevenly distributed over the different spectral conditions, as summarized in Figure 8. It has been verified that essentially the same pattern of results is obtained when the data for only those trials in which beats *were* perceived are taken into account. For example, if we combine all values obtained with relative adjusted beat frequencies between .75 and 1.25 Hz, the mean shifts in the patterns for the fifth and major third, as depicted in Figure 9, equal .20 and .26 dB, and standard deviations are .1 and .24 dB, respectively.

Discussion

One of the purposes of Experiment 4 was to ascertain to what extent the beat frequency of $(pf_2 - qf_1)$ Hz represents the *perceived* frequency of the beats. This was a relevant question because acoustically it can be shown (see Vos, 1982) that for complex tones interference of different pairs of just-noncoinciding harmonics results in a harmonic series of beat frequencies.

From the histograms given in Figure 8, it can be concluded that, for complete spectra of the tones

(Condition D), the most dominantly perceived beat frequency is $(pf_2 - qf_1)$ Hz. Moreover, *without* interfering pairs of just-noncoinciding harmonics, as in Condition A, beats have been perceived in about 80% of the cases with, especially for the fifth, a well-defined frequency of $(pf_2 - qf_1)$ Hz. Relative to Condition D, however, the strength of these beats was much smaller. For both the fifth and the major third, the significance of the interference of the first pair of just-noncoinciding harmonics is demonstrated by the data from Condition C, in which the p^{th} harmonic had been added to the odd harmonics. The results show that (1) the relative perceived beat frequency is 1 Hz or close to 1 Hz, and (2) that the perceived strength of the beats equals that found in the complete spectral conditions. In Condition B, in which the higher tone contained the odd plus $2p^{\text{th}}$ harmonics, dominant beats with a frequency of $2(pf_2 - qf_1)$ Hz were observed in only a few cases. In this condition, the mean perceived strength of the beats is only slightly higher than in Condition A. When the p^{th} and $2p^{\text{th}}$ harmonics are removed (Condition E), the perceived beat strength for tempered fifths is equal to that in Condition A. This suggests that, apart from the second pair, interference of higher pairs of just-noncoinciding harmonics does not contribute to the perceived strength of the beats. Most likely, this also holds for the major third. It can be assumed that, relative to Conditions A and B, the significantly higher strength of the beats in Condition E is the result of the interference of the *second* harmonic of the higher tone with the second and third harmonics of the lower tone. For discrimination, the relevance of the same mechanism is demonstrated in Experiment 3.

In four of the five conditions, the perceived strength of beats is 1 dB higher for tempered fifths than for tempered major thirds. From Figure 1, where the plotted DTs are also expressed in terms of level variation depth, it can be seen that major thirds are less sensitive to tempering than are fifths. Moreover, these differences increase from .8 to 2.3 dB for tone durations decreasing from 1,000 to 250 msec. For a similar tone duration of 1 sec and for beat frequencies within a range of 1 to 8 Hz, this difference is .5 dB. These corresponding effects of musical interval once more suggest that DTs are mainly determined by sensitivity to beats.

GENERAL CONCLUSIONS

From Experiment 1, we may conclude (1) that for fifths consisting of two complex tones with spectral envelope slopes of -6 dB/octave sensitivity to tempering is determined mainly by the interference of the *first* pair of just-noncoinciding harmonics. The re-

sults from Experiment 4 indicate (2) that the dominantly perceived beat frequency of these tempered fifths is $(pf_2 - qf_1)$ Hz. These two facts lead us to conclude (3) that the thresholds for the discrimination between pure and tempered fifths, as given in Figure 1, are also adequately represented from a perceptual point of view.

The data from Experiment 4 show (4) that for the major thirds the dominantly perceived beat frequency is also equal to $(pf_2 - qf_1)$ Hz. From Experiments 2 and 3, we may conclude (5) that, for major thirds, discrimination is *partly* determined by the interference of the first pair of just-noncoinciding harmonics. The results from Experiment 3 suggest (6) that, for major thirds, DTs are determined by the combined effect of different interfering sources. Moreover, thresholds from Experiment 3 revealed (7) that, in addition to interference from just-noncoinciding harmonics, interference due to the combination of the second harmonic of the higher tone with harmonics of the lower tone plays a role as well. With respect to the ordinate, it is therefore (8) not justified to apply the third conclusion to this interval.

Restricting ourselves to corresponding conditions and the same range of beat frequencies, the spectral content of fifths had, qualitatively speaking, the same effect on sensitivity to tempering as on perceived strength of the beats. This also held for the major third in the two extreme conditions in which the higher tone comprised the odd or all harmonics, and in the condition in which both the p^{th} and the $2p^{\text{th}}$ harmonics were deleted. However, the large difference between the beat strengths found for the conditions in which either the $2p^{\text{th}}$ or the p^{th} harmonics were added to the odd harmonics was not predicted by the discrimination thresholds.

For all spectral conditions, the beats perceived as dominant usually had a frequency equal to that of the first pair of just-noncoinciding harmonics. Perceptual dominance of beats resulting from a specific source of interference is in agreement with the fact that beats are a reliable tool for tuning various musical instruments. This finding does not mean that, under certain conditions and when deliberately scrutinized, beats resulting from other sources of interference cannot be perceived.

The relatively important contribution of the second harmonic of the higher tone of the major third suggests that, for other musical intervals, harmonics different from just-noncoinciding harmonics may also play a role in discrimination and perceived strength of beats. This may be true for the second harmonic of the higher tone of the fourth ($p=3$; $q=4$) and the major sixth ($p=3$; $q=5$) and for the second, third, and fourth harmonics of the higher tone of the minor third ($p=5$; $q=6$) and the minor

sixth ($p=5$; $q=8$). This hypothesis should be tested in future research. It is also interesting to determine to what extent our conclusions are independent of the spectral envelope slopes of the tones.

REFERENCES

- BARNES, W. H. (1959). *The contemporary American organ* (pp. 95-101), Glen Rock, NJ: Fisher.
- ELLERHORST, W. (1936). *Handbuch der Orgelkunde*. Einsiedeln, Switzerland: Benziger.
- ELLISTON, T. (1916). *Organs and tuning*. London: Weekes.
- HELMHOLTZ, H. L. F. VON. (1877). *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (4th ed.). Braunschweig: Vieweg. [On the sensations of tone as a physiological basis for the theory of music (A. J. Ellis, Trans.). New York: Dover, 1954.]
- KAMEOKA, A., & KURIYAGAWA, M. (1969a). Consonance theory, Part II: Consonance of complex tones and its calculation method. *Journal of the Acoustical Society of America*, **45**, 1460-1469.
- KAMEOKA, A., & KURIYAGAWA, M. (1969b). Consonance theory, Part I: Consonance of dyads. *Journal of the Acoustical Society of America*, **45**, 1451-1459.
- KLING, J. W., & RIGGS, L. A. (1972). *Woodworth and Schlosberg's experimental psychology* (3rd ed.). New York: Holt, Rinehart and Winston.
- KUILE, TH. E. TER (1902). Einfluss der Phasen auf die Klangfarbe. *Archiv für die gesamte Physiologie*, **89**, 332-426.
- LEVITT, H. (1971). Transformed up-down methods in psychoacoustics. *Journal of the Acoustical Society of America*, **49**, 467-477.
- PIERCE, J. R. (1966). Attaining consonance in arbitrary scales. *Journal of the Acoustical Society of America*, **40**, 249.
- PLOMP, R. (1967). Beats of mistuned consonances. *Journal of the Acoustical Society of America*, **42**, 462-474.
- PLOMP, R. (1976). *Aspects of tone sensation*. London: Academic Press.
- PLOMP, R., & LEVELT, W. J. M. (1965). Tonal consonance and critical bandwidth. *Journal of the Acoustical Society of America*, **38**, 548-560.
- RAYLEIGH, LORD (1966). On the determination of absolute pitch by the common harmonium. In *Proceedings of the Musical Association* (Vol. 5; pp. 15-26). Vaduz, Liechtenstein: Kraus Reprint. (Original work published 1878-1879).
- SCHEIBLER, H. (1834). *Der physikalische und musikalische Tonmesser*. Essen: Bädeker.
- SCHEIBLER, H. (1838). Über mathematische Stimmung, Temperaturen und Orgelstimmung nach Vibrations-Differenzen oder Stößen. In *Schriften über musikalische und physikalische Tonmessungen und deren Anwendung auf Pianoforte- und Orgelstimmung*. Krefeld.
- SLAYMAKER, F. H. (1970). Chords from tones having stretched partials. *Journal of the Acoustical Society of America*, **47**, 1569-1571.
- SMITH, R. (1966). *Harmonics or the philosophy of musical sounds* (1st ed.). New York: Da Capo Press, 1966. (Original work published 1749)
- VOS, J. (1982). The perception of pure and mistuned musical fifths and major thirds: Thresholds for discrimination, beats, and identification. *Perception & Psychophysics*, **32**, 297-313.
- WARREN, R. M. (1978). Complex beats. *Journal of the Acoustical Society of America*, **64**, Suppl. 1, S38.

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