A "C" Program to Fit Herrnstein's Hyperbola Using Wilkinson's Method

The law of effect, which is a central premise of behavioral theory and the basis for much of behavior therapy relates behavior to its reinforcing consequences. Herrnstein's (1970) mathematical statement of this law is:

$$R = kr/(r + r_e), \quad (1)$$

where R is the rate of responding for a behavior, r is the rate of contingent reinforcement for that behavior, k is the maximal rate of responding attainable by the organism, and r_e is all (extraneous) reinforcement concurrently delivered to the organism exclusive of r . Also known as the matching law, this model has received widespread support over three decades of animal research (see Davison & McCarthy, 1988) and has more recently attracted interest within applied behavior analysis (see, e.g., Martens & Houk, 1989).

Fitting Herrnstein's hyperbola to data is a focal part of the methodology in this area of research. It is generally accomplished with Wilkinson's (1961) method. This is a two-step procedure for estimating the parameters of a hyperbolic function: (1) initial estimates of k and r_e are obtained by a weighted least squares regression of $1/R$ on $1/r$, where R is response rate maintained by the reinforcement rate, r ; (2) the initial estimates are then revised by fitting the bilinear equation, $R = b_1 f(r) + b_2 f'_e(r)$, where $f(r)$ is Herrnstein's hyperbola with k and r_e equal to their initial estimates, and $f'_e(r)$ is the first derivative of $f(r)$ with respect to r_e (see McDowell, 1981, for a detailed explanation).

Wilkinson's method of fitting Herrnstein's hyperbola has been available as a BASIC program (McDowell, 1981). We have written a program in C to estimate the parameters of Herrnstein's hyperbola with Wilkinson's two-step method. Unlike the earlier BASIC version, the present program does not limit the number of data pairs that may be entered. In addition, the current version provides a much better user environment. The program can also be converted into other high-level languages such as Pascal. The program outputs estimated values of k and r_e , their standard errors, associated error bars (parameter ± 1 SE), percentage of variance accounted for, and coordinates for plotting the fitted hyperbola. The second step of Wilkinson's method is repeated until no further increase in percentage of variance accounted for is obtained. The results are placed into a file named WILKSON.OUT, which is an appropriately delimited ASCII file. This format is imported into Borland's QuattroPro for final presentation and graphing, although it can also be imported into most spreadsheets. The presentation is handled by a macro file named QUATRO.WQ1, which should be stored with the WILKSON program. It should be noted that Herrnstein's hyperbola is constrained to pass through the origin and that any nonzero intercept will therefore diminish the hyperbolic fit. Martens and Houk (1989) have suggested transforming the data in such cases

by a constant before fitting the hyperbola. This procedure can be adopted before using the present program, but with the caveat that there are theoretical ramifications for correcting nonzero y intercepts.

The source code and a compiled version of the program are available on a 5.25- or 3.5-in. diskette (along with user instructions), for \$10. Correspondence should be addressed to Ephrem Fernandez, Psychology Dept., University of Queensland, Queensland, Australia 4072 (email: efenez@psych.psy.uq.oz.au).

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R2: A Computer Program for Interval Estimation, Power Calculations, Sample Size Estimation, and Hypothesis Testing in Multiple Regression

Although multiple correlation coefficients can be computed with all major statistical packages, these packages do not offer a full range of inferential procedures for multiple correlation. Most notable by its absence is a confidence interval on the population coefficient of determination, P^2 .

Exact confidence intervals for P^2 , though in principle simple enough to obtain, require several iterations, each in turn requiring calculations of the cumulative distribution function of the sample squared multiple correlation, R^2 . This calculation is difficult to perform both quickly and reliably. Numerical routines must also be constructed very carefully so that they will remain stable under a variety of parameter choices. Perhaps this explains why this and several related procedures involving P^2 have not yet achieved popularity.

R2 is a simple menu-driven computer program that provides several useful procedures involving P^2 .