— METHODS & DESIGNS —

Computing data for color-vision modeling

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The program PCSF uses published mathematical equations that approximate color-sensitivity functions for computing data for simulations of color vision. It can compute tables of receptor absorptions, ocular media density, fundamental response and color-matching functions, and chromaticity coordinates. It also may be used to evaluate spectral-radiance data with respect to these sensitivity functions. The source code is available and may easily be incorporated into other programs.

Color-vision modeling usually starts from color matching functions. A set of such functions has been standardized by the CIE for the CIE 1931 Standard Colorimetric System and may be found tabulated in several sources (e.g., Wyszecki & Stiles, 1982). The CIE color-matching functions include $V(\lambda)$, the luminous efficiency function for the CIE 1924 standard observer. Later data showed deficiencies of the original $V(\lambda)$ data and led to a revision by Judd (1951, cited in Wyszecki & Stiles, 1982), which now is generally referred to as the "Judd 1951 modified $V(\lambda)$." Judd's modification of $V(\lambda)$ also led to a modification of the CIE 1931 standard observer color-matching function. This new system is called the "Judd 1951 modified" CIE standard observer and has been the basis of most work in color-vision theory involving color-matching functions.

An example is the derivation of cone-fundamental-sensitivity functions by Smith and Pokorny (1975). This derivation is based on color- and brightness-matching data of dichromats and the hypothesis of reduction dichromacy. The Smith and Pokorny results found rather strong confirmation by recent results of Baylor, Nunn, and Schnapf (1987), who were able to directly measure response functions of primate cones. They found a close similarity between their physiological measurements of the cones' spectral sensitivities with the psychophysically derived fundamental sensitivities of Smith and Pokorny. This comparison had to take into account the optical density of the lens and macular pigmentation, since these modify absorption (Pokorny & Smith, 1986).

All the spectral functions mentioned may be found tabulated somewhere. Computer simulations of color-vision models, however, are easier to program with functional

descriptions of the sensitivity data. Examples are Brainard and Wandell's Color Analysis Package (cited in Wandell, 1987), which used discrete tables of the Smith and Pokorny (1975) sensitivity functions to analyze the colors of complex surfaces, and Siminoff (1991), who did a simulation of lateral interactions of color-selective cells in the human retina. He used piecewise polynomial approximations of sensitivity functions.

PCSF contains modules that may be used for similar purposes. They are based on functional descriptions of the Smith and Pokorny (1975) fundamentals, which have been derived by Boynton and Wisowaty (1980), and of the cone-spectral-sensitivity functions of Baylor et al. (1987). The program may be used as a stand-alone program for computing and printing the built-in functions, or its single modules may be incorporated into other programs. The following functions are provided:

1. The Smith and Pokorny (1975) long- (LWS), middle- (MWS), and short-wave (SWS) sensitive cone spectra according to the equations derived by Boynton and Wisowaty (1980) (Figure 1A).
2. The Judd 1951 modified $V(\lambda)$ function, which is the sum of the LWS and the MWS functions.
3. A smooth, combined optical-media-absorption function based on an equation also derived by Boynton and Wisowaty (1980) (Figure 1B).
4. LWS, MWS, and SWS cone sensitivities, according to an equation derived by Baylor et al. (1987) (Figure 1C).
5. The Baylor et al. (1987) cone sensitivities corrected for absorption by the optical-media-absorption function.
6. The Judd 1951 modified CIE color-matching functions (Figure 1D).
7. Chromaticity coordinates for the Judd 1951 modified CIE chromaticity diagram.
8. Spectral coordinates for the cone-excitation diagram proposed by MacLeod and Boynton (1979).

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EQUATIONS

The equations developed by Boynton and Wisowaty (1980) are based on polynomial powers. The following shorthand notation for a polynomial power of the variable x with degree k and coefficients $a_0, \dots, a_k$ will simplify notation:

$$P(x, a, k) = \text{pow}_{10} \left(\sum_{i=0}^k a_i x^i \right),$$

with $\text{pow}_{10}(x) = 10^x$. The equation for computing the MWS (green) fundamental is given as a function of wavelength λ by

$$G(\lambda) = [P(\lambda, a, 4)^n + P(\lambda, b, 4)^n]^{1/n}. \quad (1)$$

The equations for computing the LWS and SWS (red and blue) fundamentals are more complicated, but both have the same form. They are not given as functions of wavelength but as quantum sensitivity of frequency. The general equation is

$$Q(\nu) = \{[P(\nu, a, 3)^{n_1} + P(\nu, b, 3)^{n_1}]^{n_2/n_1} + P(\nu, c, 3)^{n_2}\}^{1/n_2}. \quad (2)$$

The energy based sensitivity functions $R(\lambda)$ and $B(\lambda)$ may be obtained from quantum sensitivity by substituting $\nu = 2.998/\lambda$ and setting them equal to the expression¹

$$Q(\lambda) \frac{\lambda}{555}, \quad (3)$$

with appropriate parameter values a_i, b_i, c_i for $i = 0, \dots, 3$.

The values of all the coefficients and powers of Equations 1, 2, and 3 are given in Boynton and Wisowaty (1980, p. 1472, Table 1) or Pokorny and Smith (1986, p. 8-37, Table 8.16). Boynton and Wisowaty (1980) also give printed tables of the functions R, G , and B , which may be used to validate the correctness of PCSF's output. Note, however, that PCSF's output for the SWS fundamental is scaled differently from Boynton and Wisowaty's Table 2. They choose to make $B(\lambda)$ identical to the $\bar{z}(\lambda)$ function of the Judd 1951 modified CIE color-matching functions. The default output of PCSF follows Smith and Pokorny (1975) more closely and uses a scale that approximately preserves the relative sensitivity between cone types. PCSF will print the original Boynton and Wisowaty (1980) blue fundamental if the option "u" is given.

Figure 1. Graphical displays of tables produced by PCSF. (A) Output resulting from the option line "-rgbl." These are the Smith and Pokorny (1975) fundamental-sensitivity functions on a logarithmic scale. (B) Output from "-m," the smooth optical media density function. (C) Output of "-DPTI," the Baylor et al. (1987) cone-sensitivity functions on a logarithmic scale. (D) Output of "-xyz," the Judd 1951 modified CIE color-matching functions $\bar{x}$, $\bar{y}$, and $\bar{z}$.

The smooth optical-media-absorption function is also given as a function of frequency:²

$$M(v) = \{[(P(v,a,3)^{n_1} + P(v,b,3)^{n_1})^{n_2/n_1} + P(v,c,3)^{n_2}]^{n_3/n_2} + P(v,d,3)^{n_3}\}^{1/n_3}. \quad (4)$$

The coefficients and the powers of the polynomials may be found in Boynton and Wisowaty (1980). Note that $\log_{10}$ values of absorption result in optical density. This may be printed by PCSF with the log-output option.

The main difference between the Smith and Pokorny (1975) fundamentals and the Baylor et al. (1987) receptor sensitivities is that the former are determined by psychophysical considerations and the latter have been measured physiologically at receptor level. Baylor et al. also use polynomial powers to describe their sensitivity functions. The LWS, MWS, and SWS functions are assumed to have the same shape when scaled along the frequency axis such that their maxima coincide. The polynomial power $P(\mu,a,6)$ gives the shape of the function. To compute the sensitivity for cone type C , with C standing for R, G, or B, for some wavelength λ , the function is scaled by the substitution

$$\mu = \log_{10}\left(\frac{1000}{\lambda} \frac{\lambda_C}{\lambda_R}\right), \quad (5)$$

where λ_C is the wavelength of maximum sensitivity for cone type C with $\lambda_R = 561$, $\lambda_G = 531$, and $\lambda_B = 430$.

Note that all mathematical expressions have been selected by Boynton and Wisowaty (1980) and by Baylor et al. (1987) not for substantial psychological or physiological reasons, but simply because they give an excellent fit to the empirical data. All functions available in PCSF but not explicitly mentioned in the present section are derived from functions given here either by spectral multiplication or by linear transformations. The matrix equation for transforming the Smith and Pokorny (1975) fundamentals to Judd 1951 modified CIE color-matching functions is

$$\begin{bmatrix} \bar{x}'(\lambda) \\ \bar{y}'(\lambda) \\ \bar{z}'(\lambda) \end{bmatrix} = \begin{bmatrix} 2.9448 & -3.5010 & 13.1722 \\ 1.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 62.18905 \end{bmatrix} \begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix}. \quad (6)$$

PROGRAM USAGE

Printing Function Tables

The program's functions may either be requested directly from the command line by specifying UNIX-style command-line options, or they may be selected from a menu that is printed on the screen if no function-generating command-line options have been given. The command

```
pcsf -rgb -s 1
```

will print three consecutive tables containing the Smith and Pokorny (1975) LWS, MWS, and SWS fundamentals. Wavelength step size is 1 nm. Simply typing

```
pcsf
```

will print the menu, which is shown in Figure 2, and then the user is expected to type in option letters. Input is finished by the ENTER key. An input line, which is equivalent to the above command line example, is

```
rgb1.
```

Note that no special character separators are necessary in menu mode. Sequences of option letters on the command line have to be preceded by a minus sign as shown in the example.

Computing Spectrum Coordinates

In addition to printing values of sensitivity functions at arbitrary wavelength steps, the program may also be used to evaluate stimulus spectra by computing color coordinates. The radiance spectrum has to be tabulated in equal wavelength steps in an input file. This has to be an ASCII text file with a single pair of wavelength and radiance data per line. The spectrum data are read, and the products of each radiance value with the requested color-matching-function value for the corresponding wavelength are summed according to Wyszecki and Stiles (1982, Sect. 3.3.8) and printed to the output. To get stimulus coordinates instead of function tables, the user gives one or more names of stimulus spectrum files in the command line when calling PCSF. The type of coordinates computed is determined by the options given in Figure 2. Thus, the command line

```
pcsf -rgb STIMULUS.DAT
```

will compute the LWS, MWS, and SWS cone coordinates for the stimulus spectrum that is tabulated in the file STIMULUS.DAT. Suppose STIMULUS.DAT contains the following spectrum values:

```
435.8  1.0
546.1  1.38
700    72.1 ,
```

then the command line

```
pcsf -C STIMULUS.DAT
```

will return the Judd 1951 modified CIE chromaticities of the stimulus: (0.3440, 0.3317). There will also be a warning message in this case, since the spectral data are not equally spaced and thus color-matching coordinates may be distorted.

Output Control

Figure 2 shows that most of the options are used to request special sensitivity functions. Some options are used to specify output modes. The option "n" results in normalized output values. This only affects sensitivity functions and color-matching coordinates, but not chromaticity coordinates. If sensitivity function tables are requested, then normalization of the functions is with respect to their maximum. If color-matching coordinates for a radiance spectrum are requested, then normalization is such that Y ,

PCSF: Print Cone Sensitivity Functions (Vers. 2.0)
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Function Type	Option Letter(s)
Smith & Pokorny Fundamental Sensitivity Functions:	r, g, b
V(L) as modified by Judd (1951):	v
CIE chromaticities as modified by Judd (1951):	x, y, z
Smooth combined optical media absorption function:	m
Cone receptor sensitivity functions:	P, D, T
Cone sensitivity corrected by optical media absorption:	p, d, t
CIE chromaticity diagram as modified by Judd (1951):	C
MacLeod & Boynton's cone excitation diagram:	c
Logarithmic output values:	l
Normalize output:	n
Use z'-scaling for blue fundamental:	u
Wavelength step size (default is 5 nm):	number
Add string after each curve:	"string"
Give letter(s), string, or number to select option(s):	

Figure 2. This screen menu is shown if PCSF is called from the command line without giving one of the function-generating options. All but the last five options generate functions. The last five options control the output mode. After showing the menu, PCSF will wait until some option characters are typed in and the input is finished by pressing the ENTER key.

V , and $(R + G)$ approximate ordinary luminance in $\text{cd} \cdot \text{m}^{-2}$ if the input spectrum is given in $\text{watt} \cdot \text{sr}^{-1} \cdot \text{m}^{-2} \cdot \text{nm}^{-1}$ and is equally spaced. This is not possible if the spectrum is given in terms of a spectral-density function. Chromaticity coordinates are independent of the scaling of the radiance data. A lowercase letter "l" leads to $\log_{10}$ output values instead of the usual linear values. This option does not affect spectrum coordinates computed from radiance data. The option "u" scales the blue-sensitive fundamental such that it is identical to the $\bar{z}'(\lambda)$ function. In command-line mode, the option "s" followed by a number will set the wavelength step size. In menu mode, the letter "s" need not be given; a number is sufficient. If PCSF prints more than a single table, it may be useful to add a mark at the end of each table. In command-line mode, a marker string may be given after the option letter "a." In menu mode, it is sufficient to enter a string, which is enclosed between double quotes. This string will be printed at the end of the last line of each table. Note that double quotes on the command line have to be preceded by a backward slash (\); otherwise, they will not be passed to PCSF by the operating system.

The output of PCSF is written to the standard output device, and it may easily be redirected into a file by command-line redirection. The command

pcsf > OUT.TAB

will call PCSF in menu mode and write the resulting data output into the file OUT.TAB.

SYSTEM REQUIREMENTS AND AVAILABILITY

The program is available both in source and in executable form. It is written in ANSI C and may be compiled with any ANSI C compiler for any operating system. Executable versions have been created with Microsoft C 6.0 and Turbo C 2.0 for MS-DOS and with the non-ANSI UNIX C compiler cc. No special hardware is required. The single functions are written in a modular manner and may easily be incorporated into other programs. The files are available from the author via electronic mail by sending a request to: hans.irtel@rpss3.psychologie.uni-regensburg.de.

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NOTES

1. Both Boynton and Wisowaty (1980, p. 1472, Equation 8) and Pokorny and Smith (1986, p. 8-34, Equation 62) have a typographical

error in the equation for converting from quantum catch to energy-based sensitivity.

2. There is an ambiguity in Boynton and Wisowaty's (1980) description of their equation for the optical-media function (their Equation 13). They do not give density, but the power of density that corresponds to absorption. Thus, optical density is the base 10 logarithm of their Equation 13.

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